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Photography in elementary education : constructing the meaning of the concept of symmetry in the process of interiorization and exteriorization

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Summary

The article presents an example of the application of the concept of photoeducation in teaching symmetry at the early school education level. The process of constructing knowledge is presented on the basis of two mutually complementary processes: interiorization and exteriorization.

Keywords: elementary mathematics education, cognitive photography, mathematical photoeducation, interiorization and exteriorization

Introduction

The goal of this article is to present an outline of the concept of the propaedeutic teaching of mathematics with the support of photography. The example of symmetry was selected purposefully because of its broad references extending beyond mathematics (such as nature, art, music, technology, and architecture), as well as in view of the fact that the perception and understanding of symmetry are particularly important for the development of the spatial imagination and the mathematical culture of the student. This is emphasized in the general education curriculum currently in force in Poland.

The photoeducation method[1] outlined in this article, which relies on the constructivist paradigm, is based on children's natural curiosity about the world and their bold perception and formulation of problems. Its primary objective is to support the development of mathematical activity and to awaken the scientific creativity of the student within the field of mathematics. The article presents selected examples of photographic references to the concept of symmetry inferred on several different levels – from a mirror reflection to a time and space metaphor. The photographs selected were submitted to the International Photographic Competition MATHEMATICS IN FOCUS[2] by children, youth, and adult participants.

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[1] The article incorporates fragments of the following thesis: M. Makiewicz (2013), *About photography in mathematical education. How to develop mathematical culture of students (O fotografii w edukacji matematycznej. Jak kształtować kulturę matematyczną uczniów)*. Szczecin, SKNMDM US.

[2] The project under the title of MATHEMATICS IN FOCUS was launched in year 2010 by the University of Szczecin with the primary objective of promoting mathematics and mathematical culture through photography, as well as promoting a cognitive lifestyle.

The educational nature of photography

The history of photography, beginning with the invention of Louis Daguerre presented for the first time at the Palais de l'Institut on August 19th 1839, may be viewed as an example of the dynamic development of technology. It is also possible, and maybe even more interesting, to assume a social approach revealing two breakthroughs: recognition of photography as one of the high arts and its incorporation into the tools used in the exploration of the world. Consideration of the place and the significance of photography in art by experts begins usually with extreme opinions according to which photography is a craft that accurately captures images of objects selected by the photographer. *Communicating by means of metaphors which balance emotions experienced at a given moment, i.e. equivalents, raised photography, among others thanks to Alfred Stieglitz, to the rank of so-called high art* (Makiewicz 2010:17). *The concept of photography as an equivalent understood as an attempt to create a system of symbols equivalent to nature relies on communication by means of symbols and media of aesthetic experiences of viewers including their experiences, emotions, and even subconsciousness* (Stiegler 2009: 72).

The application of photography in school education makes possible the reorientation of the teaching process *by delineating the paths of exploration from perception of works of art* (Wojnar 1971: 307) to independent artistic expression, as a result of which concepts, relationships, and even inventions are explored mathematically. The creative process may be accomplished only under favourable didactic circumstances. Remaining in agreement with E. P. Torrance, all children possess creative abilities, but the development of those abilities may be impeded by inappropriate school education. The Torrance Incubation Model, the purpose of which is to inculcate students with creative inclinations and skills (Szmidt 2005: 176), refers to subjective discoveries and local creativity accomplished at school. The discussion will be limited to two basic levels: fluid level (including elementary cognitive, emotional, and motivational processes) and crystallized level (achieved by pursuing a goal, solving a problem while understanding its structure, significance, and context) (Nęcka 2001). It should be noted, (Limont 2012: 176) however, that creative activity on elementary levels is considered by C. Rogers, A.J. Cropley and J.C. Kaufman to be a prerequisite for higher levels of creativity (Makiewicz 2012b).

Photography is popular and omnipresent. Its usefulness delights those who wish to capture memories, to record significant events, and to document phenomena. *The popularity of photographic cameras is reaching its apex of saturation. They are available in our phones, advertising gadgets, and street photo booths. After the initial fascination with technological novelties it is worth getting a deeper understanding and inquiring about the purpose, the applications, and the limits of usefulness of this popular medium. Questions about the social functions of contemporary photography including integration, refinement, archivization, and aesthetics are asked, among others, by Pierre Bourdieu, Luc Boltansky, Robert Castel, Jean-Christoph Chamboredon, and Dominique Schnapper* (Makiewicz 2010: 18). Piotr Sztompka emphasizes the exceptional popularity of photography among young people: *it is difficult today to find anyone who would not have any personal experiences of photography. Hence the significant popularity of photographic images* (Sztompka 2012: 36). In view of the above, photography could be used in education of young children.

