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# APPLICATION OF GENERALIZED STUDENT'S T-DISTRIBUTION IN MODELING THE DISTRIBUTION OF EMPIRICAL RETURN RATES ON SELECTED STOCK EXCHANGE INDEXES

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## Abstract

This paper examines the application of the so called generalized Student's t-distribution in modeling the distribution of empirical return rates on selected Warsaw stock exchange indexes. It deals with distribution parameters by means of the method of logarithmic moments, the maximum likelihood method and the method of moments. Generalized Student's t-distribution ensures better fitting to empirical data than the classical Student's t-distribution.

**Keywords:** Student's t-distribution, generalized Student's t-distribution, estimation of distribution parameters.

**JEL classification:** C02, C12, C13, C46, E43.

## Introduction

While determining the risk of investment in stocks, the analysis of a random variable is conducted, namely the analysis of the rate of return on stocks. And by determining a number of parameters describing this variable, the measurement of stock risk is conducted, which is described in detail in the work[1]. The issue of modeling the distribution of empirical return rates on Warsaw Stock Exchange stocks has received a wide coverage in the literature. One of the first to cover this issue was a work[2] which presented the results of studies on time series of return rates on 33 stocks and two indexes for daily, weekly and monthly quotations in the period between 1994 and 2000. In the case of daily data, the hypothesis of fitting empirical data distribution to the normal distribution had to be rejected. In paper[3], weekly rates of return on WIG index in the period of 1991–2000 were analyzed. The authors conducted three goodness-of-fit tests for the distribution ($\chi^2$, Shapiro-Wilk, Kolmogorov), obtaining a negative result in each case.

Therefore, there is a need for modeling empirical return rates by means of different distributions with the so called ‘fat tails’. For modeling empirical distributions of return rates on indexes and stocks the following distributions are most commonly applied: Gaussian, GED, Student’s, stabilized, hyperbolic, generalized hyperbolic, NIG[4].

One of the distributions used in paper[5] for modeling empirical return rates on Warsaw Stock Exchange indexes was standardized Student’s t-distribution with one parameter to be estimated (number of degrees of freedom *n*), where this parameter *n* took real values[6]:

$$ft(x) = \frac{\Gamma\left(\frac{n+1}{2}\right)}{\sqrt{n \cdot \pi} \cdot \Gamma\left(\frac{n}{2}\right)} \cdot \left(1 + \frac{x^2}{n}\right)^{-\frac{n+1}{2}} \qquad (1)$$

where: $\Gamma(z)$ – gamma Euler function.

In the literature, the following unstandardized Student’s t-distribution is considered $ft\ (x, \mu, \sigma, n)$[7]:

$$ft(x, \mu, \sigma, n) = \frac{\Gamma\left(\frac{n+1}{2}\right)}{\sigma\sqrt{n \cdot \pi} \cdot \Gamma\left(\frac{n}{2}\right)} \cdot \left(1 + \frac{1}{n}\left(\frac{x-\mu}{\sigma}\right)\right)^{-\frac{n+1}{2}} \qquad (2)$$

where:

$\mu$ – location parameter,

$\sigma$ – scale parameter,

$n$ – number of degrees of freedom.

## 1. Estimation of parameters of generalized Student's t-distribution

To simplify calculations, it is assumed that the estimation of location parameter $\hat{\mu}$ is expressed by the arithmetic mean $\hat{\mu} = \bar{x}$, or else by the median $\hat{\mu} = med(x)$.

As a result of centering (subtracting the estimation of parameter $\hat{\mu}$ from the series of observations), a distribution dependent on two parameters is obtained: scale parameter $\sigma$ and the number of degrees of freedom *n*.

Assuming in (2) $\mu = 0$ and substituting

$$b = \frac{1}{n\sigma^2} \tag{3}$$

we obtain:

$$fut(x) = \frac{\Gamma\left(\frac{n+1}{2}\right) \cdot \sqrt{b}}{\sqrt{\pi} \cdot \Gamma\left(\frac{n}{2}\right)} \cdot \left(1 + b \cdot x^2\right)^{-\frac{n+1}{2}} \tag{4}$$

In paper[8], the distribution described by the density expressed by formula (4) was examined, which was referred to as generalized Student's t-distribution (GST for short).

