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ON NOTIONS OF ASSERTION, KNOWLEDGE AND OPINION IN EPISTEMIC LOGIC

Key words: epistemic logic, belief, truth

At the beginning of my address I will present the outline of the most important achievements in epistemic logic, which can be established on the basis of monographic studies. Next – pursuant to the subject matter of the Conference – I will raise the issue of the cognitive value of beliefs. Since it is possible to assess the degree of cognitively limiting beliefs only in theory, including the nature of beliefs in their diversity, the proposal of a stand-alone version of epistemic logic constitutes a fundamental part of this address.

As part of my report, I will firstly present a set used in literature rules, axioms and Kripke's principles of semantics in the systems K, T, S4, S5, K45 and KD45 for epistemic functors : K (I know that...) and B (I believe that ...).

Rules: modus ponens, necessitation rule: $\varphi/K\varphi$

Axioms:
CPL (classical propositional logic)
(K) $K(\varphi \rightarrow \psi) \rightarrow (K\varphi \rightarrow K\psi)$
(T) $K\varphi \rightarrow \varphi$............the accessibility relation R is reflexive
(4) $K\varphi \rightarrow KK\varphi$.........R is transitive
(5) $\sim K\varphi \rightarrow K\sim K\varphi$.......R is symmetric
(K/B) $B(\varphi \rightarrow \psi) \rightarrow (B\varphi \rightarrow B\psi)$
(4/B) $B\varphi \rightarrow BB\varphi$.........R is transitive

(5/B) $\sim B\varphi \rightarrow B\sim B\varphi$.......R is Euclidean: $\forall s,t,u \in S\ (sRt \wedge sRu \rightarrow tRu)$
(D) $\sim B(p \wedge \sim p)$.......R is serial: $\forall s \in S\ \exists t \in S\ sRt$

Systems:
K = {(K)}
T = {(K), (T)} R is reflexive
S4 = {(K), (T), (4)} R is reflexive and transitive
S5 = {(K), (T), (4), (5)} R is reflexive, transitive and symmetric
K45 = {(K/B), (4/B), (5/B)} R is transitive and Euclidean
KD45 = {(K/B), (4/B), (5/B), (D)} R is transitive, Euclidean and serial

Any scholarly scientific activity takes place in an epistemic space of belief – knowledge – opinion. But what are these three „dimensions" of knowledge and in which reciprocal relationships do they stay? In this case, logical syntax, semantics and pragmatics provide many important suggestions. However, at the outset, they demand a strong separation of the concepts of truth and belief. Although it is good to be sure that we have the truth, truth neither enforces certainty, nor does certainty guarantee possession of the truth. And therein lies our problem of cognition.

Following Aristotle we repeat: „To say that what is, is, and what is not, it is not, it's true"[1], and with Thomas Aquinas: "Veritas est rei et intellectus adequatio"[2]. Alfred Tarski taught us that: "Sentence `p' is true if and only if p"[3]. And hence, the examplary sentence „There is life on Mars" is only true if there is life on Mars. It would be enough to have access to life on Mars, to conclude that the sentence about its existence is true. But we do not have such access.

With all these non-diagnostic expressions we can only understand what is the truth. In contrast to the long history of the search for the criteria of truth, of great demand remained only the obviousness of the fact, especially the intersubjective one (paradigm). Is the theory of

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[1] Aristotle, *Metaphysics*, G7, 1011 b 26-27.

[2] This formula is probably derived from the Arabic philosophers, as indicated by the link of Thomas Aquinas in the Quaestiones disputatae de veritatae I, 1.

[3] „x is a true sentence if and only if p". A. Tarski, *Pojęcie prawdy w językach nauk dedukcyjnych*, Warszawa 1933, 5.

knowledge threatened by postmodern destruction, that the truth is what people believe to be true? Or maybe the Kantian perspective of the categorical break with the idea of reproducing the truth, because „only this we know *a priori* on a thing what we put in it"[4]?

Contemporary logical semantics introduced a new interpretation of the classical theory of truth in the notion of „the truth in a model." Sentences are often or even usually ambiguous, with a shaky sense, and sometimes even contradictory. A sentence „There is a smallest number" is true in the model of natural numbers and false in the model of real numbers. And what is there in the model of prime numbers, rational, irrational, integers, complex, imaginary, in the different intervals open or closed ...? Therefore, there is no truth per se, but only the truth in the model. After all this is not a great discovery, because 'to determine the truth of the sentence in the model', is nothing more than 'to confirm the truth of one of its many meanings'.