Photoeducation. An outline of the concept

Photographing (both taking photos and interpreting them) reinforces and multiplies children's contacts with their social surroundings. At the same time, it is also the means of mathematization and interpretation. Reaching hidden, invisible meanings and structures by following their visible footprints, it enables a better understanding of the causative and the functional mechanisms underlying social life. It also helps *determine the area of the lack of knowledge and ask accurate questions about things that remain hidden and invisible* (Sztompka 2012: 37).

The concept of mathematical photoeducation relies on the cognitive concept of the human being (Kozielecki 1976), in which students develop their own cognitive structures which transcend information received from external sources and pursue higher levels of generalization by gradually distancing themselves from the specifics of surrounding objects. The essence of photoeducation does not consist in embellishing textbooks, activity books, or notebooks with colourful images, even though this aspect is still important, especially for the so-called aestheticians. The introduction of photography into the teaching of mathematics has a higher purpose.

Mathematical photoeducation is a set of carefully considered and planned activities of the teacher and students, focusing on mathematical education supported by photography. Photoeducation focuses primarily on cognitive, aesthetic, and creative values, and its intended results are achieved through various activities undertaken by students and teachers: from reading (Jeffrey 2009) (experiencing rationally) photographs taken by others (observing photographic images, understanding their titles, descriptions and author's interpretation) by interpreting and commenting to creative statement of the problem (for example under the impact of a cognitive conflict between what the observer sees and what the observer becomes aware of), or from activity consisting in the statement of a problem on the basis of current knowledge, taking one's own photographs in order to provide visualization of the problem, classification, assigning titles, preparing comments, and posing new problems.

The abstract nature of mathematics constitutes a challenge for ambitious cognitive objectives. Our senses let us experience the properties of real, concrete objects rather than the idea of a function, an equation, or a geometric transformation. The triangle is invisible, even though we are familiar with and can indicate triangular objects. The cube has no colour, yet when we reach for dice, we experience a concrete representation, the attribute of which is, for example, the mass. We do not touch angles, straight lines, and line segments, but we get closer to their idea with the help of *shadows projected on a wall of the cave in the light of a fire...* (Plato 1958).

Photography offers a perfect medium that facilitates the transfer between the world of ideas and the world of concrete objects. This transfer is a two-way process, progressing from what is visible to what is conceived (interiorization) and from what is conceived to what is seen (exteriorization). Photography captures perfectly visual representations, or *imagens*, created in our imagination. These representations may be influential in the generation of new representations of a verbal character. *Logogens* in this case are primarily related to naming, classification, exploration of the meaning, evaluation, and posing questions.

Photography may be understood in a physical sense (a print or a file) or in a broader way as an idea, art, action or technique. The three modes of representation (enactive, iconic, and symbolic) proposed years ago by J. Bruner correspond to the three modes of learning (enactive, iconic, and symbolic respectively) (Van Dijk 2009: 295). The concept of teaching mathematics on the basis of photography takes into account the compromise between direct cognition and indirect perception consisting in a fluid and harmonious transition from the action-based level to levels of immersion in the visual aspects of reality and symbolic expression of states and relations. Photoeducation incorporates all the actions undertaken by students and teachers focusing on: reading, interpreting, and commenting on photographs, which generate new questions, hypotheses, tasks, problems, or activity, which originate in a statement of the problem that relies on current knowledge and a pursuit of photographic visualization enriched by ordering, classification, and naming.

Interiorization in photoeducation progresses from concrete to imaginary activities and includes four levels of student's behaviour and four corresponding interventions of the teacher: *The reading of photographs* is preceded by elementary activity of the teacher (N-0), which consists in organizing a didactic situation (awakening student's interest and initiating photoeducation). At first, reading involves the process of scanning the dominant properties of the image. *The eye itself makes conclusions about the world* (Bronowski 1984:134*)*, which constitutes reading at the concrete level and does not include any abstract names, metaphors and does not reveal any relationships and correlations. Reconstruction of the formal composition of a photographic image, as stated by R. Bohnsack, *should generate the primary framework of interpretation* (Bohnsack 2004). The role of the teacher at the level of reading of a photograph (N-1) is to organize a field of perception and to direct interpretation towards knowledge. Students enter the level at which they start to perceive and name mathematical objects presented in a photographic image and progress from realistic interpretation towards formal interpretation.

