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Investment model

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INVESTMENT MODEL

Investment Model

Let us have a firm that needs investments to continue its functioning. We also have n investment companies offering theirs investment projects. Let $N_1^0, N_2^0, ..., N_n^0$ be the initial investments of the companies, $\varepsilon_1, \varepsilon_2, ..., \varepsilon_n$ – the increase coefficients of $N_1, N_2, ...N_n$, and $\gamma_1, \gamma_2, ...\gamma_n$ – the loss coefficients. Let us have the loss at the unit time function $F(N_1, N_2, ..., N_n)$, which turns into zero together with the sum $N_1 + N_2 + ... + N_n = 0$.

Then we can represent the change $N_i, i = \overline{1,n}$ in short period of time with the following system of differential equations:

$$\begin{cases} \dfrac{dN_1(t)}{dt} = (\varepsilon_1 - \gamma_1 F(N_1, N_2, ..., N_n))N_1(t) \\ \dfrac{dN_2(t)}{dt} = (\varepsilon_2 - \gamma_2 F(N_1, N_2, ..., N_n))N_2(t) \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \dfrac{dN_n(t)}{dt} = (\varepsilon_n - \gamma_n F(N_1, N_2, ..., N_n))N_n(t) \end{cases} \tag{1}$$

Solving this set of equations we assume, that if $t = t_0$:

$$N_i(t_0) = N_i^0, i = \overline{1,n}. \tag{2}$$

Suppose that one of the investment companies is a big firm with long-term experience. Its activities aren't influenced by the activities of the other companies (the decisions are made independently). Without loss of generality we

might assume it is the first firm. The inner policy and the future prognosis are supposed to be known. Thus the evolution of this company's activities we may present as continuous function or continuous clutching of functions.

Assume, that

$$N_1(t) = f(t), \tag{3}$$

is a piecewise continuous function of time variable t, defined on the interval $t \geq 0$.

The task is to describe the evolution of other companies.

We will solve the system (1) eliminating the loss at the unit time function $F(N_1, N_2, ..., N_n)$. The solutions are sought as the dependence of the i-th project on the 1st.

The solution is obtained as a set:

$$\begin{cases} \dfrac{N_1^{\gamma_2}(t)}{N_2^{\gamma_1}(t)} = C\exp\{(\varepsilon_1\gamma_2 - \varepsilon_2\gamma_1)t\} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \dfrac{N_1^{\gamma_n}(t)}{N_n^{\gamma_1}(t)} = C\exp\{(\varepsilon_1\gamma_n - \varepsilon_n\gamma_1)t\} \end{cases},$$

where C – integration constant.

Using the initial assumptions on the initial amount of the investment of every investment company, we obtain:

$$\begin{cases} \dfrac{N_1^{\gamma_2}(t)}{N_2^{\gamma_1}(t)} = \dfrac{(N_1^0)^{\gamma_2}}{(N_2^0)^{\gamma_1}}\exp\{(\varepsilon_1\gamma_2 - \varepsilon_2\gamma_1)(t - t_0)\} \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \dfrac{N_1^{\gamma_n}(t)}{N_n^{\gamma_1}(t)} = \dfrac{(N_1^0)^{\gamma_n}}{(N_n^0)^{\gamma_1}}\exp\{(\varepsilon_1\gamma_n - \varepsilon_n\gamma_1)(t - t_0)\} \end{cases}. \tag{4}$$

Let's find the form of evolution equation for $N_i, i = \overline{2,n}$:

$$N_i(t) = \left(\frac{N_1(t)}{N_1^0}\right)^{\frac{\gamma_i}{\gamma_1}} N_i^0 \exp\{(\varepsilon_i - \varepsilon_1\frac{\gamma_i}{\gamma_1})(t - t_0)\}.$$

Using the assumption about the appearance of the function $N_1(t)$, we get:

$$N_i(t) = \left(\frac{f(t)}{N_1^0}\right)^{\frac{\gamma_i}{\gamma_1}} N_i^0 \exp\{(\varepsilon_i - \varepsilon_1\frac{\gamma_i}{\gamma_1})(t - t_0)\} \quad i = \overline{2,n}. \tag{5}$$

Hence the solutions of the system of differential equations with the initial assumptions (2) and (3) turns into:

$$\begin{cases} N_1(t) = f(t) \\ N_i(t) = \left(\dfrac{f(t)}{N_1^0}\right)^{\frac{\gamma_i}{\gamma_1}} N_i^0 \exp\{(\varepsilon_i - \varepsilon_1 \dfrac{\gamma_i}{\gamma_1})(t - t_0)\}, i = \overline{2,n}. \end{cases} \tag{6}$$

