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New considerations on the Liar' paradox

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Wojciech Żelaniec

New Considerations on The 'Liar' Paradox

They Argue no corrupted Mind
In him—the Fault Is with Man-Kind.

Swift, Verses on The Death of Dr. Swift

INTRODUCTION

This is going to be a piece of philosophy concerning the famous or infamous 'Liar', the sentence 'This sentence is false'. I strongly hope that these 'new considerations' argue no corrupted mind in myself, the fault of whatever could therein be found fault with being on the part of the subject-matter, i. e. ordinary language, itself.¹

As we have all known the 'Liar' is paradoxical, in the sense that whichever truth-value you try to assign to it, you end up with the truth value other than the one you started with. Tarski has shown why this should be so—as we know, chiefly for level-of-language-related reasons.

I shall have nothing to say on this aspect of things, though. Nor do I think that what I shall have to say will in any direct way be relevant to the Tarskian aspect of things.

It will, however, certainly be vaguely in line with what I take to be the main thrust and main moral from the justly celebrated Tarskian study, to wit, that 'you can't say anything like the 'Liar''.

¹ I owe the main idea of this piece to a study by John Leslie Mackie, 'Logical Paradoxes', ch. 6 in his *Truth, Probability And Paradox*, Oxford 1973, Clarendon Press. I have refounded his stuff, though, and recast it in quite a different mould.

These morals had, one can't forbear remarking, been well-heeded, long before Tarski told the story from which it was drawn. For we have, as far as we know, hardly any anecdotal, let alone scholarly, evidence that anyone ever said 'This sentence is false'. One starts feeling uneasy when one tries to imagine what would happen to a woman or man who went saying 'This sentence is false'. Without doubt he would be taken to Bedlam before long.

Here, it is perhaps in good order to make the following observation, slightly pedantical, you might protest, but nonetheless, it seems to me, not quite beside-the-mark: To lie is not the same as not to tell the truth.

To get things straight:

A liar (or a liadress, if you will) is a person who says things they don't believe. These things *can* be true, though.

How come? Small wonder—for lying is, more often than not, evidence of a kind of moral ignorance, but moral ignorance frequently goes hand in hand with factual ignorance. A liadress might be telling things she does not believe, because, in her ignorance (which is moral as much as factual in the case in hand), she believes things that just aren't true.

St. Paul was presumably not aiming at an investigation in the style of the celebrated *Pojęcie prawdy w naukach dedukcyjnych* when he said (Authorized Version):

One of themselves [the Cretians, W. Ż.], even a prophet of their own, said, The Cretians are always liars, evil beasts, slow bellies. This witness is true [...].²

Well, St. Paul was certainly not fishing for paradoxes; he just wanted to say that the 'prophet' claimed that all Cretians lied and was damned right, all the while lying himself. Because he (the prophet) need not have believed that all Cretians always lied, and thus was saying what he did not believe true; yet it *was* true. 'This witness is true'—'*Testimonium hoc verum est*' as St. Jerome translates the Apostle's words. It is clear that no paradox is looming large here.

Thus, I conclude, no-one was ever tempted to say, except perhaps tongue in cheek, 'This sentence is false'.

However, it is probably quite useful to remind others and be reminded that certain things must not be done, even though they, as a matter of fact, have never been done

² Tit 1 12–13. In the Greek *New Testament* we read: 'εἶπεν τις ἐξ αὐτῶν ἴδιος αὐτῶν προφήτης, Κρήτες ἀεὶ ψεύδοιαι, κακὰ θηρία, γαστέρες ἀργαί. ἡ μαρτυρία αὕτη ἐστὶν ἀληθής.' Ascribed (by Aland) to Epimenides, *De oraculis*. Quoted after: Nestle-Aland, *Novum Testamentum Graece et Latine*, Kurt Aland, Barbara Aland (eds.), Stuttgart 1979, Deutsche Bibelgesellschaft. The Authorized Version translation seems to be, as far as I can tell, a pretty literal translation of the Greek original, but to shed more light on the issue, here is the Polish translation by Father Wujek: 'Powiedział niektóry z nich, własny ich prorok: Kreteńczykowie zawsze kłamliwi, złe bestie, brzuchowie leniwi. To świadectwo jest prawdziwe.' Cf. Diogenes Laertios, I, 109, II, 108, Diels-Kranz 3B, fragment 1. The real inventor of the 'Liar' appears to be Eubulides the Megarean, not Epimenides.

before; Tarski has made patent one kind of reason why we mustn't ever say 'This sentence is false', those related to the level-of-language issue; I shall put forward another kind.