The ability to make the transition from a literal description of a photograph to a story about its meaning by means of anticipation of causes and effects paves the way for, in Max Kozioff's opinion, *breathing in the space created by photography* (Kozioff 2009: 7). *Interpretation* of photography, in the context of its didactic application, should be understood in a broad sense as an analysis of content by means of a formal representation of mathematical objects, while *commentary should be thought of* as capturing relationships and correlations between concepts and as an attempt to experience the unique language of photography. Only then do we have a chance of understanding photography to the best of our ability, which is pointed out by J. Berger: *we learn to read photographs just as we learn to read cardiograms or tracks. The language used by photography is the language of events* (Berger 2011: 206). On one hand all references to photography are essentially external, and on the other presented objects imply things that are not presented and stimulate our imagination, directing our thoughts and feelings. The process of reading photography resembles the concept formulated by Erwin Panowsky, as it incorporates *immanent* and *domentary* threads and makes a distinction between the primary and the secondary level (Zdanowicz – Kucharczyk 2010). Every image may be read with *a recognizing glance*, but also with *a seeing glance*. It is the role of the teacher to teach rational perception.

Another intervention of the teacher (N-2) consists in posing leading questions that follow into the footsteps of the Socratic Heuristic. *Good questions are questions that* *formulate problems, undermine generally accepted or canonical "truths", and direct the focus of our attention towards discrepancies* (Bruner 2006: 177). This leads to commentary, which includes assigning objects with names, matching those names with captions of descriptions of photographic images, and initiating a discussion. The intervention of the teacher at the highest level (N-3) consists in stimulating the student to identify, formulate, and comment about problems. At this level students may experience difficulties with undertaking creative activity, which originates in *semiotic situations* that lead to a comprehensive consideration of *the representation system* (Lewandowska-Tomaszczyk 2009: 22).

At this point the teacher's interventions, referred to by Edyta Gruszczyk-Kolczyńska as *wrapping up in good emotions* (Gruszczyk-Kolczyńska 2012: 128-130) are especially valuable. Nevertheless it is important for the teacher, both at this level and at all other levels, to let students lead and let them make their own attempts to overcome cognitive difficulties, as well as to propose *active negotiations of meanings* (Klus-Stańska 2010: 313) in the place of mere acquisition of concepts. Application of photography in the process of interiorization is presented in Figure 1 below.

Figure 1. Application of photography in the process of interiorization (Makiewicz 2013: 72)

The sources of difficulties in learning mathematics include: the immaturity of the student, which impedes generalization, abstract thinking, and identification of analogies, lowered emotional buoyancy of the student, and inadequate didactic skills of the teacher. *An objective difficulty is the abstract specificity of mathematical concepts* (Siwek 2005:192). Mathematical photoeducation has a positive impact on the facilitation of overcoming difficulties in transitioning from concrete to abstract mathematical concepts. *Abstraction [...] does not constitute a culmination of an uninterrupted chain of preceding concrete events. The real reason behind failure in the field of formal education is the fact that formal education begins with language (accompanied by a drawing, a fictional action, or a narrative about it, etc.) instead of with an actual, concrete activity. Teaching mathematics should be introduced in preschool, through a series of manipulations pertaining to sets, numbers, and the concept of length and area, etc.* (Piaget 1977: 87). Activity, on the other hand, is an individual characteristics of every person and its form depends on several factors such as on the level of personal development. *A child progresses gradually from physical activity, i.e. activities performed on physical objects, through imaginary activity to logical and mathematical activity, which is manifested in the ability to carry out mental operations, i.e. reversible mental activities* (Piaget 1966a: 565).

A photograph received, read, and interpreted by the student initiates a significant transformation in the way in which problems are perceived and stated. The process of interiorization demonstrates the precedence of *concrete actions (manipulations) on material objects or their figurative and symbolic representations over actions undertaken on the imaginary plane, and then on the verbal and conceptual plane. A child, however, possesses at the same time the ability to externalize (exteriorization) iconic signs or verbal symbols* *in practical activities or during play* (Przetacznik-Gierowska 1993). Therefore, equally important as interiorization for the concept of photoeducation is the progression from what is conceived of to what is seen. Its beginnings are related to the current mathematical knowledge of students. The primary intervention of the teacher (N'-1) is to organize a discussion about newly introduced mathematical objects and their properties. Its objective is to state the problem and direct students towards the formulation of problems concerned with the visualization of previously discussed concepts and theses. At stage (N'-2) the teacher organizes a school outing, a photographic competition, assigns homework that will lead to independent exploration by children (in nature and in creations of human hand and mind) of concepts that have been previously discussed in class. The teacher's Intervention (N'-3) consists mainly in *discrete supervision of understanding, thinking, and the way children* learn (Filipiak 2012: 150) with the purpose of supporting students' ingenuity, encouraging divergent solutions of problems, an original approach to a given topic, perfecting one's own ideas, as well as taking photographs in an efficient way. The application of photography in the process of exteriorization is presented in Figure 2 below.

Figure 2. Application of photography in the process of exteriorization (Makiewicz 2013: 75)

Photoeducation incorporates two essential processes: mathematization consisting in a transition from a concrete to an idea by way of the interiorization and visualization of abstract problems with the aid of a photographic metaphor are considered equal in photoeducation, they may occur simultaneously, be intertwined, or constitute a cycle.