Knowing these functions we may find terms of repayment for every investment project. All we need is to solve the following system:

$$\begin{cases} N_1^0 = f(t_1^*) \\ N_i^0 = \left(\dfrac{f(t_i^*)}{N_1^0}\right)^{\frac{\gamma_i}{\gamma_1}} N_i^0 \exp\{(\varepsilon_i - \varepsilon_1 \dfrac{\gamma_i}{\gamma_1})(t_i^* - t_0)\}, i = \overline{2,n}, \end{cases}$$

simplified:

$$\begin{cases} N_1^0 = f(t_1^*) \\ \ln f(t_i^*) = \ln N_1^0 + (\varepsilon_1 - \dfrac{\gamma_1}{\gamma_i}\varepsilon_i)(t_i^* - t_0), i = \overline{2,n}. \end{cases} \tag{7}$$

Among all the solutions we choose the smallest non-negative. Then vector $(t_1^*, t_2^*, ..., t_n^*)$, where $t_i^*, i = \overline{1,n}$ - terms of repayment of every project, solves the system. If some of $t_i^*, i = \overline{1,n}$ equals zero, the project is considered irrepayable and we shall not pay any attention to it.

Let us cast aside nearly impossible case, when $\varepsilon_i\gamma_j - \varepsilon_j\gamma_i = 0$, and assume that $\varepsilon_i\gamma_j - \varepsilon_j\gamma_i > 0$, i.e. $\dfrac{\varepsilon_i}{\gamma_i} > \dfrac{\varepsilon_j}{\gamma_j}$, then in accordance with (4):

$$\lim_{t\to\infty} \frac{N_i^{\gamma_j}}{N_j^{\gamma_i}} = +\infty. \tag{8}$$

Since N_i is bounded, N_j approaches zero.

So the preference should be given to the investment company which index ε/γ is the greatest.

Compute the expected profitability of every project. To do this we compute annual profit using evolutionary equations (6).

Assign $W_i(\tau) = N_i(\tau) - N_i(\tau - 1)$, $\tau = \overline{1,T}$, $\tau, T \in \mathrm{Z}$ – profit functions of i-th project in τ-th year. Then the profitability in this year is $r_i(\tau) = \frac{W_i(\tau) - W_i(\tau-1)}{W_i(\tau-1)}$, $\tau = \overline{2,T}$. The expected profitability of the project is:

$$\mathrm{r_i} = \frac{1}{\mathrm{T}} \sum_{\tau=2}^{\mathrm{T}} \mathrm{r_i}(\tau). \tag{9}$$

Stochastic generalization of the model

Suppose that the above described model is influenced by the change of the environment. Let us have m outer factors $\{A_1, A_2, \ldots, A_m\}$ forming divisible group of pairwise independent events;

$P(A_1), P(A_2), \ldots, P(A_m)$ – their probabilities. The following equality holds:

$$\sum_{\mathrm{i}=1}^{\mathrm{m}} \mathrm{P}(\mathrm{A_i}) = 1. \tag{10}$$

Write the system of differential equations, similarly to (1), describing the change of N_i in short time period taking into account the influence of the environment. Denote $N_i(A_k)$ – cash flow of the i-th investment project on condition of the influence of k-th factor. We should mention that the loss and profit coefficients now depend on the state of environment. Denote them $\varepsilon_i(A_k)$, $\gamma_i(A_k)$, $i = \overline{1;n}$, $k = \overline{1;m}$. Assume that the capital inflows of every investment company do not depend on the state of the environment (it is the amount of money defined in advance). Then the system will look like:

$$\begin{cases} \frac{dN_1(t,A_1)}{dt} = (\varepsilon_1(A_1) - \gamma_1(A_1)F(N_1, N_2, \ldots, N_n))N_1(t,A_1) \\ \frac{dN_1(t,A_2)}{dt} = (\varepsilon_1(A_2) - \gamma_1(A_2)F(N_1, N_2, \ldots, N_n))N_1(t,A_2) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \frac{dN_1(t,A_m)}{dt} = (\varepsilon_1(A_m) - \gamma_1(A_m)F(N_1, N_2, \ldots, N_n))N_1(t,A_m) \\ \frac{dN_2(t,A_1)}{dt} = (\varepsilon_2(A_1) - \gamma_2(A_1)F(N_1, N_2, \ldots, N_n))N_2(t,A_1) \\ \frac{dN_2(t,A_2)}{dt} = (\varepsilon_2(A_2) - \gamma_2(A_2)F(N_1, N_2, \ldots, N_n))N_2(t,A_2) \end{cases} \tag{11}$$