But when – thus far – model theories are presented in the language of well-defined theory sets, the model theory of truth is successful in its application to formalized languages. With respect to fuzzy natural languages, it only warns against its insolvability, because the sentence is not unequivocal, i.e . it is not a projection onto one intended model; it does not allow for a comparison between „intellect" and „thing".

Finally, identifying truth with fact, we emphasize its non-gradable nature and independence from the subject of cognition, i.e. its objectivity. However, it is obvious that we are, in varying degrees, occasionally convinced of the truth of each sentence, speech, or thoughts Since beliefs are continuously graded from weakest to strongest, from the state of suspension to certainty, and largely depend on various conditions of cognitive apparatus, as well as perceived by consciousness as primarily their own experience, which is subjective.

In my presentation, a distinction is made between the assertive (strong), hypothetical (conjectures) and supposition (admission) courts. I firmly believe that the earth revolves around the sun, (I suppose that natural satellites orbit around each planet in the solar system) and I just admit that the number of stars in the universe is even.

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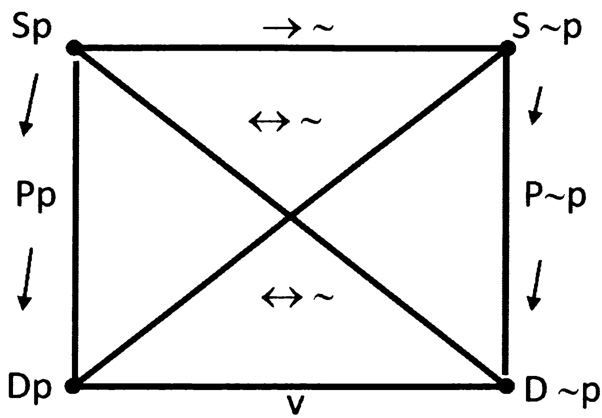
[4] I. Kant, *Krytyka czystego rozumu*, Volume I, trans. R. Ingarden, Kraków 1957, 32.

Abbreviations:
Sp = (I believe firmly that p),
Pp =: (I suppose that p),
Dp = (I assume that p),
Wp =: (I know that p),
Mp =: (It is my opinion that p).

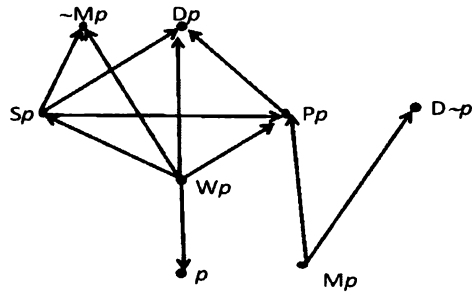
From the position of pragmatics, let us focus now on some of the epistemic phenomena. One of them is that the power of believing is gradated in a continuous manner between two extremes in belief: assertion and admition. Assertion is the end of the strongest convictions, and admition of the weakest. This weakest belief, which is only the allowing of the logical possibility of non-contradiction, is called the threshold of rationality. At this threshold, the forces of opposing propositions ~ p and p, may nevertheless be equated, which was known to the ancient skeptics (third century AD) under the name of isosteni (balance of beliefs), where Dp ∧ D~p (eg, for p =: ("The number of stars in the universe is even")).

Let's graphically illustrate the relationship between beliefs: Sp, S~p, Pp, P~p, Dp, D~p in the form of a logical square:



Aristotle, in Analytics II, 89a, compares the concepts of knowledge and opinion: „Nobody considers that he believes if he thinks that it can not be otherwise, but then considers that he knows. The other hand, considers that thinks when thoughts that although it is so and so, it could be otherwise”[5]. Entailment relations between distinguished types of beliefs and truth (Wp, Sp, Pp Mp, p) are illustrated in the next graph (where the arrows define the relation of logical consequence):

Sp = (I believe firmly that p),
Pp = (I suppose that p),
Dp =: (I assume that p),
Wp =: (I know that p),
Mp =: (It is my opinion that p).