The activity of the teacher and the students in photoeducation is focused on creative endeavours, the reach of which become extended over the territory traditionally reserved for art. Recognizing the cognitive network of students, the teacher has no expectation of ready-made recipes, scenarios, or templates that need to be followed, but rather tries to independently construct tasks on the basis of personal interests, the interests of his or her students, immediate surroundings, and current events. By doing so, the teacher is trying to remove barriers and familiarize students with mathematical thinking, raise the awareness of its usefulness, promote understanding of mathematics as a tool for learning about and understanding the world, and shape geometric perception and imagination, as well as an uninhibited use of the language of mathematics on the basis of cognitive aesthetics, the sense of harmony, and sensitivity to beauty.

Photoeducation allows for the inclusion of all forms of work: individual work (such as commenting on photographs individually, solving problems based on a photograph, taking photographs), group work (such as long-term projects focused on searching for mathematics in art or an analysis of shadows cast by objects on a flat surface), and collective work (such as school trips aimed at exploration of mathematical patterns in the surroundings). Photoeducation encourages a pragmatic approach to classical teaching methods while maintaining a balance between observation-based, action-based, and verbal methods. Group organization creates also an opportunity to teach cooperation, responsibility towards group members, conscientiousness, anticipation, and work planning. Practical actions and operations are genetically primal as opposed to mental processes and are a source of the creation of all mental images and intellectual processes. The transition from the former to the latter occurs by means of interiorization of practical actions and *is manifested through gradual changes in the form of actions and objects on which they are performed* (Cackowska 1985: 439).

Examples of activity of students and teachers in the development of the concept of symmetry through the process of interiorization

The source of the process of mathematization is a prizewinning photograph in the second edition of Mathematics in Focus competition (Photo 1). In this case interiorization progresses from the source, i.e. the photograph, to the formulation of abstract mathematical problems. Photography assumes the role of a constant stimulus that generates an array of *incessantly transforming readings* (Eco 1972). The length and the originality of the list of mathematical problems unravelled by students as a result of an analysis of an image is dependent on the fluency and agility of their thought process, geometric imagination, and ability to create and understand metaphors.

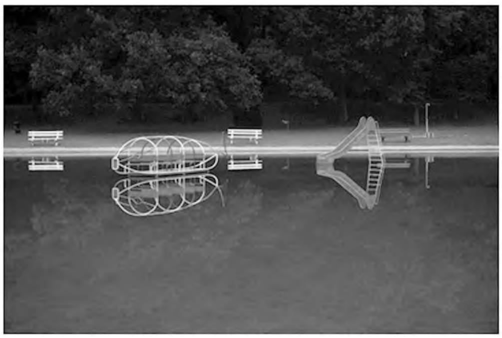

Photo 1. Symmetry

Table 1. The activities of the teacher and students during the process of interiorization in mathematical photoeducation based on the example of symmetry of reflection off the lake

Activities of the teacher	Activities of the student
(N-1) Presents a photograph. Organizes reading of a photograph. *What do you see in the photograph?*	Becomes interested, scanning the dominant properties of the photograph, names objects with their proper names. *Benches and water play structures for children. I see an edge of a lake.*
(N-2) Organizes the field of perception and directs the interpretation towards knowledge.	Names individual elements with the help of names of abstract ideas.

Does what you see in the photograph remind you of any geometric figures?	*Yes, I can see rectangles, triangles, and a sector, as well as another figure that I cannot name – something like a fish.*
(N-3) Asks leading questions. *Do you see anything interesting in this photograph? Any regularities?* *What can you say about the dimensions of the rectangles – benches, slide platform?* *What do we call that figure and its reflection?* *All right, and in the language of mathematics we say that those two figures are…?* *Can you perhaps ask any mathematical questions related to this photograph? Can you think of any?*	 Answers questions, states own problems. *The rectangles at the bottom are the reflection of the ones on top. The section, the edge of the lake, seems to divide the image into two parts, which look very similar. As a matter of fact, they look identical.* *The actual and the reflected objects are of equal length. Their size remains unchanged after reflection.* *A mirror?* *Symmetrical.* *How many benches are there?* *How many steps are there in the ladder and how many are there in the reflection?*

Source: Own case study on the basis of an interview with a second grade student of a school of primary education.

Examples of activity of students and teachers in the development of the concept of symmetry through the process of exteriorization

A double coded metaphor (with an image and a word) is perfectly suitable for the construction of a bridge *between the language, the thought, and the reality* (Ogonowska 2006: 40). With regard to mathematical ideas (such as orthogonality, ordered pairs, and infinity), the educational power of a metaphor helping with the transition from ignorance to knowledge is emphasized among others by George Lakoff (2008). Independent creation and communication of metaphors, both figurative and semantic, requires the student to undertake independent cognitive activity. The creation of and references to the *identifying foundation of meanings* and the replacement of the reproduction of a definition or a proof of a thesis with spiral inoculation with new concepts establishes an ambience conducive to the *activation of personal mathematical knowledge based on freedom of expression, independent exploration, and intuition* (Klus-Stańska 2005). This is accomplished by the juxtaposition of objects that are subjectively new and unknown with familiar observations and experiences.