$$\begin{cases} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \dfrac{dN_2(t,A_m)}{dt} = (\varepsilon_2(A_m) - \gamma_2(A_m)F(N_1, N_2, \ldots, N_n))N_2(t,A_m) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \dfrac{dN_n(t,A_1)}{dt} = (\varepsilon_n(A_1) - \gamma_n(A_1)F(N_1, N_2, \ldots, N_n))N_n(t,A_1) \\ \dfrac{dN_n(t,A_2)}{dt} = (\varepsilon_n(A_2) - \gamma_n(A_2)F(N_1, N_2, \ldots, N_n))N_n(t,A_2) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \dfrac{dN_n(t,A_m)}{dt} = (\varepsilon_n(A_m) - \gamma_n(A_m)F(N_1, N_2, \ldots, N_n))N_n(t,A_m) \end{cases}$$

and the initial conditions:

$$N_i(t_0, A_k) = N_i^0, i = \overline{1,n}, k = \overline{1,m}. \tag{12}$$

Like in previous case, we solve this system, using the initial conditions, and the assumption about the 1st company's functioning, which evolution function now turns into:

$$N_1(t, A_k) = f(t, A_k), k = \overline{1,m}. \tag{13}$$

The solution:

$$\begin{cases} N_1(t,A_k) = f(t,A_k) \\ N_i(t,A_k) = \left(\dfrac{f(t,A_k)}{N_1^0}\right)^{\frac{\gamma_i(A_k)}{\gamma_1(A_k)}} N_i^0 \exp\{(\varepsilon_i(A_k) - \varepsilon_1(A_k)\dfrac{\gamma_i(A_k)}{\gamma_1(A_k)})(t - t_0)\}, i = \overline{2,n}. \end{cases} \tag{14}$$

The selection of the best investor now depends on the state of environment. So, having the predicted indexes $\varepsilon_i(A_k)$, $\gamma_i(A_k)$, $i = \overline{1;n}$, $k = \overline{1;m}$, we can define most attractive project.

Suppose that the case when $\varepsilon_i(A_k)\gamma_j(A_k) - \varepsilon_j(A_k)\gamma_i(A_k) = 0$, $k = \overline{1;m}$ is impossible, and $\varepsilon_i(A_k)\gamma_j(A_k) - \varepsilon_j(A_k)\gamma_i(A_k) > 0$, which means $\dfrac{\varepsilon_i(A_k)}{\gamma_i(A_k)} > \dfrac{\varepsilon_j(A_k)}{\gamma_j(A_k)}$, $k = \overline{1;m}$, then in response to (14) we have

$$\lim_{t \to \infty} \frac{N_i^{\gamma_j}(A_k)}{N_j^{\gamma_i}(A_k)} = +\infty, \quad k = \overline{1;m}. \tag{15}$$

As N_i is bounded, then N_j follows zero.

So when the *k*-th factor influences, we should prefer the investment company which index $\dfrac{\varepsilon(A_k)}{\gamma(A_k)}$ is the largest.

Now we will realize the outer factor averaging.

$$\begin{cases} \overline{N_1(t)} = \sum_{k=1}^{m} P(A_k) f(t, A_k) \\ \overline{N_i(t)} = \sum_{k=1}^{m} P(A_k) \left(\frac{f(t, A_k)}{N_1^0} \right)^{\frac{\gamma_i(A_k)}{\gamma_1(A_k)}} N_i^0 \exp\{(\varepsilon_i(A_k) - \varepsilon_1(A_k) \frac{\gamma_i(A_k)}{\gamma_1(A_k)})(t - t_0)\}, i = \overline{2,n}. \end{cases} \tag{16}$$

Denote

$$\overline{\varepsilon_i} = \sum_{k=1}^{m} P(A_k) \varepsilon_i(A_k), \quad \overline{\gamma_i} = \sum_{k=1}^{m} P(A_k) \gamma_i(A_k), \ i = \overline{1,n}. \tag{17}$$

Substitute the coefficients of increases and losses in (1) for their average indexes.