SYNTAX
Axioms and definitions:
KRZ (classical propositional logic)
A1. $S(p \rightarrow q) \rightarrow (Sp \rightarrow Sq)$
A2. $Sp \rightarrow SSp$
Df.W: $Wp \leftrightarrow Sp \wedge p$
A3. $Wp \rightarrow SWp$ (In this set of axioms the H3 is unnecessary.)
A4. $Sp \rightarrow Pp$
Df.D: $Dp \leftrightarrow {\sim}S{\sim}p$
A5. $Pp \rightarrow Dp$
Df.M: $Mp \leftrightarrow Pp \wedge D{\sim}p$

Rules of inference: RP (rule of detachment), RO (modus ponens), RG (Gödel’s rule: I– α, therefore I– $S\alpha$
Theses:
T1. $D{\sim}p \leftrightarrow {\sim}Sp$, from Df.D
T2. $Sp \leftrightarrow {\sim}D{\sim}p$, from T1
T3. $S{\sim}p \leftrightarrow {\sim}Dp$, from T2
T4. $Sp \rightarrow Dp$, from A4, A5

[5] Aristotle, *Analityki pierwsze i wtóre*, transl. by K. Leśniak, Warszawa 1973.

T5. $S{\sim}p \to D{\sim}p$, from T4
T6. $S{\sim}p \to P{\sim}p$, from A4
T7. $Sp \to {\sim}S{\sim}p$, from T4, Df.D
T8. $Pp \to {\sim}S{\sim}p$, from A5, Df.D
T9. $S{\sim}p \to {\sim}Sp$, from T5, T1
T10. $P{\sim}p \to {\sim}Sp$, from A5, T1
T11. ${\sim}Dp \to D{\sim}p$, from T4, T1
T12. ${\sim}D{\sim}p \to Dp$, from T5, Df.D
T13. ${\sim}Pp \to D{\sim}p$, from A4, T1
T14. ${\sim}P{\sim}p \to Sp$, from T6, Df.D
T15. ${\sim}SSp \leftrightarrow DD{\sim}p$, from T1, therefore ${\sim}SSp \leftrightarrow D{\sim}Sp$, T1
T16. $SSp \leftrightarrow {\sim}DD{\sim}p$, from T15
T17. $DD{\sim}p \to {\sim}Sp$, from T15, A2
T18. $DDp \to {\sim}S{\sim}p$, from T17
T19. $DDp \to Dp$, from T18, Df.D
T20. $S(p \wedge q) \to Sp \wedge Sq$, from $p \wedge q \to p$, $p \wedge q \to q$, RG, A1, therefore $S(p \wedge q) \to S/p$
T21. $Sp \wedge Sq \to S(p \wedge q)$, from $p \to [q \to (p \wedge q)]$, RG, therefore $Sp \to S[q \to (p \wedge q)]$, A1, therefore $Sp \to [Sq \to S(p \wedge q)]$
T22. $S(p \wedge q) \leftrightarrow Sp \wedge Sq$, from T20, T21
T23. $(Sp \vee Sq) \to S(p \vee q)$, from $p \to (p \vee q)$, $q \to (p \vee q)$, RG, A1
T24. $D(p \vee q) \to (Dp \vee Dq)$, from T21, therefore $S{\sim}p \wedge S{\sim}q \to S({\sim}p \wedge {\sim}q)$, therefore ${\sim}S({\sim}p \wedge {\sim}q) \to ({\sim}S{\sim}p \vee {\sim}S{\sim}q)$,T1
T25. $(Dp \vee Dq) \to D(p \vee q)$, from T20, therefore $S({\sim}p \wedge {\sim}q) \to (S{\sim}p \wedge S{\sim}q)$, therefore $({\sim}S{\sim}p \vee {\sim}S{\sim}q) \to {\sim}S{\sim}(p \vee q)$, Df.D
T26. $D(p \vee q) \leftrightarrow (Dp \vee Dq)$, from T24, T25
T27. $D(p \wedge q) \to (Dp \wedge Dq)$, from T23, therefore $(S{\sim}p \vee S{\sim}q) \to S({\sim}p \vee {\sim}q)$, therefore ${\sim}S({\sim}p \vee {\sim}q) \to {\sim}S{\sim}p \wedge {\sim}S{\sim}q$, T1, Df.D
T28. $Wp \to Sp$, from Df.W
T29. $Wp \to p$, from Df.W
T30. $W{\sim}p \to {\sim}Wp$, from T28, therefore $W{\sim}p \to S{\sim}p$, T9, therefore $S{\sim}p \to {\sim}Sp$, T28, therefore ${\sim}Sp \to {\sim}Wp$
T31. $Wp \to {\sim}W{\sim}p$, from T28, T7, therefore $Wp \to {\sim}S{\sim}p$, T28, therefore $W{\sim}p \to S{\sim}p$
T32. $Wp \to WSp$, from T28, A2, therefore $Wp \to SSp \wedge Sp$, Df.W
T33. $WWp \to Wp$, from T29