The process of exteriorization is illustrated by a series of photographs (Photos 2-13) taken by students under the influence of their mathematical knowledge about symmetry acquired at school.

Table 2. The activities of the teacher and students during the process of exteriorization in mathematical photoeducation based on the example of symmetry

Activities of the teacher	Activities of the student
(N'-1) Creates a problem situation and directs students towards the formulation of problems concerning visualizations of previously discussed notions. *What have we discussed recently in class?* *What is symmetrical around us?* *Do you remember the name of the transformation we discussed when we made an impression of an ink blot on a piece of paper?* *Do you remember the name of the transformation we discussed when we cut holes in folded paper?* *Do you remember examples of centrally symmetric figures?* *Can you give me an example of a capital letter that has an axis of symmetry?* *And a capital letter that has a centre of symmetry?* *What types of symmetry are you familiar with? – you must be thinking of planar symmetry.*	 Becomes interested, retrieves names of familiar mathematical objects and their properties from memory. *Symmetry.* *The blackboards, windows, lamps, a ring, a ball, the monument in front of the school, a kite, a butterfly.* *Axial symmetry.* *Reflection symmetry.* *A kite, a ball, a marble, an umbrella.* **M, W, U, H, K** **I, X, Z** *I know axial symmetry, central symmetry, as well as symmetry reflected off the lake.*
(N'-2) Organizes students' activity. *Today is our last class of mathematics before the holidays. I would like you to take pictures illustrating symmetry during your trips and travels. The most interesting photographs will be presented at an exhibition at our school during the next school year. Please remember to add titles and descriptions to your photographs.* *Please take a moment to think about what you can photograph and how you will take a photo.*	Searches for photographic associations, presents ideas, and takes photographs.
(N'-3) Supports students' ingenuity, encourages students to solve a given problem students to solve a given problem (visualization) in	Perfects own idea and successfully takes a photograph. Identifies the problem.

many different ways and to approach the task in an original way. *Let me guess: your photograph presents central symmetry – a sphere is indeed the perfect symmetry and your photograph shows symmetry in reflection in water.*	A student takes a photograph, gives it a title and description.

Source: Own case study.

After their summer school break, students had a meeting with their teacher. Many of them brought along photographs showing examples of symmetry. All of them were provided with titles and descriptions. The most interesting ones were selected for the school newsletter[3].

Bartłomiej noticed that the axis of symmetry may be traced along the body of a true fly (Photo 2). Alicja captured symmetry during her trip to Western Norway. Her photograph presents historic houses reflected in a pond (Photo 3). Katarzyna noticed oval rings created by reflection of arches of a bridge in water (Photo 4). Katarzyna captured the harmony of the Brooklyn Bridge, emphasizing at the same time the choice of a symmetric route by a runner appearing in the centre of the image. According to the author such a trajectory evokes the feeling of inner peace and harmony in runners (Photo 5). Marek noticed the almost perfectly symmetrical vault of a church (Photo 6). Klaudia noticed the wooden intarsias on the ceiling of a chapel in Kazimierz Dolny (Photo 7). Jerzy noticed the numerous axis of symmetry, the centre of symmetry, and the rotations in the rose of the Basilica of Saint Clare in Assisi (Photo 8). Dominika noticed an example of central symmetry in an open umbrella (Photo 9). Anna took a photograph of a circular flower bed (Photo10). Jadwiga was enraptured by the architecture of a castle of the Teutonic Order in which she found central symmetry (Photo11). Daniel presented a photograph of a ball – a model of the perfect symmetry of the sphere (Photo 12). Kasia presented a metaphor of symmetry departing from its traditional geometric representation (Photo 13).

Photo 2. Axis of symmetry along the body of an insect

Photo 3. Clean symmetry in Laerdal

Photo 4. Rings on water

[3] The article presents photographs selected from the collection of competition entries in order to illustrate the process. The authors of photographs did not attend the same school.

