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Studia i Prace Wydziału Nauk Ekonomicznych i Zarządzania 4, 135-158

2008

STUDIA I PRACE WYDZIAŁU NAUK EKONOMICZNYCH I ZARZĄDZANIA NR 4

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Introduction

The Government Order dated 2004-08-11, about the detailed method of computation of state aid granted in various ways (Dz. U. Nr 194, pos. 1983) determines in a detailed way the rules of conversion of state aid value, granted in different ways, into the value of subsidy expressed in the gross subsidy equivalent (EDB) or the net subsidy equivalent (EDN). In the Order the following phrases are defined:

- EDB (the gross subsidy equivalent) – the value of financial support that the beneficiary or the entity applying for the state aid, would receive if it was granted a subsidy, excluding the income tax,
- EDN (the net subsidy equivalent) – the value of financial support that the beneficiary or the entity applying for the state aid, would receive if it was granted a subsidy, including the income tax,
- reference rate – the interest rate periodically set by the European Committee on the basis of objective criteria,
- discounting – including the changes of money value in time by multiplying future payments by the discounting factor;
- Discounting factor – factor equals $(1 + r)^{-i}$ where:

- r – reference rate valid at the day of grant,
- i – subsequent repayment period.

– Tax rate – the highest possible income tax rate, valid for the beneficiary or entity applying for subsidy.

The gross subsidy equivalent (EDB) for the preferential loan or credit equals the difference between the discounted value of interests of loan or credit lent on market terms and discounted value of interests of preferential loan or credit. In turn, the net subsidy equivalent (EDN) is expressed by the following formula:

$$EDN = EDB \times (1 - t), \tag{1}$$

where t is the tax rate.

Further, in the next part of this paragraph, the algorithms of computation of the amount of the gross subsidy equivalent (EDB) will be presented and explained for the following cases[1]:

– the equal principal installments repayment,
– the equal principal installments repayment with a grace period for capital repayment,
– the equal principal installments repayment with a grace period for both capital and interest repayment,

pondered as the difference between the installments calculated for the loan (credit) lent on the market terms and the installments for analogical preferential loan (credit).

[1] In this paper authors used methods of mathematical analysis described in: Nowak, 1997; Bień A., Bień W., 1996; Ostasiewicz, Ronka-Chmielowiec, 1994; Piszczała 2000; Skałba, 1999; Sobczyk, 2000.

Interests in the equal principal installments system

The principal installments calculated for consecutive periods $i = 1, 2, \ldots, N$ in case of the loan (credit) lent on the market terms and analogical preferential loan (credit) equal:

$$K = \frac{S}{N}, \tag{2}$$

where the following symbols stand for:

S – loan (credit) amount,

N – number of repayment periods.

Interests on the loan (credit) lent on the market terms and analogical preferential loan (credit) are presented in the table 1.

Table 1

Interests on the loan lent on the market terms and preferential loan

Consecutive payment period	Interests on the loan (credit) lent on the market terms	Interests on the preferential loan (credit)
1	$D_1 = Sr$	$D_1 = Sr_p$
2	$D_2 = \left(S - \frac{S}{N}\right)r$	$D_2 = \left(S - \frac{S}{N}\right)r_p$
...	...	...
N	$D_N = \left[S - (N-1)\frac{S}{N}\right]r$	$D_N = \left[S - (N-1)\frac{S}{N}\right]r_p$
Total for i form 1 to N	$\sum_{i=1}^{N} D_i = \sum_{i=1}^{N}\left[S - (i-1)\frac{S}{N}\right]r$	$\sum_{i=1}^{N} D_i = \sum_{i=1}^{N}\left[S - (i-1)\frac{S}{N}\right]r_p$

Source: own compilation.

Discounted value of the interests on the loan (credit) lent on the market terms and analogical preferential loan (credit) is presented in the table 2.

Table 2

Discounted value of the interests on the loan lent on the market terms and analogical preferential loan

Consecutive payment period	Discounted value of interests on the loan (credit) lent on the market terms	Discounted value of interests on the preferential loan (credit)
1	$Sr\frac{1}{(1+r)^1}$	$Sr_p\frac{1}{(1+r)^1}$
2	$\left(S-\frac{S}{N}\right)r\frac{1}{(1+r)^2}$	$\left(S-\frac{S}{N}\right)r_p\frac{1}{(1+r)^2}$
...	...	...
N	$\left[S-(N-1)\frac{S}{N}\right]r\frac{1}{(1+r)^N}$	$\left[S-(N-1)\frac{S}{N}\right]r_p\frac{1}{(1+r)^N}$
Total for i form 1 to N	$\sum_{i=1}^{N}\left[S-(i-1)\frac{S}{N}\right]r\frac{1}{(1+r)^i}$	$\sum_{i=1}^{N}\left[S-(i-1)\frac{S}{N}\right]r_p\frac{1}{(1+r)^i}$

Source: own compilation.

Using the above formulas the gross subsidy equivalent for the loan or credit repaid in the equal principal installments system can be calculated as follows:

$$EDB=\sum_{i=1}^{N}S\left(1-\frac{i-1}{N}\right)(r-r_p)\frac{1}{(1+r)^i}. \tag{3}$$

Interests in the equal principal installments repayment with a grace period for capital repayment

The principal installments calculated for consecutive periods $i = T + 1$, $T + 2$, ..., N in case of the loan (credit) lent on the market terms with the grace period for capital repayment and analogical preferential loan (credit) with the grace period for capital repayment equal:

$$K=\frac{S}{N-T}, \tag{4}$$

where the following symbols stand for:

- S – loan (credit) amount,
- N – number of repayment periods,
- T – number of grace periods.

Interests on the loan (credit) lent on the market terms with the grace period for capital repayment and analogical preferential loan (credit) with the grace period for capital repayment are presented in table 3.