Mine is related to an observation that you frequently make when teaching the 'Liar' to beginners, or talking about it to a general (as distinct from technically sophisticated philosophical) audience. The observation is that such minds find the 'Liar' objectionable, not because it is paradoxical, but because it appears to them 'empty', and that in a rather queer way. Why so queer a way? Because sentences felt to be 'empty' often turn out to be tautological, as does the famous song of Monsieur de La Palisse, who *un quart d'heure avant sa mort était encore en vie*, was still alive a quarter of an hour before he died. However, the 'Liar' is not like that at all. What is it like, then?

WHAT IS REALLY WRONG WITH THE 'LIAR'?

At this stage, the matter begins to look pretty much like a philosophical Gordian knot. I shall cut through this knot by taking recourse to the principle which says that not sentences (*qua* linguistic entities) are actual truth-bearers, but propositions, 'ideal' entities (whatever that might mean). The propositions that I mean are, for instance, like the *Sätze an sich* in Bernard Bolzano, who set forth a theory on which not sentences, as linguistic entities but the underlying propositions (*Sätze an sich*) are either true or false and that in an absolute sense (i. e. not relative to a language).³ They are, as Mark Textor has put it, 'primary bearers of truth-values'.

The theory implies that, among other things, lots of sentences (as linguistic entities) are 'one and the same thing', as far as the underlying proposition, or *Satz an sich*, is concerned. The most obvious examples for the correctness of that implication include sentences with expletives (so characteristic of Polish popular speech) and without them, such as 'The cat is lying on the mat' and 'The f* cat is lying on the f* mat'; but also, as an English scholar has claimed, the famous phrase of Nelson's:

England expects every man to do his duty

is (i. e., expresses) the same proposition as:

England anticipates that, as regards the current emergency, personnel will face up to the issue, and exercise appropriately the functions allocated to their respective occupational groups.⁴

³ *Wissenschaftslehre*, I, § 19, 24, II, § 125. See Wolfgang Künne, *Propositions in Bolzano and Frege*, 'Grazer Philosophische Studien' 53 (1998), 203–240; Textor, Mark, *Bolzano's Propositionalism*, Berlin–New York 1996, De Gruyter, in particular section 1.1.4: 'Sätze als primäre Wahrheitswertträger'.

⁴ Joseph F. Littel (ed.) 1971. *Dialects and Levels of Language*. Evanston, Ill. McDougal Littel & Co, 1971. Quoted after: Bogusław Lawendowski, James Pankhurst. *British And American English. A Comparison of The Grammar And Vocabulary*. Warszawa 1975, Państwowe Wydawnictwo Naukowe, p. 117.

which sounds a bit American to our ears. And, whatever can be said on the difficulties, judged most of the time insuperable, of translating from English to any other language, you will probably agree that there *are* safe examples of perfect translations, i. e. cases where two completely different sentences express one and the same proposition. For example:

The Earth revolves around the Sun

is 'the same' (proposition, *Satz an sich*) as:

Ziemia obraca się dookoła Słońca,

or, for that matter:

Die Erde dreht sich um die Sonne,

albeit the English sentence has only a rather faint phonetic similarity to the German, and next to none to the Polish one.

Assuming the existence of propositions as distinct from sentences, the matter would be to undig the former from under the layers of linguistic rubbish making up the latter. In the eyes of a Bolzanian, that would be an enterprise doomed to failure, because Bolzanian propositions are ideal, not real (no spatial-temporal), not even potentially real entities.⁵ That means that we can never arrive at *the* formulation of a proposition, any formulation being just one amongst many.

However, I think that, if for a sentence (as linguistic entity) there is, in fact, a proposition (*Satz an sich*) that it expresses, then there must be a number of legitimate ways of reformulating it so that it might express the *very same* proposition. In other words, there is always a way of saying the very same thing *in other words*. This seems to me to be pretty obvious, although I can't see any reason why there should not be exceptions to it. Maybe there are exceptions; I, however, can't think of any.