Photo 5. A bridge

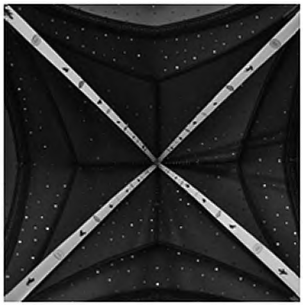

Photo 6. Symmetrical vault of Saint John the Baptist Church in Szczecin

Photo 7. A flower inscribed into a square

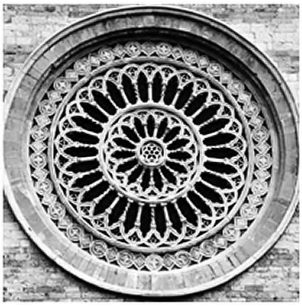

Photo 8. An ornament with numerous axes of symmetry

Photo 9. Regular octagons

Photo 10. The harmony of nature's rings

Photo11. A regular octagon

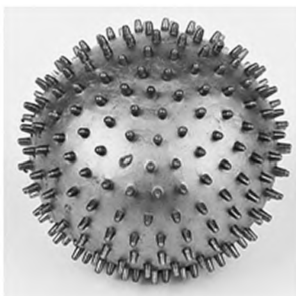

Photo 12. Perpendicular

Photo 13. Symmetry in time

It is worth noting a very significant didactic moment, i.e. the so-called critical event in teaching mathematics. The process of exteriorization led to the creation of a compilation of captivating photographs which, along with comments, provide a valuable resource for the beginning of a new cycle, the aim of which is to pursue knowledge externalized at a higher level. The consolidation of the process of exteriorization and (new) interiorization in the case of symmetry discussed here consists in a discussion carried out by the teacher about the classification (*What groups can we assign your photographs to?*) and the perception of differences between images and properties of the discussed transformations (*How do the photographs differ from the mathematical concepts discussed in class? Which of the photographs best represents the concept of symmetry?*). This is how students, by means of negotiation, redefine the concept of symmetry, evaluate discrepancies between an abstract object and its visualization expressed in a photograph and a caption. At this stage the concept of symmetry becomes interiorized, which is expressed by students in the following way: *"I can finally feel it. Now it starts to make sense. I finally get it."*

Comments

The outlined examples of the activity of students and teachers in the development of the concept of symmetry through interiorization and exteriorization illustrate selected paths of external influence directed at the cognitive development of the student in the area of mathematics in early school education. Both mathematization consisting in a transition (through interiorization) from the concrete to the idea, and the visualization of abstract problems by means of a photographic and semantic metaphor play significant roles in the broader didactic proposal referred to as *photoeducation* (Makiewicz 2013). The essence of this didactic concept is how these two processes mutually complete each other and intertwine together. The final stage of visualization (the photograph) becomes a source of a new cycle of interiorization. The student's cognitive development, supported by the two pillars: *assimilation and accommodation, gradually leads to the equilibrium of the state of adaptation* (Piaget 1966b: 17). On one hand photoeducation *enables interpretation of the surroundings in accordance with already internalized meanings (...), and on the other a change of cognitive structures as a result of the influx of new information* (Klus-Stańska 2004: 27). As a result of combining these two processes, students' progress from a visible object to an idea, mathematizing an independently created image. This road, through the creation of their own cognitive network, leads to the interiorization of knowledge at a higher level. By experiencing cognitive conflicts on the border of what is thought of and of what is seen, the student not only interprets the components of the perceived surroundings, but also experiences changes in his or her own cognitive structures (Makiewicz 2013).

Posing questions by the student (See Fig. 1) constitutes the culmination of the process of interiorization. E. Gruszczyk-Kolczyńska describes this process in detail by distinguishing the following intermediate steps: establishing activity by means of audible speech (words reflect activity), transforming audible speech into internal speech (activity on the internal plane, using metalanguage and arbitrary signs), the final stage of exteriorization (reduction, quick conclusion, and generalization leading to mature internalized knowledge) (Gruszczyk-Kolczyńska 1992).

The above steps may be observed in actual activities of the teacher and students in the form of negotiations leading from the description of actual actions towards the determination of differences between images and their properties and an ideal abstract concept. The effectiveness of the development of a child's mathematical activity in the surroundings closest to its growth is related to the exceptional susceptibility of an organism to external stimuli (Gruszczyk-Kolczyńska 1992). From the point of view of further mathematical education, the common way of understanding the essence of mathematic objects, which proves to be still correct, paves the cognitive way to the mature comprehension of scientific phenomena. Relying on the authority of L. S. Wygotski, the process of paving the way that occurs in early school education *resembles the propaedeutic of their* development (Wygotski 1989: 167).

The End

The application of photography in mathematical education at elementary level responds to the need to support the cognitive, aesthetic, and creative development of the student. It is an attempt, as postulated by Dorota Klus-Stańska, to develop *the colloquial activity consisting in the construction of meanings* (Klus-Stańska 2009: 70). The child's activity related to the reading (Jeffrey 2009) of a photograph leads to rational experiencing, and then to negotiation of the meaning of mathematical concepts and patterns. Photographic visualization in connection with semantic interpretation opens before students and their teacher a field of creative activity. This is how they are transported to the *second degree level* (Luhmann 2006: 77), at which they become both authors and observers.