Table 3

Interests on the loan (credit) lent on the market terms and analogical preferential loan (credit) with the grace period for capital repayment

Consecutive payment period	Interests on the loan (credit) lent on the market terms with the grace period for capital repayment	Interests on the preferential loan (credit) with the grace period for capital repayment
1	$D_1 = Sr$	$D_1 = Sr_p$
2	$D_2 = Sr$	$D_2 = Sr_p$
...	...	...
T	$D_T = Sr$	$D_T = Sr_p$
Total for i form 1 to T	$\sum_{i=1}^{T} D_i = TSr$	$\sum_{i=1}^{T} D_i = TSr_p$
$T + 1$	$D_{T+1} = Sr$	$D_{T+1} = Sr_p$
$T + 2$	$D_{T+2} = \left(S - \frac{S}{N-T}\right)r$	$D_{T+2} = \left(S - \frac{S}{N-T}\right)r_p$
...	...	...
N	$D_N = \left[S - (N-T-1)\frac{S}{N-T}\right]r$	$D_N = \left[S - (N-T-1)\frac{S}{N-T}\right]r_p$
Total for i form $T + 1$ to N	$\sum_{i=T+1}^{N} D_i = \sum_{i=T+1}^{N}\left[S - (i-T-1)\frac{S}{N-T}\right]r$	$\sum_{i=T+1}^{N} D_i = \sum_{i=T+1}^{N}\left[S - (i-T-1)\frac{S}{N-T}\right]r_p$

Source: own compilation.

Discounted value of the interests on the loan (credit) lent on the market terms with the grace period for capital repayment and analogical preferential loan (credit) with the grace period for capital repayment is presented in the table 4.

Table 4

Discounted value of the interests on the loan lent on the market terms and analogical preferential loan with the grace period for capital repayment

Consecutive payment period	Discounted value of interests on the loan (credit) lent on the market terms with the grace period for capital repayment	Discounted value of interests on the preferential loan (credit) with the grace period for capital repayment
1	$Sr\frac{1}{(1+r)^1}$	$Sr_p\frac{1}{(1+r)^1}$
2	$Sr\frac{1}{(1+r)^2}$	$Sr_p\frac{1}{(1+r)^2}$
...	...	...
T	$Sr\frac{1}{(1+r)^T}$	$Sr_p\frac{1}{(1+r)^T}$
Total for i from 1 to T	$\sum_{i=1}^{T} Sr\frac{1}{(1+r)^i}$	$\sum_{i=1}^{T} Sr_p\frac{1}{(1+r)^i}$
$T+1$	$Sr\frac{1}{(1+r)^{T+1}}$	$Sr_p\frac{1}{(1+r)^{T+1}}$
$T+2$	$\left(S-\frac{S}{N-T}\right)r\frac{1}{(1+r)^{T+2}}$	$\left(S-\frac{S}{N-T}\right)r_p\frac{1}{(1+r)^{T+2}}$
...	...	...
N	$\left[S-(N-T-1)\frac{S}{N-T}\right]r\frac{1}{(1+r)^N}$	$\left[S-(N-T-1)\frac{S}{N-T}\right]r_p\frac{1}{(1+r)^N}$
Total for i from $T+1$ to N	$\sum_{i=T+1}^{N}\left[S-(i-T-1)\frac{S}{N-T}\right]r\frac{1}{(1+r)^i}$	$\sum_{i=T+1}^{N}\left[S-(i-T-1)\frac{S}{N-T}\right]r_p\frac{1}{(1+r)^i}$

Source: own compilation.

Using the above formulas the gross subsidy equivalent for the loan or credit repaid in the equal principal installments system with the grace period for capital repayment may be calculated as follows:

$$EDB=\sum_{i=1}^{T}S(r-r_p)\frac{1}{(1+r)^i}+\sum_{i=T+1}^{N}S\left[1-\frac{i-(T+1)}{N-T}\right](r-r_p)\frac{1}{(1+r)^i}. \quad (5)$$

Interests in the equal principal installments repayment with a grace period for capital and interests repayment

The principal installments calculated for consecutive periods $i = T + 1$, $T + 2, \dots, N$ in case of the loan (credit) lent on the market terms with the grace period for capital and interests repayment equal:

$$K = \frac{S(1+r)^T}{N-T}. \tag{6}$$

The principal installments calculated for consecutive periods $i = T + 1$, $T + 2, \dots, N$ in case of analogical preferential loan (credit) with the grace period for capital and interests repayment equal:

$$K = \frac{S(1+r_p)^T}{N-T}. \tag{7}$$

Interests on the loan (credit) lent on the market terms with the grace period for capital and interests repayment and analogical preferential loan (credit) with the grace period for capital and interests repayment are presented in table 5.

Table 5

Interests on the loan (credit) lent on the market terms and preferential loan (credit) with the grace period for capital and interests repayment

Consecutive payment period	Interests on the loan (credit) lent on the market terms with the grace period for capital and interests repayment	Interests on the preferential loan (credit) with the grace period for capital and interests repayment
1, 2, ..., T	0	0
Total for i from 1 to T	0	0
$T + 1$	$D_{T+1} = S(1+r)^T r$	$D_{T+1} = S(1+r_p)^T r_p$
$T + 2$	$D_{T+2} = \left(S - \frac{S}{N-T}\right)(1+r)^T r$	$D_{T+2} = \left(S - \frac{S}{N-T}\right)(1+r_p)^T r_p$
...	...	...
N	$D_N = \left[S - (N-T-1)\frac{S}{N-T}\right](1+r)^T r$	$D_N = \left[S - (N-T-1)\frac{S}{N-T}\right](1+r_p)^T r_p$
Total for i from $T + 1$ to N	$\sum_{i=T+1}^{N} D_i = \sum_{i=T+1}^{N}\left[S - (i-T-1)\frac{S}{N-T}\right](1+r)^T r$	$\sum_{i=T+1}^{N} D_i = \sum_{i=T+1}^{N}\left[S - (i-T-1)\frac{S}{N-T}\right](1+r_p)^T r_p$

Source: own compilation.