By applying maximum likelihood method in work[9] the following equations were obtained:

$$gW(b) = \Psi\left(\frac{n'(b)+1}{2}\right) - \Psi\left(\frac{n'(b)}{2}\right) - \frac{1}{N}\sum_{k=1}^{N} \ln\left(1 + b \cdot x_k{}^2\right) = 0 \tag{5}$$

where:

$$n'(b) = \frac{N}{\sum\limits_{k=1}^{N} \frac{b \cdot x_k{}^2}{1 + b \cdot x_k{}^2}} - 1 \tag{6}$$

$$\Psi(z) = \frac{d}{dz}\left[\ln \Gamma(z)\right] \tag{7}$$

By solving equation (5), the value of estimation $bW$ of parameter $b$ is obtained. And subsequently from equation (6), estimation $nW = n'(bW)$ is obtained.

In the above mentioned work the method of logarithmic moments was adopted to obtain the following system of equations:

$$\frac{1}{4}\left[\Psi'\left(\frac{n}{2}\right)+\Psi'\left(\frac{1}{2}\right)-\Psi\left(\frac{n}{2}\right)^2-\Psi\left(\frac{1}{2}\right)^2\right]=VL \tag{8a}$$

$$\ln(b)=\Psi\left(\frac{1}{2}\right)-\Psi\left(\frac{n}{2}\right)-2\cdot E_{\ 1} \tag{8b}$$

where:

$$\widehat{EL}_1=\frac{1}{N}\cdot\sum_{k=1}^{N}\ln(|x_k|)\ ; \qquad V\hat{L}=\frac{1}{N}\cdot\sum_{k=1}^{N}\left[\ln(|x_k|)-E_{\ 1}\right]^2 \tag{9}$$

$\Psi(z)$ is determined by equation (7); $\Psi'(z)=\Gamma''(z)\cdot\Gamma(z)^{-1}$.

While determining estimators $nL$ and $bL$ with the method of logarithmic moments, equation (9) should be applied and then the system of equations (8a, 8b) should be solved.

Instead of solving equation (8), the following approximate equations are proposed:

$$nLp=\left(3.90145-11.8288\cdot VL^{-1}+15.0446\cdot VL^{-2}-7.8803\cdot VL^{-3}\right)^{-1} \tag{10}$$

$$bLp=\exp(EP-2\cdot EL_1) \tag{11}$$

where:

$$EP=\left(-0.1904-1.0864\cdot nLp^{-1}+1.3242\cdot nLp^{-2}-2.5507\cdot nLp^{-3}\right)^{-1} \tag{12}$$

Application of equation (10) provides an approximate value nLp burdened with relative error $\leq 0.0003$ for parameter $n \in (2.9, 10)$. The same value of the relative error of expression $EP$ (equation (12)) can be observed for the range of variability of parameter $n \in (2.6, 9.7)$.

In the case of Student's t-distribution described by the density expressed by equation (1), MLM leads to the following equation:

$$gW(n)=\Psi\left(\frac{n+1}{2}\right)-\Psi\left(\frac{n}{2}\right)-\frac{1}{N}\sum_{k=1}^{N}\ln\left(1+\frac{{x_k}^2}{n}\right)-\frac{1}{n}+\frac{n+1}{n\cdot N}\sum_{k=1}^{N}\frac{{x_k}^2}{n+{x_k}^2}=0 \tag{13}$$

The solution of equation (13) determines the estimation of the number of degrees of freedom labeled as *nt*. Apart from MLM and the method of logarithmic moments, the method of moments can be applied to determine the estimation of parameters GST.

For the distribution described by equation (4), the second degree moment (variance) and the fourth degree moment are given:

$$E_2 = \frac{1}{b\cdot(n-2)}; \quad E_4 = \frac{3}{b^2\cdot(n-2)(n-4)} \tag{14}$$

Estimator of the moment of degree *m* is obtained from equation:

$$\widehat{E}_m = \frac{1}{N}\cdot\sum_{k=1}^{N}|\,x_k\,|^m \tag{15}$$

On the basis of relation (14), kurtosis K is obtained:

$$K = \frac{E_4}{{E_2}^2} = \frac{3(n-2)}{n-4} \tag{16}$$

From equations (14) and (16), estimation of parameters GST is obtained:

$$\hat{n} = \frac{4K-6}{K-3}; \quad \hat{b} = \frac{K-3}{2\cdot K\cdot E_2} \tag{17}$$

In the case of Student's t-distribution (equation (1)), the second degree moment (variance) is given by:

$$E_2 = \frac{n}{n-2} \tag{18}$$

From equation (18) the estimation of parameter *n* is obtained:

$$\hat{n} = \frac{2\cdot E_2}{E_2 - 1} \tag{19}$$

The quality assessment of the derived estimators will be conducted in two ways.

The first criterion is the value of the mean-squared error. Namely, the mean-squared error of the estimator is the sum of the square of its bias and its variance[10].