$$\begin{cases} \frac{dN_1(t)}{dt} = (\overline{\varepsilon_1} - \overline{\gamma_1} F(N_1, N_2, ..., N_n)) N_1(t) \\ \frac{dN_2(t)}{dt} = (\overline{\varepsilon_2} - \overline{\gamma_2} F(N_1, N_2, ..., N_n)) N_2(t) \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \frac{dN_n(t)}{dt} = (\overline{\varepsilon_n} - \overline{\gamma_n} F(N_1, N_2, ..., N_n)) N_n(t) \end{cases} \tag{18}$$

Solving this system with the initial conditions (2) and assumption (3), we obtain:

$$\begin{cases} N_1(t) = f(t) \\ N_i(t) = \left(\frac{f(t)}{N_1^0} \right)^{\frac{\overline{\gamma_i}}{\overline{\gamma_1}}} N_i^0 \exp\{(\overline{\varepsilon_i} - \overline{\varepsilon_1} \frac{\overline{\gamma_i}}{\overline{\gamma_1}})(t - t_0)\}, i = \overline{2,n}. \end{cases} \tag{19}$$

The natural question is arising: when the average solutions converge to the solutions of averaged system? Graphical analysis of the concrete cases showed that only the first project's function given in system (16) converges to the respective function of the system (19). For the other projects there is no convergence. One might suppose that this convergence is possible under some conditions on the increase coefficients $\varepsilon_1, \varepsilon_2, ..., \varepsilon_n$ and loss coefficients $\gamma_1, \gamma_2, ...\gamma_n$. Theoretical provement of this convergence remains open.

Similarly to the previous case, one may compute expected profitability of every project. To do it we compute annual incomes for every outer factor.

Let $W_i(\tau, A_k) = N_i(\tau, A_k) - N_i(\tau - 1, A_k)$, $\tau = \overline{1,T}$, $\tau, T \in Z, k = \overline{1,m}$ be the profit of i-th project in k-th year within the state of the environment A_k. Then

the profitability is $r_i(\tau, A_k) = \dfrac{W_i(\tau, A_k) - W_i(\tau - 1, A_k)}{W_i(\tau - 1, A_k)}$, $\tau = \overline{2,T}$ and the expected profitability of the project under conditions of the affect of *k*-th factor:

$$r_i(A_k) = \frac{1}{T}\sum_{\tau=2}^{T} r_i(\tau, A_k). \tag{20}$$

We should mark here, that one may compute the expected profitability of the project using this formulas and taking into account that among the numbers $r_i(\tau, A_k)$ there are both coefficients of profitability and loss.

Using the developed model

Example 1.

Consider a simple case, when the evolution of the investment projects is not being affected by the environment. Let us have 10 companies offering their money. The task is to find the most attractive project, even if it is known there is at least one reliable investor among them with good reputation and well thought-out investment policy. First we suppose the amount of capital inflow of every investor is equal. We know indexes ε_i and γ_i, $i = \overline{1,10}$ predicted by the projects.

Table 1.

i	1	2	3	4	5	6	7	8	9	10
N_i^0	1500000	1500000	1500000	1500000	1500000	1500000	1500000	1500000	1500000	1500000
ε_i	0,38	0,28	0,38	0,42	0,3	0,45	0,44	0,5	0,34	0,41
γ_i	0,89	0,85	0,98	0,8	0,92	0,89	0,97	0,95	0,99	0,85

Suppose the evolution of the first investment project is given by the function.

$$N_1(t) = \begin{cases} N_1^0\left(1 + \varepsilon_1 t - \gamma_1 t + \dfrac{(\varepsilon_1 t)^2}{2}\right), t \le t_0 \\ N_1^0(-1 + \gamma_1 t) + N_1^0\left(2 + \varepsilon_1 t_0 - 2\gamma_1 t_0 + \dfrac{(\varepsilon_1 t_0)^2}{2}\right), t \ge t_0 \end{cases}$$

and $t_0 = 7$ years. For further calculations and plot construction we will use package *Mathematica*. Graphically evolution can be shown:

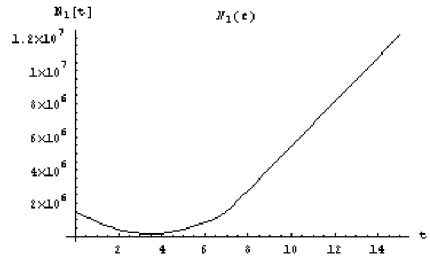

Fig. 1. Evolution in time of $N_1(t)$.