Discounted value of the interests on the analogical loan (credit) lent on the market terms with the grace period for capital and interests repayment and preferential loan (credit) with the grace period for capital and interests repayment is presented in the table 6.

Table 6

Discounted value of the interests on the loan lent on the market and preferential loan with the grace period for capital and interests repayment

Consecutive payment period	Discounted value of interests on the loan (credit) lent on the market terms with the grace period for capital and interests repayment	Discounted value of interests on the preferential loan (credit) with the grace period for capital and interests repayment
1, 2, ..., T	0	0
Total for i from 1 to T	0	0
T + 1	$S(1+r)^T r \frac{1}{(1+r)^{T+1}}$	$S(1+r_p)^T r_p \frac{1}{(1+r)^{T+1}}$
T + 2	$\left(S-\frac{S}{N-T}\right)(1+r)^T r \frac{1}{(1+r)^{T+2}}$	$\left(S-\frac{S}{N-T}\right)(1+r_p)^T r_p \frac{1}{(1+r)^{T+2}}$
...	...	...
N	$\left[S-(N-T-1)\frac{S}{N-T}\right](1+r)^T r \frac{1}{(1+r)^N}$	$\left[S-(N-T-1)\frac{S}{N-T}\right](1+r_p)^T r_p \frac{1}{(1+r)^N}$
Total for i from T + 1 to N	$\sum_{i=T+1}^{N}\left[S-(i-T-1)\frac{S}{N-T}\right](1+r)^T r \frac{1}{(1+r)^i}$	$\sum_{i=T+1}^{N}\left[S-(i-T-1)\frac{S}{N-T}\right](1+r_p)^T r_p \frac{1}{(1+r)^i}$

Source: own compilation.

Using the above formulas the gross subsidy equivalent for the loan or credit repaid in the equal principal installments system with the grace period for capital and interests repayment may be calculated as follows:

$$EDB = \sum_{i=T+1}^{N} S(1+r)^T\left[1-\frac{i-(T+1)}{N-T}\right] r \frac{1}{(1+r)^i} - \sum_{i=T+1}^{N} S(1+r_p)^T\left[1-\frac{i-(T+1)}{N-T}\right] r_p \frac{1}{(1+r)^i}. \qquad (8)$$

The analysis of the interests value paid in case of preferential credit as well as the credit received on the market terms is made above. Further, one tried to answer the question of the value of the principal installments for the preferential credit in comparison with the market terms credit.

Credit installments for credit or loan repaid in the equal installments system

Principal installments for the loan (credit) lent on the market terms and analogical preferential loan (credit) are presented in the table 7.

Table 7

Principal installments value for the credit lent on market terms and for preferential credit

Consecutive payment period	Value of principal installments for the (loan) credit lent on market terms	Value of principal installments for the preferential (loan) credit
1	K_1	K_1
2	$K_2 = (1+r)K_1$	$K_2 = (1+r_p)K_1$
...	...	...
N	$K_N = (1+r)K_{N-1} = (1+r)^{N-1}K_1$	$K_N = (1+r_p)K_{N-1} = (1+r_p)^{N-1}K_1$
Total for i from 1 to N	$\sum_{i=1}^{N} K_i = \sum_{i=1}^{N}(1+r)^{i-1}K_1 = S$	$\sum_{i=1}^{N} K_i = \sum_{i=1}^{N}(1+r_p)^{i-1}K_1 = S$

Source: own compilation.

The total of principal installments equals the amount of the credit (loan). In case of the analogical loan (credit) granted on market terms such relation is presented by the equation below:

$$K_1 + (1+r)K_1 + (1+r)^2 K_1 + (1+r)^3 K_1 + ... + (1+r)^{N-1} K_1 = S\,. \quad (9)$$

The total of such geometric sequence can be expressed as follows:

$$K_1 \frac{(1+r)^N - 1}{(1+r) - 1} = S\,. \quad (10)$$

Therefore, the result is following:

$$K_1 = S\frac{r}{(1+r)^N - 1}. \tag{11}$$

Similarly, in case of the preferential loan (credit) one may receive:

$$K_1 = S\frac{r_p}{(1+r_p)^N - 1}. \tag{12}$$

The principal installments for the loan (credit) lent on the market terms and for analogical preferential loan (credit) were presented, using the formulas above, in the table 8.

Table 8

Principal installments value for the credit lent on market terms and for preferential credit

Consecutive payment period	Value of principal installments for the (loan) credit lent on market terms	Value of principal installments for the preferential (loan) credit
1	$K_1 = S\frac{r}{(1+r)^N - 1}$	$K_1 = S\frac{r_p}{(1+r_p)^N - 1}$
2	$K_2 = S\frac{r(1+r)}{(1+r)^N - 1}$	$K_2 = S\frac{r(1+r_p)}{(1+r_p)^N - 1}$
...	...	...
N	$K_N = S\frac{r(1+r)^{N-1}}{(1+r)^N - 1}$	$K_N = S\frac{r(1+r_p)^{N-1}}{(1+r_p)^N - 1}$
Total for i from 1 to N	$\sum_{i=1}^{N} K_i = K_1\frac{(1+r)^N - 1}{r} = S$	$\sum_{i=1}^{N} K_i = K_1\frac{(1+r_p)^N - 1}{r_p} = S$

Source: own compilation.