It may be assumed that since the *process of extracting information incorporates two strategies, i.e. verbal and imaginary* (Hankała 2009), teaching mathematics with the help of photography will emerge as a universal way of stimulating the memory processes and cognitive activation of students, irrespective of the dominant learning style or the dominant cerebral hemisphere. The purpose of combining the process of interiorization and exteriorization into a cycle is to enable a *harmonious functioning of both cerebral hemispheres. A continuous synchronization of left- and right-brain processes is one of the prerequisites of the superteaching* discussed by J. Gnitecki (1988).

From the very first days of school, students are constantly hurried and they do not have enough time to read books or to meet friends. It is easier to send an MMS or post a photo online than to exchange a few sentences. Images reach all of us significantly faster than sounds, touch or smell. It is estimated that at the perception level the number of visual stimuli equals approximately 10^7 bit/s of information units (Śpiewak 2013), which constitutes the top number of all the channels of perception. The recently accelerated *technological innovations, their availability, appeal, price, as well as the economic logic of the consumptionistic capitalism amplify our society's saturation with visual images* (Sztompka 2012: 22-24). It is now time for pedagogic reflection on how the media may support cognitive process in a meaningful and pragmatic way.

The popularity, availability, and the relative ease of use of photography offer an opportunity to overcome *the vicious circle in the teaching of mathematics* described by B. Butterworth (1999), in which fear leads to avoidance of didactic activities (classes), which results in resignation from education. Skills are then insufficient to satisfy requirements and students get poor results and are subject to punishment (a poor grade, opinion of their peers, parents, and teachers). As a result, they experience frustration, which creates fear. The vicious circle closes here. With the help of photoeducation discussed in this article using the example of symmetry, the student's joy derived from the sense of discovery and creative activity generates a desire for even more intensive learning. As a result, the student's skills exceed the standard requirements and the student is rewarded with good results. The success acts as reinforcement and gives satisfaction with mathematics.

References

Berger J. (2011), *Zrozumieć fotografię*, translated by K. Olechnicki. In: M. Frąckowiak, M. Krajewski, K. Olechowski, *Badania wizualne w działaniu. Antologia tekstów*. Warszawa, Fundacja Nowej Kultury Bęc Zmiana.

Bohnsack R. (2004), *Dokumentarna interpretacja obrazu - w stronę rekonstrukcji ikonicznych zasobów wiedzy.* In: S. Krzychała (ed.) (2004), *Społeczne przestrzenie doświadczenie – metoda interpretacji dokumentarnej*. Wrocław, WN DSWE.

Bronowski J. (1984), *Źródła wiedzy i wyobraźni*, translated by S. Amsterdamski. Warszawa, PIW.

Bruner J. (2006), *Kultura edukacji,* translated by T. Brzostowska-Tereszkiewicz. Kraków, Universitas.

Butterworth B. (1999), *Neuroscience: a head for figures*. Science, 284.

Cackowska M. (1985), *Koncepcja nauczania czynnościowego i możliwości jej realizacji na szczeblu nauki początkowej*, „Życie Szkoły", 9.

Eco U. (1972), *Dzieło otwarte. Forma i nieokreśloność w poetykach współczesnych*, translated by J. Gałuszka, L. Eustachiewicz, A. Kreisberg i M. Olesiuk. Warszawa, Czytelnik.

Filipiak E. (2012), *Rozwijanie zdolności uczenia się. Z Wygotskim i Brunerem w tle*. Sopot, GWP.

Gnitecki J. (1988), *Supernauczanie. Perspektywy nowej edukacji.* Poznań, Wydawnictwo Naukowe Polskiego Towarzystwa Pedagogicznego.

Gruszczyk-Kolczyńska E. (ed.) (2012), *O dzieciach matematycznie uzdolnionych. Książka dla rodziców i nauczycieli*. Warszawa, Nowa Era.

Gruszczyk-Kolczyńska E. (1992), *Dzieci ze specyficznymi trudnościami w uczeniu się matematyki*. Warszawa, WSiP.

Hankała A. (2009), *Aktywność umysłu w procesie wydobywania informacji pamięciowych*. Warszawa, WUW.

Jeffrey I. (2009), *Jak czytać fotografię. Lekcje mistrzów,* translated by J. Jedliński. Kraków, Universitas.

Klus-Stańska D. (2004), *Światy dziecięcych znaczeń - poszukiwanie kontekstów teoretycznych.* In: D. Klus-Stańska (ed.), *Światy dziecięcych znaczeń.* Warszawa, Żak.

Klus-Stańska D. (2010), *Dydaktyka wobec chaosu pojęć i zdarzeń*. Warszawa, Żak.