T34. $Wp \rightarrow WWp$, from A3, therefore $Wp \rightarrow SWp \wedge Wp$, Df.W
T35. $WWp \leftrightarrow Wp$, from T33, T34
T36. $Wp \rightarrow Pp$, from T28, A4
T37. $Wp \rightarrow Dp$, from T36, A5
T38. $W(p\rightarrow q) \rightarrow (Wp \rightarrow Wq)$, from T29, therefore $W(p\rightarrow q) \rightarrow (p\rightarrow q)$, T29, therefore $W(p\rightarrow q) \rightarrow (Wp \rightarrow q)$, T28, therefore $W(p\rightarrow q) \rightarrow S(p\rightarrow q)$, A1, therefore $W(p\rightarrow q) \rightarrow (Sp \rightarrow Sq)$, T28, therefore $W(p\rightarrow q) \rightarrow (Wp\rightarrow Sq)$, therefore $W(p\rightarrow q) \rightarrow (Wp \rightarrow Sq\wedge q)$, Df.W
RG*: $I\!-\!\alpha$, therefore $W\alpha$, from $I\!-\!\alpha$, RG, therefore $I\!-\!S\alpha$, therefore $I\!-\!S\alpha\wedge\alpha$, Df.W
T39. $W(p\wedge q) \rightarrow Wp \wedge Wq$, from $p\wedge q \rightarrow p$, $p\wedge q \rightarrow q$, RG*, therefore $W(p\wedge q \rightarrow p)$, $W(p\wedge q \rightarrow q)$, T38, therefore $W(p\wedge q) \rightarrow W p$, $W(p\wedge q) \rightarrow Wq$
T40. $(Wp \wedge Wq) \rightarrow W(p\wedge q)$, from T28, therefore $Wp \rightarrow Sp$, $Wq \rightarrow Sq$, therefore $Wp \wedge Wq \rightarrow Sp \wedge Sq$, T21, therefore $Wp \wedge Wq \rightarrow S(p\wedge q)$, T29, therefore $Wp \rightarrow p$, $Wq \rightarrow q$, therefore $Wp \wedge Wq \rightarrow (p\wedge q)$, therefore $Wp \wedge Wq \rightarrow S(p\wedge q) \wedge (p\rightarrow q)$, Df.W
T41. $W(p\wedge q) \leftrightarrow (Wp \wedge Wq)$, from T39, T40
T42. $Mp \rightarrow (Dp \wedge D{\sim}p)$, from Df.M, A5
T43. $Sp \rightarrow {\sim}Mp$, from T2, T42
T44. $Wp \rightarrow {\sim}Mp$, from T28, T43

SEMANTICS

Let Φ denote the set of all propositional formulas the language of the presented epistemic logic and $\alpha\in\Phi$.

Each record in the type V (`α', t) = ..., and: F α t, is elliptical: always refers to the user of the language (e.g. "I") at time T his existence.

Dictionary

$t,s,k,\ldots \in T$, $p,q,r,.. \in\Phi$

Fpt = the situation p is for someone a fact at the time t (p is one situation which I accept at the time t as the fact). Epistemic functors express the readiness of recognition of a situation as fact.