The interests for the consecutive payment periods i will equal: $R - K_i$ ($i = 1, 2, ..., N$). In order to calculate these interests, one firstly needs to calculate the total amount of the installment. In case of the loan (credit) lent on the market terms it will equal:

$$R = D_1 + K_1 \tag{13}$$

$$R = Sr + S\frac{r}{(1+r)^N - 1} \tag{14}$$

$$R = Sr\frac{(1+r)^N}{(1+r)^N - 1}. \tag{15}$$

It is similar for the preferential loan. The total installment will equal:

$$R = Sr_p\frac{(1+r_p)^N}{(1+r_p)^N - 1}. \tag{16}$$

The interests on the loan (credit) lent on the market terms as well as the preferential loan (credit) are presented in the table 9.

Table 9

The interests on the loan (credit) lent on the market term and the preferential loan (credit)

Consecutive payment period	Value of interests for the (loan) credit lent on market terms	Value of interests for the preferential (loan) credit
1	$D_1 = R - K_1 = Sr\left[\frac{(1+r)^N - 1}{(1+r)^N - 1}\right]$	$D_1 = R - K_1 = Sr_p\left[\frac{(1+r_p)^N - 1}{(1+r_p)^N - 1}\right]$
2	$D_2 = R - K_2 = Sr\left[\frac{(1+r)^N - (1+r)}{(1+r)^N - 1}\right]$	$D_2 = R - K_2 = Sr_p\left[\frac{(1+r_p)^N - (1+r_p)}{(1+r_p)^N - 1}\right]$
...	...	...
N	$D_N = R - K_N = Sr\left[\frac{(1+r)^N - (1+r)^{N-1}}{(1+r)^N - 1}\right]$	$D_N = R - K_N = Sr_p\left[\frac{(1+r_p)^N - (1+r_p)^{N-1}}{(1+r_p)^N - 1}\right]$
Total for i From 1 to N	$\sum_{i=1}^{N} D_i = \sum_{i=1}^{N} Sr\left[\frac{(1+r)^N - (1+r)^{i-1}}{(1+r)^N - 1}\right]$	$\sum_{i=1}^{N} D_i = \sum_{i=1}^{N} Sr_p\left[\frac{(1+r_p)^N - (1+r_p)^{i-1}}{(1+r_p)^N - 1}\right]$

Source: own compilation.

The discounted value of interest on the analogical loan (credit) lent on the market terms as well as the preferential loan (credit) is presented in the table 10.

Table 10

The discounted value of interest on the analogical loan (credit) lent on the market terms as well as the preferential loan (credit)

Consecutive payment period	Discounted value of interests for the (loan) credit lent on market terms	Discounted value of interests for the preferential (loan) credit
1	$Sr\left[\frac{(1+r)^N-1}{(1+r)^N-1}\right]\frac{1}{(1+r)^1}$	$Sr_p\left[\frac{(1+r_p)^N-1}{(1+r_p)^N-1}\right]\frac{1}{(1+r)^1}$
2	$Sr\left[\frac{(1+r)^N-(1+r)}{(1+r)^N-1}\right]\frac{1}{(1+r)^2}$	$Sr_p\left[\frac{(1+r_p)^N-(1+r_p)}{(1+r_p)^N-1}\right]\frac{1}{(1+r)^2}$
...	...	...
N	$Sr\left[\frac{(1+r)^N-(1+r)^{N-1}}{(1+r)^N-1}\right]\frac{1}{(1+r)^N}$	$Sr_p\left[\frac{(1+r_p)^N-(1+r_p)^{N-1}}{(1+r_p)^N-1}\right]\frac{1}{(1+r)^N}$
Total for i from 1 to N	$\sum_{i=1}^{N}Sr\left[\frac{(1+r)^N-(1+r)^{i-1}}{(1+r)^N-1}\right]\frac{1}{(1+r)^i}$	$\sum_{i=1}^{N}Sr_p\left[\frac{(1+r_p)^N-(1+r_p)^{i-1}}{(1+r_p)^N-1}\right]\frac{1}{(1+r)^i}$

Source: own compilation.

Using the above formulas the gross subsidy equivalent for the loan or credit repaid in the equal installments system may be calculated as follows:

$$EDB=\sum_{i=1}^{N}S\left[\frac{(1+r)^N-(1+r)^{i-1}}{(1+r)^N-1}\right]r\frac{1}{(1+r)^i}-\sum_{i=1}^{N}S\left[\frac{(1+r_p)^N-(1+r_p)^{i-1}}{(1+r_p)^N-1}\right]r_p\frac{1}{(1+r)^i}. \quad (17)$$

Principal credit installments for credit repaid in the equal installments system with the grace period for the capital repayment

Principal installments for the loan (credit) lent on the market terms with the grace period for the capital repayment as well as analogical preferential loan (credit) with the grace period for the capital repayment are presented in the table 11.

Table 11

Principal installments value for the credit lent on market terms
and for preferential credit with the grace period for the capital repayment

Consecutive payment period	Value of principal installments for the (loan) credit lent on market terms with the grace period for the capital repayment	Value of principal installments for the preferential (loan) credit with the grace period for the capital repayment
1, 2, ..., T	0	0
Total for i from 1 to T	0	0
$T + 1$	K_{T+1}	K_{T+1}
$T + 2$	$K_{T+2} = (1+r)K_{T+1}$	$K_{T+2} = (1+r_p)K_{T+1}$
...	...	...
N	$K_N = (1+r)K_{N-1} = (1+r)^{N-T-1} K_{T+1}$	$K_N = (1+r_p)K_{N-1} = (1+r_p)^{N-T-1} K_{T+1}$
Total for i from $T + 1$ to N	$\sum_{i=T+1}^{N} K_i = \sum_{i=T+1}^{N} (1+r)^{i-T-1} K_{T+1} = S$	$\sum_{i=T+1}^{N} K_i = \sum_{i=T+1}^{N} (1+r_p)^{i-T-1} K_{T+1} = S$

Source: own compilation.