Bernard Bolzano, incidentally, has things to say on paraphrasing sentences in such a way that they should express the very same proposition. He concerns himself with this topic under the title of '*Auslegung*' or 'interpretation' of sentences, to which paraphrasing them is a means.⁶

My impression is that what we call 'simplifying' the formulation of a sentence is a paraphrase that has the proposition expressed by the sentence in question as an invariant. For instance, we shall agree that 'Peter is taller than John' expresses the same proposition as 'Peter excels John as far as stature is concerned' or some such gobble-dygook. Bolzano's favourite examples seem to include such sentences as 'There are unicorns' and 'The expression 'unicorn' has objective reality', which are not likely to be *our* favourite examples, I am afraid, and that for a number of reasons. Sometimes, to be honest, it might not be clear whether a paraphrase expresses or does not express

⁵ *Wissenschaftslehre*, I, § 19, 68; II, § 122.

⁶ *Wissenschaftslehre*, II, § 169. See also Roger Schmit, *Über Bolzanos Begriff der Auslegung*, 'Grazer Philosophische Studien' 47 (1994), pp. 1–29.

the same proposition. For instance, 'God is' and 'there is a God' or 'there is someone called God' or some such. Those who (like Arthur Prior) say that 'God is' is a syntactically faulty expression, in the first place (God is *what?*) are eager to conclude that the existence of God (Him being quite literally neither here nor there) has thereby been disproven semantically, much as they once thought they could prove it ontologically. But a theist might say that as God is quite an unusual kind of entity, we are doomed to talk about Him in faulty phrases. Judge for yourselves; there is, no doubt, a problem here.

Be that as it may, in a good many cases there is a safe way of paraphrasing a sentence so that the result might express what we cannot but identify as the very same proposition. In the sequel I shall propound two such legitimate, in my view, paraphrasing procedures. Then, I shall try to apply them to the 'Liar'; you will see that the result will be disastrous in a yet-to-be-specified sense (see 'A rule' below). The upshot will be that 'you can't say anything like the 'Liar'' simply because the 'Liar' does not, contrary to appearances (if any...), express any proposition at all. In other words, the moral will be that if you say 'This sentence is false', the sentence referred to as 'this' being not, say, 'Whales are kind of mosquitos' but 'This sentence is false', you are saying quite literally nothing. In the final section, however, a shadow of doubt will be cast on this devastating conclusion, because, besides the 'Liar', there is also the 'Truth-Teller' which some people think is simply true.

FIRST PROCEDURE

One procedure is that of replacing, in a phrase such as 'This sentence is (something or other)' the expression 'this sentence' by a quotation of the very sentence that is being referred to by means of the demonstrative pronoun 'this'. For clarity, it is obviously OK to prefix the expression 'this sentence' to the quoted sentence in question. So, for instance, instead of saying 'This sentence is short', if the sentence referred to as 'this' is the sentence 'God is', it is all right to say 'The sentence 'God is' is short'. This replaceability, if it really (as I propose) obtains *salva veritate* and maybe *salva* a number of other things, is something peculiar to sentences and possibly other *linguistic* items, but not necessarily (but in the context of *viva voce* verbal exchange, where ostension is allowed to play a part) to non-linguistic items. Replacing words by the corresponding non-linguistic items used to be common, if we are to believe Dr. Swift, in the Kingdom of Laputa where, as he reports

[a]n expedient was [...] offered, that since words are only names for *things*, it would be more convenient for all men to carry about them such *things* as were necessary to express the particular business they are to discourse on. [...] [M]any of the most learned and wise adhere to the new scheme of expressing themselves by *things*, which hath only this inconvenience attending it, that if a man's business be very great, and of various kinds, he must be obliged in proportion to carry a greater bundle of *things* upon his back [...] I have often beheld two of those sages almost sinking under the weight of their packs, like pedlars among us; who, when they met in

the streets, would lay down their loads, open their sacks, and hold conversation for an hour together.⁷

Well—this expedient would certainly be most ridiculous if applied to such phrases as ‘this kingdom might soon have a better’ where an entire kingdom would have to be presented instead of the expression ‘this kingdom’, but, as I fancy to think, the expedient works reasonably well for *linguistic* items, where, by the very nature of thing, the name and the object named itself is made of the same kind of stuff.