The second criterion, resulting from the fact that the paper is concerned with modeling empirical distributions of return rates on indexes and stocks, is the value of chi-squared test statistic. The latter criterion is superior, since the aim of the proposed method is the approximation of empirical distributions of return rates on stock indexes.

## 2. Computer simulations

Computer simulations were conducted using a random number generator with Student's t-distribution.

$$X = \sqrt{n\left[\left(U_1\right)^{\frac{-2}{n}} - 1\right]} \cdot \cos\left(2\pi U_2\right) \tag{20}$$

where:

$U_1 ; U_2 \in (0, 1)$ – independent random variables with uniform distribution,

X – random variable with Student's t-distribution with *n* degrees of freedom.

In the case of random variable Y with distribution GST (equation (4)), the generator is given by:

$$Y = \frac{X}{\sqrt{nb}} \tag{21}$$

where:

random variable X given by equation (23),

Y – random variable with GST distribution with *n* and *b.*

A numerical experiment was conducted for samples of length N = 100, 200, 300, ..., 1000 performing M = 10,000 repetitions. Among the methods of distribution parameter estimation examined so far (method of logarithmic moments, MLM and the method of moments) the smallest mean-squared error is yielded by MLM. In order to compare the results for variable *X* (equation (23)) and variable *Y* (equation (24)), the study was limited to the mean-squared error of the number of degrees of freedom *n.* In the case of the generator described by equation (23), estimator *nt* (equation (13)) yields smaller error, however for variable *Y* (generator given by (24)), smaller mean-squared error is obtained for the estimator derived from equations (5), (6), (7). Therefore the computer simulations did not decide which method: MLM given by (13) or

MLM given by (5), (6), and (7) is more useful in the estimation of the number of degrees of freedom *n*.

## 3. Results of chi-squared test

In view of the situation described in point 2, the calculations were made for a daily return rate derived from:

$$r = \frac{I_{n+1} - I_n}{I_n} \cdot 100\% \tag{22}$$

Percentage rate was applied on account of Student's t-distribution – in the case of the return rate not expressed as a percentage, equation (13) led to estimators burdened with a very large error. The following return rates on stock indexes were considered: WIG, WIG20, MWIG40 and SWIG80. As the test result, a normalized statistic was given which was the ratio of the statistic of $\chi^2$ test to the critical value. In the case of the value of normalized statistic *h* larger than 1, the hypothesis of fitting Student's t-distribution to empirical distribution had to be rejected.

Table 1 contains the results of calculations for a daily return rate on WIG index for the data from the period 31.03.1995–28.03.2013 – the period of 18 years. Number of observations N = 4505; mean value $E = \bar{x} = 0.0557$; median $E1 = med(x) = 0.0566$ .

The table contains the results of two centering methods: subtracting the mean value and subtracting the median. Application of the method of moments – equations (17) or (19) – resulted in the estimators burdened with a large error, hence these results were not presented. The last column includes the name of the method together with the equations used to estimate distribution parameters. In the case of the method of logarithmic moments, approximate equations were applied (9), (10), (11), and (12).

The results of calculations were given in rows numbered 1–2; 5–6; 9–10. Rows 3–4; 7–8; 11–12 include the results of 'minimum-value-of-statistic' method. Namely, the determined values of estimations $\hat{n}$ and $\hat{b}$ were corrected (made slightly larger or smaller) so as to obtain a minimum value of normalized statistic h.

Table 1 clearly shows that the method of logarithmic moments leads to the values of statistic *h* > 1. MLM based on equation (13) leads to the positive result of the chi-squared test: $h_{\bar{x}} = 0.82 < 1$; $h_{med} = 0.805 < 1$. MLM for equations (5), (6), (7) leads to the values of statistic h > 1. Application of 'minimum-value-of-statistic' method leads to the positive test result:

$h_{\bar{x}}$ = 0.84 < 1; $h_{med}$ = 0,817 < 1. There is no significant difference between the value of statistic $h_{\bar{x}}$ and $h_{med}$ .