The total of principal installments equals the amount of the credit (loan). In case of the analogical loan (credit) granted on market terms with the grace period for the capital repayment, such relation is presented by the equation below:

$$K_{T+1} + (1+r)K_{T+1} + (1+r)^2 K_{T+1} + (1+r)^3 K_{T+1} + \dots + (1+r)^{N-T-1} K_{T+1} = S \,. \tag{18}$$

The total of such geometric sequence can be expressed as follows:

$$K_{T+1} \frac{(1+r)^{N-T} - 1}{(1+r) - 1} = S \,. \tag{19}$$

Therefore, the result is following

$$K_{T+1} = S \frac{r}{(1+r)^{N-T} - 1} \,. \tag{20}$$

Similarly, in case of the preferential loan (credit) with the grace period for the capital repayment, one may receive:

$$K_{T+1} = S\frac{r_p}{(1+r_p)^{N-T}-1}. \tag{21}$$

Using the above formulas one can calculate the principal installments for analogical loan (credit) lent on the market terms with the grace period for the capital repayment as well as the preferential loan (credit) with the grace period for the capital repayment, which is shown in the table 12.

Table 12

Principal installments value for the credit lent on market terms and for preferential credit with the grace period for the capital repayment

Consecutive payment period	Value of principal installments for the (loan) credit lent on market terms with the grace period for the capital repayment	Value of principal installments for the preferential (loan) credit with the grace period for the capital repayment
1, 2, ..., T	0	0
Total for i from 1 to T	0	0
$T+1$	$K_{T+1} = S\frac{r}{(1+r)^{N-T}-1}$	$K_{T+1} = S\frac{r_p}{(1+r_p)^{N-T}-1}$
$T+2$	$K_{T+2} = S\frac{r(1+r)}{(1+r)^{N-T}-1}$	$K_{T+2} = S\frac{r_p(1+r_p)}{(1+r_p)^{N-T}-1}$
...	...	...
N	$K_N = S\frac{r(1+r)^{N-T-1}}{(1+r)^{N-T}-1}$	$K_N = S\frac{r_p(1+r_p)^{N-T-1}}{(1+r_p)^{N-T}-1}$
Total for i from $T+1$ to N	$\sum_{i=T+1}^{N} K_i = K_{T+1}\frac{(1+r)^{N-T}-1}{r} = S$	$\sum_{i=T+1}^{N} K_i = K_{T+1}\frac{(1+r_p)^{N-T}-1}{r_p} = S$

Source: own compilation.

The interests for the consecutive payment periods i will equal: Sr (for $i = 1, 2, ..., T$) and $R - K_i$ ($i = T + 1, T + 2, ..., N$). In order to calculate the interests for periods $i = T + 1, T + 2, \dots, N$, one needs to calculate the amount of the installment all together at first. In case of the analogical loan (credit) lent on the market terms with the grace period for the capital repayment it will equal:

$$R = D_{T+1} + K_{T+1} \tag{22}$$

$$R = Sr + S\frac{r}{(1+r)^{N-T} - 1} \tag{23}$$

$$R = Sr\frac{(1+r)^{N-T}}{(1+r)^{N-T} - 1}. \tag{24}$$

Similarly, in case of the preferential loan (credit) with the grace period for the capital repayment, the total installment will equal:

$$R = Sr\frac{(1+r_p)^{N-T}}{(1+r_p)^{N-T} - 1}. \tag{25}$$

The interests on the analogical loan (credit) lent on the market terms with the grace period for the capital repayment as well as the preferential loan (credit) with the grace period for the capital repayment are presented in the table 13.

Table 13

The interests on the loan (credit) lent on the market terms and the preferential loan (credit) with the grace period for the capital repayment

Consecutive payment period	The interests on the (loan) credit lent on market terms with the grace period for the capital repayment	The interests on the preferential (loan) credit with the grace period for the capital repayment
1	2	3
1	$D_1 = Sr$	$D_1 = Sr_p$
2	$D_2 = Sr$	$D_2 = Sr_p$
...	...	...
T	$D_T = Sr$	$D_T = Sr_p$

1	2	3
Total for *i* form 1 to *T*	$\sum_{i=1}^{T} D_i = TSr$	$\sum_{i=1}^{T} D_i = TSr_p$
$T+1$	$D_{T+1} = Sr\left[\frac{(1+r)^{N-T}-1}{(1+r)^{N-T}-1}\right]$	$D_{T+1} = Sr_p\left[\frac{(1+r_p)^{N-T}-1}{(1+r_p)^{N-T}-1}\right]$
$T+2$	$D_{T+2} = Sr\left[\frac{(1+r)^{N-T}-(1+r)}{(1+r)^{N-T}-1}\right]$	$D_{T+2} = Sr_p\left[\frac{(1+r_p)^{N-T}-(1+r_p)}{(1+r_p)^{N-T}-1}\right]$
…	…	…
N	$D_N = Sr\left[\frac{(1+r)^{N-T}-(1+r)^{N-T-1}}{(1+r)^{N-T}-1}\right]$	$D_N = Sr_p\left[\frac{(1+r_p)^{N-T}-(1+r_p)^{N-T-1}}{(1+r_p)^{N-T}-1}\right]$
Total for *i* from $T+1$ to N	$\sum_{i=T+1}^{N} D_i = \sum_{i=T+1}^{N} Sr\left[\frac{(1+r)^{N-T}-(1+r)^{i-T-1}}{(1+r)^{N-T}-1}\right]$	$\sum_{i=T+1}^{N} D_i = \sum_{i=T+1}^{N} Sr_p\left[\frac{(1+r_p)^{N-T}-(1+r_p)^{i-T-1}}{(1+r_p)^{N-T}-1}\right]$

Source: own compilation.