$mt = \{p: Fpt\}$ for $t\in T$

$M = <\{mt\}t\in T, R>$, where $R\subseteq \{mt\}t\in T \times \{mt\}t\in T$

and $mt\ R\ ms \leftrightarrow t<s$, $mt = ms \leftrightarrow t=s$

$V(`p',t)=1 \leftrightarrow p \in mt$ $V(`p',t)=0 \leftrightarrow p\notin mt$

VN1: $V(`\sim\alpha', t)=1 \leftrightarrow V(\alpha,t)=0$
VN0: $V(`\sim\alpha', t)=0 \leftrightarrow V(\alpha,t)=1$
VI1: $V(`\alpha\to\beta', t)=1 \leftrightarrow [V(`\alpha', t)=0 \vee V(`\beta', t)=1]$
VI0: $V(`\alpha\to\beta', t)=0 \leftrightarrow [V(`\alpha', t)=1 \wedge V(`\beta', t)=0]$
VS1: $V(`S\alpha', t) = 1 \leftrightarrow \forall s\ (t<s \to V(`\alpha', s)=1)$
VS0: $V(`S\alpha', t) = 0 \leftrightarrow \exists s\ (t<s \wedge V(`\alpha', s)=0)$
VW1: $V(`W\alpha', t) = 1 \leftrightarrow \forall s\ (t\leq s \to V(`\alpha', s)=1)$
VW0: $V(`W\alpha', t) = 0 \leftrightarrow \exists s\ (t\leq s \wedge V(`\alpha', s)=0)$
VD1: $V(`D\alpha', t) = 1 \leftrightarrow \exists s\ (t<s \wedge V(`\alpha', s)=1)$
VD0: $V(`D\alpha', t) = 0 \leftrightarrow \forall s\ (t<s \to V(`\alpha', s)=0)$
VP1: $V(`P\alpha', t)=1 \leftrightarrow \exists k\ [t<k \wedge \forall s\ (t<s<k \to V(`\alpha', s)=1)$
VP0: $V(`P\alpha', t)=0 \leftrightarrow \forall k\ [t<k \to \exists s\ (t<s<k \wedge V(`\alpha', s)=0)$
VM1: $V(`M\alpha', t)=1 \leftrightarrow V(`P\alpha', t)=1 \wedge V(`D\sim\alpha', t)=1$
VM0: $V(`M\alpha', t)=0 \leftrightarrow V(`P\alpha', t)=0 \vee V(`D\sim\alpha', t)=0$

Evidence of the validity of axioms:
A1. $S(p\to q) \to (Sp \to Sq)$
1. $V(`S(p\to q) \to (Sp \to Sq)', t) = 0$, a.i.p. (assumption of indirect proof)
2. $V(`S(p\to q)', t) =1$, VI0: 1
3. $V(`Sp', t)=1$, VI0: 1
4. $V(`Sq', t)=0$, VI0: 1
5. $\forall s\ (t<s \to V(`p\to q', s)=1)$, VS1: 2
6. $\forall s\ (t<s \to V(`p', s)=1)$, VS1: 3
7. $\exists s\ (t<s \wedge V(`q', s)=0)$, VS0: 4
8. $t<a,\ a\in T$, 7
9. $V(`q', a)=0$, 7
10. $V(`p', a)=1$, 6,8
11. $V(`p\to q', a)=1$, 5,8
12. $V(`q', a)=1$, VI1: 11,10

contr. (contradiction): 9, 12

A2. $Sp \to SSp$
1. $V(`Sp \to SSp', t)=0$, a.i.p
2. $V(`Sp', t)=1$, VI0: 1
3. $V(`SSp'', t)=0$, VI0: 1

4. $\forall s\ (t<s \rightarrow V(\text{'p'},s)=1)$, VS1: 2
5. $\exists s\ (t<s \wedge V(\text{'Sp'},s)=0)$, VS0: 3
6. $t<a,\ a\in T$, 5
7. $V(\text{'Sp'},a)=0$, 5
8. $\exists s\ (a<s \wedge V(\text{'p'}, s)=0$, VS0: 7
9. $a<b,\ b\in T$, 8
10. $V(\text{'p'},b)=0$, 8
11. $t<b$, 6,9
12. $V(\text{'p'},b)=1$, 4, 11

contr.: 10, 12

A3. $Wp \rightarrow SWp$

1. $V(\text{'Wp} \rightarrow \text{SWp'},t)=0$, a.i.p
2. $V(\text{'Wp'},t)=1$, VI0: 1
3. $V(\text{'SWp'},t)=0$, VI0: 1
4. $\exists s\ (t<s \wedge V(\text{'Wp'},s)=0)$, VS0: 3
5. $t<a,\ a\in T$, 4
6. $V(\text{'Wp'},a)=0$, 4
7. $\exists s\ (a\leq s \wedge V(\text{'p'},s)=0)$, VW0: 6
8. $a\leq b,\ b\in T$, 7
9. $V(\text{'p'},b)=0$, 7
10. $\forall s\ (t\leq s \rightarrow V(\text{'p'},s)=0)$, VW1: 2
11. $t\leq b$, 5, 8
12. $V(\text{'p'},b)=1$, 10, 11

contr.: 9, 12

A4. $Pp \rightarrow Dp$

1. $V(\text{'Pp} \rightarrow \text{Dp'}, t)=0$, a.i.p
2. $V(\text{'Pp'},t)=1$, VI0: 1
3. $V(\text{'Dp'},t)=0$, VI0: 1
4. $\exists k\ [t<k\ \wedge \forall s\ (t<s<k \rightarrow V(\text{'p'},s)=1]$, VP1: 2
5. $t<a,\ a\in T$, 4
6. $\forall s\ (t<s<a \rightarrow V(\text{'p'},s)=1]$, 4
7. $\forall s\ (t<s \rightarrow V(\text{'p'},s)=0)$, VD0: 3
8. $\forall t\forall k\ [t<k \rightarrow \exists s\ (t<s<k)]$, density of the relation accessibility: $<$