Klus-Stańska D. (2009), *Polska rzeczywistość dydaktyczna - paradygmatyczny taniec św. Wita*. In: L. Hurło, D. Klus-Stańska, M. Łojko (eds.), *Paradygmaty współczesnej dydaktyki.* Kraków, Impuls.

Klus-Stańska D., Nowicka M. (2005), *Sensy i bezsensy edukacji wczesnoszkolnej*. Warszawa, WSiP.

Kozielecki J. (1976), *Koncepcje psychologiczne człowieka*. Warszawa, PIW.

Kozioff M. (2009), Przedmowa do: I. Jeffrey, *Jak czytać fotografię. Lekcje mistrzów,* translated by J. Jedliński. Kraków, Universitas.

Lakoff G. (2008), *The Neural Theory of Metaphor*. In: R.W. Gibbs Jr. (ed.), *The Cambridge Handbook of Metaphor and Thought*. New York, Cambridge University Press.

Lewandowska-Tomaszczyk B. (2009), *Kategorie obrazowania i reprezentacja rzeczywistości*. In: A. Kwiatkowska, J. Jarniewicz (eds.), *Między obrazem a tekstem.* Łódź, WUŁ.

Limont W. (2012), *Uczeń zdolny. Jak go rozpoznać i jak z nim pracować*. Sopot, GWP.

Luhmann N. (2006), *Kultura jako pojęcie historyczne*, translated by M. Lipnicki. In: E. Kuźma, A. Skrendo, J. Madejski (eds.), *Konstruktywizm w badaniach literackich. Antologia*, Kraków, Universitas.

Makiewicz M. (2010), *Matematyka w obiektywie. Kultura matematyczna dla nauczycieli.* Szczecin, WNUS.

Makiewicz M. (2012a), *Poznawcza sieć matematycznego myślenia.* Szczecin, SKNMDM.

Makiewicz M. (2012b), *Mathematical cognition in metaphors expressed through photography*, „Chemistry-Didactics-Ecology-Metrology", 2.

Makiewicz M. (2013), *O fotografii w edukacji matematycznej. Jak kształtować kulturę matematyczną uczniów*. Szczecin, SKNMDM US.

Nęcka E. (2001), *Psychologia twórczości.* Gdańsk, GWP.

Ogonowska A. (2006), *Twórcze metafory medialne. Baudrillard – McLuhan – Goffman*. Kraków, Universitas.

Piaget J. (1966a), *Narodziny inteligencji dziecka,* translated by M. Przetacznikowa. Warszawa, PWN.

Piaget J. (1966b), *Studia z psychologii dziecka*, translated by T. Kołakowska. Warszawa, PWN.

Piaget J. (1977), *Dokąd zmierza edukacja,* translated by M. Domańska. Warszawa, PWN.

Platon (1958), *Państwo*, ks. VII, r. I-III, translated by W. Witwicki. Warszawa, PWN.

Przetacznik-Gierowska M. (1993), *Świat dziecka. Aktywność – Poznanie – Środowisko.* Kraków, UJ.

Siwek H. (2005), *Dydaktyka matematyki. Teoria i zastosowania w matematyce szkolnej*. Warszawa, WSiP.

Stiegler B. (2009), *Obrazy w fotografii. Album metafor fotograficznych*. Kraków, Universitas.

Sztompka P. (2012), *Wyobraźnia wizualna i socjologia*. In: M. Bogunia- Borowska, P. Sztompka (eds.) (2012), *Fotospołeczeństwo. Antologia tekstów z socjologii wizualnej*. Kraków, Znak.

Sztompka P. (2006), *Socjologia wizualna. Fotografia jako metoda badawcza.* Warszawa, WN PWN.

Szmidt K. J. (2005), *Paula Torrance'a inkubacyjny model kształcenia uczniów zdolnych.* In: W. Limont, J. Cieślikowska (eds.), *Wybrane zagadnienia edukacji uczniów zdolnych*, t.1. Kraków, Impuls.

Śpiewak S. (2013), *Rozgrzewanie uwagi - wyczerpywanie woli - uległość. Mechanizmy adaptacji umysłu do wysiłku poznawczego.* Warszawa, Wydawnictwo Naukowe Scholar.

Van Dijk J. (2010), *Społeczne aspekty nowych mediów. Analiza społeczeństwa sieci*, translated by J. Konieczny. Warszawa, WN PWN.

Wojnar I. (1971), *Estetyka i wychowanie.* Warszawa, PWN.

Wygotski L. (1989), *Myślenie i mowa*, translated by E. Flesznerowa, J. Fleszner. Warszawa, PWN.

Zdanowicz-Kucharczyk K. (2010), *Fotografia oknem na świat społeczny.* In: H. Kędzierska (ed.) (2010), *Jakościowe inspiracje w badaniach edukacyjnych.* Olsztyn, WN UWM.