In turn, the discounted value of the interests on the analogical loan (credit) lent on the market terms with the grace period for capital repayment and the preferential loan (credit) with the grace period for capital repayment is presented in the table 14.

Table 14

Discounted value of the interests on the loan (credit) lent on the market terms and preferential loan (credit) with the grace period for capital repayment

Consecutive payment period	Discounted value of interests for the (loan) credit lent on market terms with the grace period for the capital repayment	Discounted value of interests for the preferential (loan) credit with the grace period for the capital repayment
1	2	3
1	$Sr\,\frac{1}{(1+r)^1}$	$Sr_p\,\frac{1}{(1+r)^1}$
2	$Sr\,\frac{1}{(1+r)^2}$	$Sr_p\,\frac{1}{(1+r)^2}$
…	…	…
T	$Sr\,\frac{1}{(1+r)^T}$	$Sr_p\,\frac{1}{(1+r)^T}$
Total for *i* from 1 to *T*	$\sum_{i=1}^{T} Sr\,\frac{1}{(1+r)^i}$	$\sum_{i=1}^{T} Sr_p\,\frac{1}{(1+r)^i}$

1	2	3
$T+1$	$Sr\left[\frac{(1+r)^{N-T}-1}{(1+r)^{N-T}-1}\right]\frac{1}{(1+r)^{T+1}}$	$Sr_p\left[\frac{(1+r_p)^{N-T}-1}{(1+r_p)^{N-T}-1}\right]\frac{1}{(1+r)^{T+1}}$
$T+2$	$Sr\left[\frac{(1+r)^{N-T}-(1+r)}{(1+r)^{N-T}-1}\right]\frac{1}{(1+r)^{T+2}}$	$Sr_p\left[\frac{(1+r_p)^{N-T}-(1+r_p)}{(1+r_p)^{N-T}-1}\right]\frac{1}{(1+r)^{T+2}}$
...	...	...
N	$Sr\left[\frac{(1+r)^{N-T}-(1+r)^{N-T-1}}{(1+r)^{N-T}-1}\right]\frac{1}{(1+r)^{N}}$	$Sr_p\left[\frac{(1+r_p)^{N-T}-(1+r_p)^{N-T-1}}{(1+r_p)^{N-T}-1}\right]\frac{1}{(1+r)^{N}}$
Total for i from $T+1$ to N	$\sum_{i=T+1}^{N} Sr\left[\frac{(1+r)^{N-T}-(1+r)^{i-T-1}}{(1+r)^{N-T}-1}\right]\frac{1}{(1+r)^{i}}$	$\sum_{i=T+1}^{N} Sr_p\left[\frac{(1+r_p)^{N-T}-(1+r_p)^{i-T-1}}{(1+r_p)^{N-T}-1}\right]\frac{1}{(1+r)^{i}}$

Source: own compilation.

Using the above formulas the gross subsidy equivalent for the loan or credit repaid in the equal installments system with the grace period for the capital repayment may be calculated as follows:

$$EDB = \sum_{i=1}^{T} Sr\frac{1}{(1+r)^i} + \sum_{i=T+1}^{N} S\left[\frac{(1+r)^{N-T}-(1+r)^{i-(T+1)}}{(1+r)^{N-T}-1}\right]r\frac{1}{(1+r)^i} - \sum_{i=1}^{T} Sr_p\frac{1}{(1+r)^i} - \sum_{i=T+1}^{N} S\left[\frac{(1+r_p)^{N-T}-(1+r_p)^{i-(T+1)}}{(1+r_p)^{N-T}-1}\right]r_p\frac{1}{(1+r)^i}. \tag{26}$$

Principal credit installments for credit repaid in the equal installments system with the grace period for the capital and interests repayment

Principal installments for the analogical loan (credit) lent on the market terms with the grace period for the capital and interests repayment, as well as the preferential loan (credit) with the grace period for the capital and interests repayment are presented in the table 15.

Table 15

Principal installments value for the credit lent on market terms
and for preferential credit with the grace period for the capital and interests repayment

Consecutive payment period	Value of principal installments for the (loan) credit lent on market terms with the grace period for the capital and interests repayment	Value of principal installments for the preferential (loan) credit with the grace period for the capital and interests repayment
1, 2, ..., T	0	0
Total for i from 1 to T	0	0
$T + 1$	K_{T+1}	K_{T+1}
$T + 2$	$K_{T+2} = (1+r)K_{T+1}$	$K_{T+2} = (1+r_p)K_{T+1}$
...	...	...
N	$K_N = (1+r)K_{N-1} = (1+r)^{N-T-1}K_{T+1}$	$K_N = (1+r_p)K_{N-1} = (1+r_p)^{N-T-1}K_{T+1}$
Total for i from $T + 1$ to N	$\sum_{i=T+1}^{N} K_i = \sum_{i=T+1}^{N}(1+r)^{i-T-1}K_{T+1} = S(1+r)^T$	$\sum_{i=T+1}^{N} K_i = \sum_{i=T+1}^{N}(1+r_p)^{i-T-1}K_{T+1} = S(1+r_p)^T$

Source: own compilation.

The total of principal installments equals the amount of the credit (loan) multiplied by the factor $(1 + r)^T$. In case of the analogical loan (credit) granted on market terms with the grace period for the capital and interests repayment, such relation is presented by the equation below:

$$K_{T+1} + (1+r)K_{T+1} + (1+r)^2 K_{T+1} + (1+r)^3 K_{T+1} + ... + (1+r)^{N-T-1} K_{T+1} = S(1+r)^T . \quad (27)$$

The total of such geometric sequence can be expressed as follows:

$$K_{T+1}\frac{(1+r)^{N-T}-1}{(1+r)-1} = S(1+r)^T . \quad (28)$$

Therefore, the result is following

$$K_{T+1} = S(1+r)^T \frac{r}{(1+r)^{N-T}-1} . \quad (29)$$

Similarly, in case of the preferential loan (credit) with the grace period for the capital and interests repayment one may receive:

$$K_{T+1} = S(1+r_p)^T \frac{r_p}{(1+r_p)^{N-T} - 1}. \tag{30}$$

The principal installments for the loan (credit) lent on the market terms with the grace period for the capita and interests repayment as well as for the preferential loan (credit) with the grace period for the capita and interests repayment were presented, using the formulas above, in the table 16.