SECOND PROCEDURE

The second procedure, which I also hold to be self-evidently defensible, is that of replacing something like:

The sentence ‘X’ is false

with

It is not the case that X

where X is not quoted (mentioned) but used. I think the application of this rule makes the input sentence simpler. There is a philosophical theory of truth (and falsity, for the science of opposites is the same, as we have known since Plotinus’ times, at least), but there is no philosophical theory of (not) being the case, not that I know of, at least. The task awaits a new Heidegger, perhaps It is true that in some languages (not in English, as far as I can tell) the disquoting of ‘X’ and using it in ‘It is not the case that X’ requires that certain things should be changed in ‘X’, such as the word-order in it, or the mood of the main verb or some such. But I think that that is just the ‘idiotism of idiom’, as Arthur Prior would say.

A RULE

I should like to propound a rule (or a metarule, as you might wish to call it) by virtue of which, if you apply a set of ‘good’ procedures to a sentence with a view to expounding the proposition that it expresses, you must never launch an infinite regress. If you do, it is a sure sign that something has gone wrong. If all of your procedures have been correct and correctly applied, the conclusion must be, I think, that the sentence you started with simply does not express any proposition at all.

However, what do I mean by ‘launching an infinite regress’, you might ask? Well, suppose you have a sentence *s*. You apply a procedure *p* to it once and you get an

⁷ Jonathan Swift, *Gulliver’s Travels*, New York 1957, Pocket Books, p. 183. Strange though this may seem, the method works sometimes in real life. Recently, while seeing someone off to the railway station, I wanted the person to make sure he had all necessary keys on him. The window in the the train-coach was locked, however, so I just produced a key from my pocket and waved it. It worked.

other sentence, s' , as the output. But to s' the very same procedure p is applicable, and if you apply it to s' , you get yet another sentence s'' , to which p is again applicable. And so on. This is what I mean by 'infinite regress'. The procedure p may be however simple or complex as you wish, except that it must not consist of two subprocedures, the one of which reverted the results of the other, because in a case like that, you would launch an infinite regress on any sentence.

I must admit that I do not know why the rule just propounded should be correct; I just think it is. I do so, in part, no doubt, because I can't think of any sensible sentence for which it wouldn't work. Sometimes, you might think that even a patently 'good' procedure applied on very contentful sentences, such as 'The Earth is a planet' will lead to an infinite regress. For instance, Bolzano himself appears to have thought that very subject-predicate sentence should be reducible to what he thought was canonical form of 'The subject S has the property of being p ' instead of the vulgar ' S is p '. Now, clearly, if 'The Earth is a planet', then (apply the rule just mentioned) 'The Earth has the property of being a planet', and this means that (apply it again) 'The Earth has the property of having the property of being a planet'; to which the said rule is again applicable, yielding: 'The Earth has the property of having the property of having the property of being a planet', and so on. Well, this is just one reason why Bolzano-scholars are divided on whether Bolzano was right on that 'having-the-property-of' business. The two procedures I stated above are, I should argue, far less controversial than the Bolzaniian one, having none of the artificiality of the tortuous 'has the property of being such and such' locution.

I think that my rule is all right also, in part, because it lays bare the emptiness of the 'Liar'; which, of course, makes my whole argument (appear) circular.

TO WORK

Our input is the 'Liar', i. e. the sentence 'This sentence is false'. Apply the first procedure to it, and you'll get:

The sentence 'This sentence is false' is false

Applying the second procedure to what we have just got, we obtain:

It is not the case that this sentence is false

whence, again by procedure one:

It is not the case that the sentence 'This sentence is false' is false

And then, by procedure two, we obtain:

It is not the case that it is not the case that this sentence is false

And then, by procedure one:

It is not the case that it is not the case that the sentence 'This sentence is false' is false
whence, again by procedure two:

It is not the case that it is not the case that it is not the case that this sentence is false.

From where, by analogous steps, you get:

It is not the case that it is not the case that it is not the case that the sentence 'This sentence is false' is false

and:

It is not the case that it is not the case that it is not the case that it is not the case that this sentence is false.

And so on, to infinity.

MORAL (SUGGESTED TO BE) DRAWN

My impression is that the above reasoning shows quite conspicuously that there is no proposition 'under' the 'Liar'. In other words: there is nothing that the 'Liar' really says, and consequently, there is nothing to attach a truth-value to.