Table 1. Estimations of parameters and values of statistic *h* for the distribution of return rates on WIG index

| No. | $\hat{n}$ | $\hat{b}$ | *h* | Centering | Method (equation) |
|---|---|---|---|---|---|
| 1 | 3.72929 | 0.232460 | 1.089 | mean $\bar{x}$ | MLM (5), (6), (7) |
| 2 | 3.72931 | 0.232467 | 1.124 | median | |
| 3 | 3.63129 | 0.232475 | 0.840 | mean $\bar{x}$ | MLM (5), (6), (7) + minimum |
| 4 | 3.63151 | 0.232467 | 0.817 | median | |
| 5 | 2.44808 | 0.447797 | 1.487 | mean $\bar{x}$ | MLOG (10), (11), (12) |
| 6 | 2.61965 | 0.402009 | 1.218 | median | |
| 7 | 2.38838 | 0.447797 | 1.215 | mean $\bar{x}$ | MLOG (10), (11), (12) + minimum |
| 8 | 2.56045 | 0.402009 | 1.168 | median | |
| 9 | 3.21342 | – | 0.820 | mean $\bar{x}$ | MLM (13) |
| 10 | 3.21342 | – | 0.805 | median | |
| 11 | 3.21550 | – | 0.810 | mean $\bar{x}$ | MLM (13) + minimum |
| 12 | 3.21600 | – | 0.791 | median | |

Source: authors' own study.

Table 2. Estimations of parameters and values of statistic *h* for the distribution of return rates on WIG20 index

| No. | $\hat{n}$ | $\hat{b}$ | *h* | Centering | Method (equaution) |
|---|---|---|---|---|---|
| 1 | 4.24148 | 0.175282 | 0.786 | mean $\bar{x}$ | MLM (5), (6), (7) |
| 2 | 4.24147 | 0.175297 | 0.890 | median | |
| 3 | 4.14328 | 0.175282 | 0.768 | mean $\bar{x}$ | MLM (5), (6), (7) + minimum |
| 4 | 4.15247 | 0.175297 | 0.815 | median | |
| 5 | 3,.58165 | 0.229189 | 0.657 | mean $\bar{x}$ | MLOG (10), (11), (12) |
| 6 | 3.50768 | 0.235324 | 0.882 | median | |
| 7 | 3.57765 | 0.229189 | 0.598 | mean $\bar{x}$ | MLOG (10), (11), (12) + minimum |
| 8 | 3.46768 | 0.235324 | 0.826 | median | |
| 9 | 3.05049 | – | 0.888 | mean $\bar{x}$ | MLM (13) |
| 10 | 3.0503 | – | 0.868 | median | |
| 11 | 3.0500 | – | 0.888 | mean $\bar{x}$ | MLM (13) + minimum |
| 12 | 3.05500 | – | 0.854 | median | |

Source: authors' own study.

Table 2 contains the results of calculations for a daily return rate on WIG20 index for the data from the period 28.03.2002–28.03.2013 – the period of 11 years. Number of observations N = 2761; mean value $E = \bar{x} = 0.0333$; median $E1 = med(x) = 0.0477$.

All the methods lead to the positive result of the chi-squared test. In the case of the generalized Student's t-distribution (rows 1–8), mean-centering yields smaller values of statistic h than by subtracting the median. The smallest value of the statistic is obtained for the method of logarithmic moments $h_{\bar{x}} = 0.657$, also after correction $h_{\bar{x}\,\min} = 0.598$.

Table 3 contains the results of calculations for a daily return rate on MWIG40 index for the data from the period 30.03.2007–28.03.2013 – the period of 6 years. Number of observations N = 1502, mean value $E = \bar{x} = -0.0302$; median $E1 = med(x) = 0.00981$.

It should be noticed that both the mean value and the median obtain different signs ($\bar{x} < 0$; $E1 > 0$).

Table 3. Estimations of parameters and values of statistic *h* for the distribution of return rates on MWIG40 index

| No. | $\hat{n}$ | $\hat{b}$ | $h$ | Centering | Method (equation) |
|---|---|---|---|---|---|
| 1 | 2.90683 | 0.538085 | 1.219 | mean $\bar{x}$ | MLM (5), (6), (7) |
| 2 | 2.91645 | 0.548081 | 0.900 | median | |
| 3 | 2.94283 | 0.538085 | 1.157 | mean $\bar{x}$ | MLM (5), (6), (7) + minimum |
| 4 | 2.80745 | 0.548081 | 0.787 | median | |
| 5 | 7.67552 | 0.153198 | 2.048 | mean $\bar{x}$ | MLOG (10), (11), (12) |
| 6 | 3.91648 | 0.352633 | 1.396 | median | |
| 7 | 7.73552 | 0.153198 | 2.015 | mean $\bar{x}$ | MLOG (10), (11), (12) + minimum |
| 8 | 3.85648 | 0.353133 | 1.324 | median | |
| 9 | 4.48143 | – | 1.739 | mean $\bar{x}$ | MLM (13) |
| 10 | 4.46815 | – | 1.941 | median | |
| 11 | 4.48000 | – | 1.737 | mean $\bar{x}$ | MLM (13) + minimum |
| 12 | 4.47000 | – | 1.935 | median | |

Source: authors' own study.