Table 16

Principal installments value for the credit lent on market terms
and for preferential credit with the grace period for the capital and interests repayment

Consecutive payment period	Value of principal installments for the (loan) credit lent on market terms with the grace period for the capital and interests repayment	Value of principal installments for the preferential (loan) credit with the grace period for the capital and interests repayment
1, 2, ..., T	0	0
Total for i from 1 to T	0	0
$T + 1$	$K_{T+1} = S(1+r)^T \frac{r}{(1+r)^{N-T} - 1}$	$K_{T+1} = S(1+r_p)^T \frac{r_p}{(1+r_p)^{N-T} - 1}$
$T + 2$	$K_{T+2} = S(1+r)^T \frac{r(1+r)}{(1+r)^{N-T} - 1}$	$K_{T+2} = S(1+r_p)^T \frac{r_p(1+r_p)}{(1+r_p)^{N-T} - 1}$
...	...	...
N	$K_N = S(1+r)^T \frac{r(1+r)^{N-T-1}}{(1+r)^{N-T} - 1}$	$K_N = S(1+r_p)^T \frac{r_p(1+r_p)^{N-T-1}}{(1+r_p)^{N-T} - 1}$
Total for i from $T + 1$ to N	$\sum_{i=T+1}^{N} K_i = K_{T+1} \frac{(1+r)^{N-T} - 1}{r} = S(1+r)^T$	$\sum_{i=T+1}^{N} K_i = K_{T+1} \frac{(1+r_p)^{N-T} - 1}{r_p} = S(1+r_p)^T$

Source: own compilation.

The interests for the consecutive payment periods i will equal: 0 (for $i = 1, 2, ..., T$) and $R - K_i$ ($i = T + 1, T + 2, ..., N$). In order to calculate the interests for periods $i = T + 1, T + 2, ..., N$, one needs to calculate the amount of the installment all together at first. In case of the analogical loan (credit) lent on the market terms with the grace period for the capital and interests repayment it will equal:

$$R = D_{T+1} + K_{T+1} \tag{31}$$

$$R = S(1+r)^T r + S(1+r)^T \frac{r}{(1+r)^{N-T} - 1} \tag{32}$$

$$R = S(1+r)^T r \frac{(1+r)^{N-T}}{(1+r)^{N-T} - 1}. \tag{33}$$

Similarly, in case of the preferential loan (credit) with the grace period for the capital and interests repayment, one may receive:

$$R = S(1+r_p)^T r_p \frac{(1+r_p)^{N-T}}{(1+r_p)^{N-T} - 1}. \tag{34}$$

The interests on the analogical loan (credit) lent on the market terms with the grace period for the capital and interests repayment as well as the preferential loan (credit) with the grace period for the capital and interests repayment are presented in the table 17.

Table 17

The interests on the loan (credit) lent on the market terms and the preferential loan (credit) with the grace period for the capital and interests repayment

Consecutive payment period	The interests on the (loan) credit lent on market terms with the grace period for the capital and interests repayment	The interests on the preferential (loan) credit with the grace period for the capital and interests repayment
1	2	3
1, 2, ..., T	0	0
Total for i from 1 to T	0	0
$T+1$	$D_{T+1} = S(1+r)^T r\left[\frac{(1+r)^{N-T} - 1}{(1+r)^{N-T} - 1}\right]$	$D_{T+1} = S(1+r_p)^T r_p\left[\frac{(1+r_p)^{N-T} - 1}{(1+r_p)^{N-T} - 1}\right]$

1	2	3
$T+2$	$D_{T+2}=S(1+r)^T r\left[\frac{(1+r)^{N-T}-(1+r)}{(1+r)^{N-T}-1}\right]$	$D_{T+2}=S(1+r_p)^T r_p\left[\frac{(1+r_p)^{N-T}-(1+r_p)}{(1+r_p)^{N-T}-1}\right]$
...	...	...
N	$D_N=S(1+r)^T r\left[\frac{(1+r)^{N-T}-(1+r)^{N-T-1}}{(1+r)^{N-T}-1}\right]$	$D_N=S(1+r_p)^T r_p\left[\frac{(1+r_p)^{N-T}-(1+r_p)^{N-T-1}}{(1+r_p)^{N-T}-1}\right]$
Total for i from $T+1$ to N	$\sum_{i=T+1}^{N} S(1+r)^T r\left[\frac{(1+r)^{N-T}-(1+r)^{i-T-1}}{(1+r)^{N-T}-1}\right]$	$\sum_{i=T+1}^{N} S(1+r_p)^T r_p\left[\frac{(1+r_p)^{N-T}-(1+r_p)^{i-T-1}}{(1+r_p)^{N-T}-1}\right]$

Source: own compilation.

The discounted value of the interests on the analogical loan (credit) lent on the market terms with the grace period for capital and interests repayment and the preferential loan (credit) with the grace period for capital and interest repayment is presented in the table 18.