But isn't there? The 'expounding' of the proposition believed to be expressed by the 'Liar' is strangely frustrated by the infinite regress in which it gets stuck. But we know that there *are* infinite regresses (or things that look like such) which we know how to handle and get along with. Don't we know, for instance, that

$$1/2 + 1/4 + 1/8 + 1/16 + 1/32 + \dots + 1/2^n + \dots = 1?$$

Yes, we certainly do know that, my answer would be, but how? Thanks to the theory of infinite sums, developed only in the XVIII century. Zeno of Elea, for instance, did not have that theory and would not have agreed that one-half and one-fourth and one-eighth and one-two-to-the-power-of-nth equals anything finite.⁸ He (sort of) thought that all infinite sums could not have finite results. And what we still *do not* understand, is an infinite application of a phrase like 'it is not the case that' within one and the same sentence. An analogy with mathematics might be illuminating for future research, yet it certainly does not, by itself, solve the problem.

⁸ I am twisting and bending here Zeno's argument for my purposes, but not quite out of shape, as you will recognize on a moment's reflection. See Edward Hussey, 'Pythagoreans and Eleatics', ch. 4 [in:] C. C. W. Taylor (ed.), *Routledge History of Philosophy*, vol. 1, *From the Beginning to Plato*, London and New York 1997, Routledge, pp. 128–174; 156f. There is an excellent study on family of topics by Tomasz Placek, (in Polish), 'Zenona Paradoxy Ruchu a Labirynt Kontinuum—Dychotomia', *Studia Filozoficzne* 4 (1989), pp. 57–73. See also his: *Paradoxy ruchu Zenona z Elei a labirynt kontinuum: „Achilles i żółw”, „strzala”, „stadion”*, 'Filozofia Nauki', R. V, 1997 nr 1(17).

A SHADOW OF (MORAL) DOUBT: THE 'TRUTH-TELLER'

What, however, are we to make of the 'Truth-teller' which is the sentence 'This sentence is true'? It is obvious that an analogous reasoning—except that for 'it is not the case that' we should have to substitute 'it is the case that'—can be carried out on it. With the same result: an infinite regress. And it, too, does appear strangely 'empty' to many who approach it for the first time. Didn't Löb, however, prove that it *was* true, after all (in his 'Solution of A Problem of Leon Henkin', *Journal of Symbolic Logic*, 20 (1955), 115-118)? Yes, he did; at least, this is what some people take him to have done, to judge from anecdotal evidence. What to make of it? I dunno. There's certainly a serious problem here; but I shan't have a great deal to say about it. It's difficult to get one's mind round it without being a genuine mathematician, methinks. My (i. e. a person's who is not a mathematician) impression is that the Löb Theorem is, as far as the mathematical nitty-gritty of it is concerned, not about itself, but about various arithmetical relations which but 'represent' (in the Gödelian sense of this word) provability and other things. For it states (stripped of all formalism and expressed in ordinary, if a bit clumsy, English) that:

φ being an arbitrary sentence of Peano Arithmetics,

it is a thesis of Peano Arithmetics that φ provided that there is a natural number carrying the arithmetical relation that represents (Gödel-style) the relation of being a proof of to the number that represents φ

if and only if

φ is itself a thesis of Peano Arithmetics⁹.

It is patent that on a higher language level the Löb theorem might be taken to say something like: A sentence is a thesis (and thereby true) if and only if it is a thesis that it follows from the assertion that it is a thesis. So, on this reading it has some vague similarity to the 'Truth-Teller'. But what it really says, is that certain arithmetical, numerical relations obtain. Being a layman in mathematics, I can't pretend this is anything more than an impression, acquired on the basis of a perusal of both proofs of the theorem, quoted in the book by Murawski mentioned in a footnote above. Should this impression of mine not be quite wrong, though, it would be correct to say that the Löbian 'Truth-Teller' is not all that 'empty' as is the 'Truth-Teller' of ordinary language, and so does not resist all attempt at extricating a Bolzianian proposition out of it. The 'vulgar' 'Truth-Teller', though, would. And, as we have (I insist) seen, so does the 'Liar'.

⁹ Roman Murawski, *Funkcje rekurencyjne i elementy metamatematyki. Problemy zupełności, rozstrzygalności, twierdzenia Gödla*, Poznań 1991, Wydawnictwa Naukowe Uniwersytetu im. Adama Mickiewicza w Poznaniu, p. 102.