The equation for computing the repayment period of the investment was deduced earlier. We will use the first one. Obtain that this project will repay itself already in 7,036 years.

Substitute the function $N_1(t)$ into equations for other projects:

$$N_i(t)=\begin{cases}(1+\varepsilon_1 t-\gamma_1 t+\frac{(\varepsilon_1 t)^2}{2})^{\frac{\gamma_i}{\gamma_1}} N_2^0 \exp\{(\varepsilon_i-\varepsilon_1\frac{\gamma_i}{\gamma_1})t\}, t\le t_0\\ (1+\gamma_1 t+\varepsilon_1 t_0-2\gamma_1 t_0+\frac{(\varepsilon_1 t_0)^2}{2})^{\frac{\gamma_i}{\gamma_1}} N_2^0 \exp\{(\varepsilon_i-\varepsilon_1\frac{\gamma_i}{\gamma_1})t\}, t\ge t_0\end{cases}$$

for $i=\overline{2,10}$. Construct plots for the rest 9 projects using data from Table 1.

Project 2: $\varepsilon_2=0.28$, $\gamma_2=0.85$.

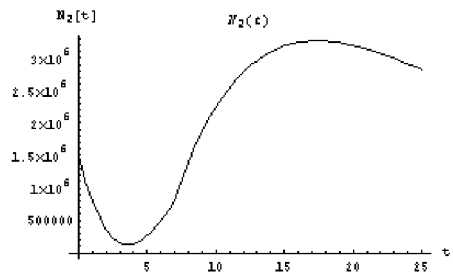

Fig. 2. Evolution in time of $N_2(t)$..

Such project repay itself in 8,13 years, but it is obvious from the plot that there comes the moment, when its profitability starts falling; the project becomes unprofitable.

Project 3: $\varepsilon_3=0.38$, $\gamma_3=0.98$.

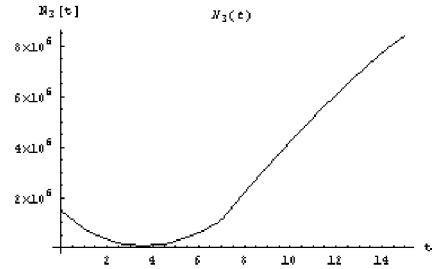

Fig. 3. Evolution in time of $N_3(t)$.

Repayment period: 7,27 years.

Project 4: $\varepsilon_4 = 0.42$, $\gamma_4 = 0.8$.

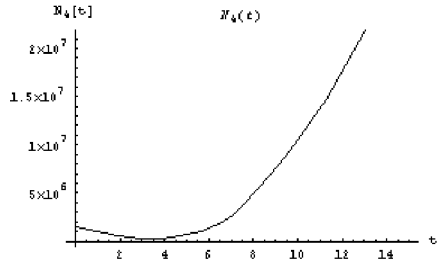

Fig. 4. Evolution in time of $N_4(t)$.

Repayment period: 6,547 years.

Project 5: $\varepsilon_5 = 0.3$, $\gamma_5 = 0.92$.

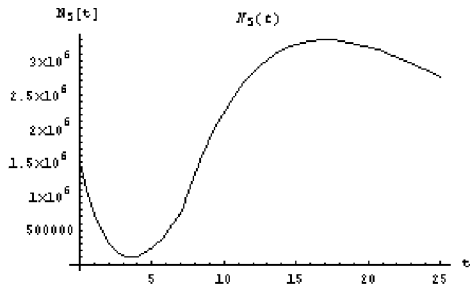

Fig. 5. Evolution in time of $N_5(t)$.

Repayment period: 8,274 years. Profits of such project decrease in time and there comes the moment when it becomes unprofitable.

Project 6: $\varepsilon_6 = 0.45$, $\gamma_6 = 0.89$.

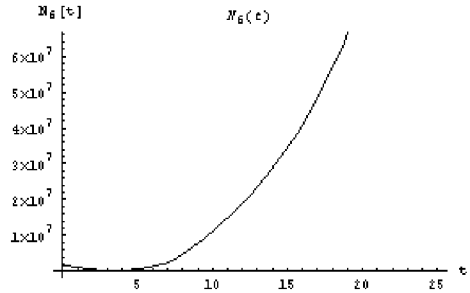

Fig. 6. Evolution in time of $N_6(t)$.