9. $\exists s\,(t<s<a)$, 8, 5
10. $t<b<a$, $b\in T$, 10
11. $V(\text{'p'},b)=1$, 6, 10
12. $V(\text{'p'},b)=0$, 7, 10
contr.: 11, 12

A5. $Sp \rightarrow Pp$
1. $V(\text{'Sp} \rightarrow \text{Pp'}, t)=0$, a.i.p
2. $V(\text{'Sp'},t)=1$, VI0: 1
3. $V(\text{'Pp'},t)=0$, VI0: 1
4. $\forall s\,(t<s \rightarrow V(\text{'p'},s)=1)$, VS1: 2
5. $\forall k\,[t<k \rightarrow \exists s\,(t<s<k \wedge V(\text{'p'},s)=0)$, VP0: 3
6. $\forall s\exists k\ s<k$, seriality of the relation accessibility: <[6]
7. $\exists k\ t<k$, 6
8. $t< a$, $a\in T$, 7
9. $\exists s\,(t<s<a \wedge V(\text{'p'},s)=0)$, 5, 8
10. $t<b<a$, $b\in T$, 9
11. $V(\text{'p'},b)=0$, 9
12. $V(\text{'p'},b)=1$, 4, 10
contr.: 11, 12

O POJĘCIACH ASERCJI, WIEDZY I MNIEMANIA W LOGICE EPISTEMICZNEJ

Streszczenie

W artykule przedstawiono propozycję sformalizowanej teorii, w której nadane oraz wzajemnie porównywane są znaczenia trzech stopni przekonań (S- sądzę stanowczo, D- dopuszczam, P- przypuszczam), funktora W rozumianego jako zwrot „wiem, że" oraz funktora M rozumianego jako zwrot „mniemam, że". Teoria ta posiada zarówno ujęcie składniowe jak i semantyczne.

Przedstawiając ujęcie składniowe tworzonej teorii zwraca się uwagę na to, że pomiędzy funktorem S oraz D zachodzą związki kwadratu logicznego. Oznacza to, że

S(p) jest sprzeczne z D(¬p), natomiast przeciwne do S(¬p).

S(¬p) jest sprzeczne z D(p).

[6] Time of loss of awareness of the fact it is a segment and not the point.

Parę wyrażeń podprzeciwnych stanowią: D(p) oraz D(¬p).

Pojęcia: „sądzę stanowczo", „przypuszczam", „dopuszczam" wyrażają różne stopnie przekonania. Najmocniejszy stopień przekonania kryje się w zwrocie „sądzę stanowczo", słabszy w zwrocie „przypuszczam", a najsłabszy w zwrocie „dopuszczam". Dlatego też z S(p) wynika logicznie P(p), a z P(p) wynika logicznie D(p). Funktor P jak i S wprowadzone są do teorii aksjomatycznie. Funktory: D (dopuszczam, że), W (wiem, że) oraz funktor M (mniemam, że) wprowadzone są poprzez kontekstowe definicje równościowe. Definicje te w przełożeniu na język naturalny brzmią następująco:

Def 1: Dopuszczam, że p wtedy i tylko wtedy, gdy nie sądzę stanowczo, iż nieprawda, że p.

Def 2: Wiem, że p wtedy i tylko wtedy, gdy zarazem p oraz sądzę stanowczo, że p.

Def 3: Mniemam, że p wtedy i tylko wtedy, gdy przypuszczam, że p, ale jednocześnie dopuszczam, że nieprawda, że p.

Zgodnie z definicją Def 2 funktor wiedzy jest silniejszy od każdego z rozpatrywanych stopni przekonań.

W ujęciu semantycznym została wyróżniona struktura (T, <, ≤) – w której T jest zbiorem punktów czasowych, < to relacja bycia wcześniejszym, natomiast ≤ jest relacją bycia nie późniejszym – i zdefiniowane indukcyjnie pojęcie prawdziwości w chwili t.

Słowa kluczowe: logika epistemiczna, przekonanie, prawda