Table 18

Discounted value of the interests on the loan (credit) lent on the market terms and preferential loan (credit) with the grace period for capital and interests repayment

Consecutive payment period	Discounted value of interests for the (loan) credit lent on market terms with the grace period for the capital and interests repayment	Discounted value of interests for the preferential (loan) credit with the grace period for the capital and interests repayment
1, 2, ..., T	0	0
Total for i from 1 to T	0	0
$T+1$	$S(1+r)^T r\left[\frac{(1+r)^{N-T}-1}{(1+r)^{N-T}-1}\right]\frac{1}{(1+r)^{T+1}}$	$S(1+r_p)^T r_p\left[\frac{(1+r_p)^{N-T}-1}{(1+r_p)^{N-T}-1}\right]\frac{1}{(1+r)^{T+1}}$
$T+2$	$S(1+r)^T r\left[\frac{(1+r)^{N-T}-(1+r)}{(1+r)^{N-T}-1}\right]\frac{1}{(1+r)^{T+2}}$	$S(1+r_p)^T r_p\left[\frac{(1+r_p)^{N-T}-(1+r_p)}{(1+r_p)^{N-T}-1}\right]\frac{1}{(1+r)^{T+2}}$
...	...	...
N	$S(1+r)^T r\left[\frac{(1+r)^{N-T}-(1+r)^{N-T-1}}{(1+r)^{N-T}-1}\right]\frac{1}{(1+r)^{N}}$	$S(1+r_p)^T r_p\left[\frac{(1+r_p)^{N-T}-(1+r_p)^{N-T-1}}{(1+r_p)^{N-T}-1}\right]\frac{1}{(1+r)^{N}}$
Total for i from $T+1$ to N	$\sum_{i=T+1}^{N} S(1+r)^T r\left[\frac{(1+r)^{N-T}-(1+r)^{i-T-1}}{(1+r)^{N-T}-1}\right]\frac{1}{(1+r)^{i}}$	$\sum_{i=T+1}^{N} S(1+r_p)^T r_p\left[\frac{(1+r_p)^{N-T}-(1+r_p)^{i-T-1}}{(1+r_p)^{N-T}-1}\right]\frac{1}{(1+r)^{i}}$

Source: own compilation.

Using the above formulas the gross subsidy equivalent for the loan or credit repaid in the equal installments system with the grace period for the capital and interests repayment may be calculated as follows:

$$EDB = \sum_{i=T+1}^{N} S(1+r)^T \left[\frac{(1+r)^{N-T} - (1+r)^{i-(T+1)}}{(1+r)^{N-T} - 1} \right] r \frac{1}{(1+r)^i} - \\ - \sum_{i=T+1}^{N} S(1+r_p)^T \left[\frac{(1+r_p)^{N-T} - (1+r_p)^{i-(T+1)}}{(1+r_p)^{N-T} - 1} \right] r_p \frac{1}{(1+r)^i}. \qquad (35)$$

Conclusion

Summarizing, the amount of the gross subsidy equivalent (EDB), which is the value of financial support that the beneficiary or the entity applying for the state aid in Poland would receive if it was granted a subsidy, equals:

$$EDB = \sum_{i=1}^{N} S \left[\frac{(1+r)^N - (1+r)^{i-1}}{(1+r)^N - 1} \right] r \frac{1}{(1+r)^i} - \sum_{i=1}^{N} S \left[\frac{(1+r_p)^N - (1+r_p)^{i-1}}{(1+r_p)^N - 1} \right] r_p \frac{1}{(1+r)^i}. \qquad (17)$$

for the repayment in the equal principal installments system with the grace period for the capital repayment:

$$EDB = \sum_{i=1}^{T} Sr \frac{1}{(1+r)^i} + \sum_{i=T+1}^{N} S \left[\frac{(1+r)^{N-T} - (1+r)^{i-(T+1)}}{(1+r)^{N-T} - 1} \right] r \frac{1}{(1+r)^i} - \sum_{i=1}^{T} Sr_p \frac{1}{(1+r)^i} - \\ - \sum_{i=T+1}^{N} S \left[\frac{(1+r_p)^{N-T} - (1+r_p)^{i-(T+1)}}{(1+r_p)^{N-T} - 1} \right] r_p \frac{1}{(1+r)^i}. \qquad (26)$$

and for the repayment in the equal principal installments system with the grace period for the capital and interests repayment:

$$EDB = \sum_{i=T+1}^{N} S(1+r)^T \left[\frac{(1+r)^{N-T} - (1+r)^{i-(T+1)}}{(1+r)^{N-T} - 1} \right] r \frac{1}{(1+r)^i} - \\ - \sum_{i=T+1}^{N} S(1+r_p)^T \left[\frac{(1+r_p)^{N-T} - (1+r_p)^{i-(T+1)}}{(1+r_p)^{N-T} - 1} \right] r_p \frac{1}{(1+r)^i}. \qquad (35)$$

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8. Government Order dated 2004-08-11, about detailed method of computation of state aid granted in various ways (Dz. U. Nr 194, pos. 1983).

THE CALCULATION METHOD OF THE STATE AID GRANTED AS A PREFERENTIAL LOAN OR CREDIT

Summary

The rules of computation of state aid granted as a preferential loan or credit in Poland were regulated in Government Order dated 2004-08-11, about the detailed method of computation of state aid granted in various ways (Dz. U. Nr 194, pos. 1983). Preferential loans and credits were introduced as a solution that should help the

beneficiaries, in other words the entities of economic activity, irrespective of its organization, legal form or way of funding.

The aim of this study is to present and explain the algorithms of computation of the amount of the gross subsidy equivalent (EDB), that is the value of financial support that the beneficiary or the entity applying for the state aid in Poland would receive if it was granted a subsidy, excluding the income tax, that was examined for the following cases:

- the equal principal installments repayment,
- the equal principal installments repayment with a grace period for capital repayment,
- the equal principal installments repayment with a grace period for both capital and interest repayment.

The gross subsidy equivalent (EDB) for the preferential loan or credit equals the difference between the discounted value of interests of loan or credit lent on market terms and discounted value of interests of preferential loan or credit.