Table 3 shows that the positive test result is obtained for parameter estimations determined using MLM for GST distribution for median-centering: $h_{med} = 0.900$ and $h_{med} \min = 0.787$.

Table 4 contains the results of calculations for a daily return rate on SWIG80 index for the data from the period 31.03.2008–28.03.2013 – the period of 5 years. Number of observations N = 1255, mean value $E = \bar{x} = -0.0148$; median $E1 = med(x) = 0.0544$.

Similarly to MWIG40 index, the mean value and the median obtain different signs.

Table 4. Estimations of parameters and values of statistic *h* for the distribution of return rates on SWIG80 index

| No. | $\hat{n}$ | $\hat{b}$ | $h$ | Centering | Method (equation) |
|---|---|---|---|---|---|
| 1 | 2.92041 | 0.825462 | 0.963 | mean $\bar{x}$ | MLM (5), (6), (7) |
| 2 | 2.94983 | 0.884760 | 0.968 | median | |
| 3 | 2.92541 | 0.825462 | 0.911 | mean $\bar{x}$ | MLM (5), (6), (7) + minimum |
| 4 | 2.82833 | 0.884760 | 0.925 | median | |
| 5 | 2.72436 | 0.928498 | 0.906 | mean $\bar{x}$ | MLOG (10), (11), (12) |
| 6 | 3.51542 | 0.647157 | 0.954 | median | |
| 7 | 2.72736 | 0.928498 | 0.893 | mean $\bar{x}$ | MLOG (10), (11), (12) + minimum |
| 8 | 3.41602 | 0.647657 | 0.878 | median | |
| 9 | 7.59028 | – | 3.688 | mean $\bar{x}$ | MLM (13) |
| 10 | 7.43401 | – | 3.674 | median | |
| 11 | 7.57000 | – | 3.688 | mean $\bar{x}$ | MLM (13) + minimum |
| 12 | 7.42000 | – | 3.670 | median | |

Source: authors' own study.

Table 4 shows that the positive test result is obtained for parameter estimations determined using MLM for GST distribution, however, median-centering yields results similar to those when subtracting the arithmetic mean.

In the cases under study the maximum number of years was taken into account for which at least one method led to the positive result of $\chi^2$ test.

Cumulative distribution function of distribution GST is given by:

$$Ftu(x) = Ft\left(x \cdot \sqrt{b \cdot n}\right) \tag{23}$$

where: $Ft(z)$ – cumulative distribution function of student's t-distribution.

If $t_\alpha$ is the quantile of Student's t-distribution, then quantile $tu_\alpha$ of GST distribution equals:

$$tu_\alpha = \frac{t_\alpha}{\sqrt{b \cdot n}} \tag{24}$$

Relations (23) and (24) enable straightforward application of freely available Student's t-distribution software.

## Conclusions

The research aim was to compare the applicability of Student's t-distribution (equation (1)) and generalized Student's t-distribution (equation (4)) in modeling empirical return rates on selected exchange indexes. Furthermore, an attempt was made to ascertain which estimator of location parameter: arithmetic mean $\hat{\mu} = \bar{x}$ , or median $\hat{\mu} = med(x)$ proves more useful in the process of centering (subtraction of parameter $\hat{\mu}$ estimation from a series of observations).

To summarize the results of modeling the distribution of return rates described in point 3, it should be noticed that GST distribution provided a positive result of the chi-squared test in all the considered cases. By contrast, Student's t-distribution proved useful in only two cases (Tables 1 and 2). It means that GST distribution ensures better fitting to empirical data than the classical Student's t-distribution.

While comparing two methods of centering, no clear advantage of either method can be observed: in one case (Table 3) the subtraction of median yields better results, in another case (Table 2) the subtraction of arithmetic mean yields smaller values of normalized statistic, and in two other cases (Tables 1 and 4) the results are similar. Therefore, the practical conclusion is that centering should be done using both methods, and as a final result the variant yielding a smaller value of normalized statistic should be chosen.

## Notes

1 Tarczyński, Mojsiewicz (2001), pp. 61–84.

2 Jajuga (2000).

3 Tarczyński, Mojsiewicz (2001), pp. 55–58.

4 Weron, Weron (1998), pp. 285–291; Mantegna, Stanley (2001); Purczyński, Guzowska (2002), pp. 105–118; Tomasik E. (2011).

5 Ibidem

6 Shaw (2006), pp. 37–73.

7 Sutradhar (1986), pp. 329–337; Jackman (2009).

8 Purczyński (2003), pp. 147.

9 Ibidem, pp. 149–150.

10 Krzyśko (1997)

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