Repayment period: 6,619 years.

Project 8. $\varepsilon_7 = 0.44$, $\gamma_7 = 0.97$.

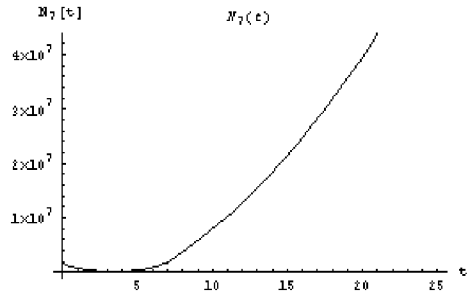

Fig. 7. Evolution in time of $N_7(t)$.

Repayment period: 6,867 years.

Project 8: $\varepsilon_8 = 0.5$, $\gamma_8 = 0.95$.

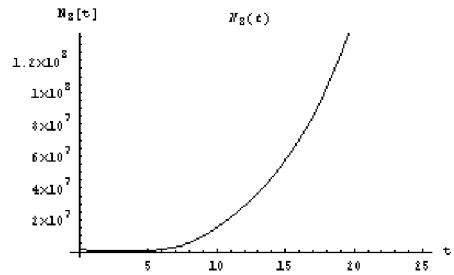

Fig. 8. Evolution in time of $N_8(t)$.

Repayment period: 6,543 years.

Project 9: $\varepsilon_9 = 0.34$, $\gamma_9 = 0.99$.

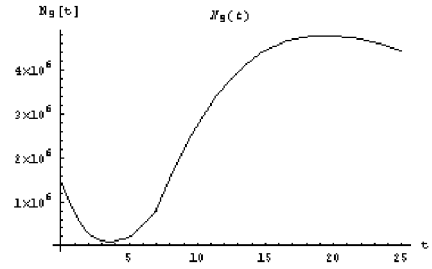

Fig. 9. Evolution in time of $N_9(t)$.

Repayment period: 7,94 years. Profits of such project decrease in time and there comes the moment when it becomes unprofitable.

Project 10: $\varepsilon_{10} = 0.41$, $\gamma_{10} = 0.85$.

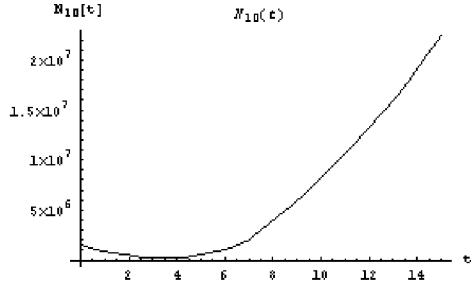

Fig. 10. Evolution in time of $N_{10}(t)$.

Repayment period: 6,719 years.

Let's form a table containing index ε_i/γ_i and repayment period t_i^*, $i = \overline{1,10}$.

Table 2.

i	1	2	3	4	5	6	7	8	9	10
ε_i/γ_i	0,427	0,329	0,388	0,525	0,326	0,506	0,454	0,526	0,343	0,482
t_i^*	7,036	8,13	7,27	6,547	8,274	6,619	6,867	6,543	7,94	6,719

It was shown that we should prefer the project with the largest ε_i/γ_i. That is why in our case we should prefer 4th and 8th projects. Moreover one can see the repayment periods of these projects are the smallest and almost equal. Find the expected profitability of every project. To do this we form a table of values of $N_i(k)$, $i = \overline{1;10}$, $k = \overline{1;25}$ (Table 3). Then we compute annual profit for every project (Table 4) and coefficients of profitability (Table 5).

Table 3. Evolution $N_i(t)$ in years

year	Number of investment project									
	1	2	3	4	5	6	7	8	9	10
0	1500000	1500000	1500000	1500000	1500000	1500000	1500000	1500000	1500000	1500000
1	843300	715427	765599	966795	753781	904446	821720	891478	727722	907129
2	403200	287042	326923	538691	320387	463790	377294	445691	294837	469955
3	179700	106845	129209	281786	126638	221692	160466	206724	110474	227669
4	172800	90375,4	119091	294235	110836	228637	157788	217886	97371,1	229884
5	382500	186143	274901	650049	229665	542793	384949	559205	216959	514676
6	808800	365112	603369	1378260	453900	1230960	893447	1366770	459423	1102990
7	1451700	601218	1105650	2521950	757273	2369630	1734450	2804440	810731	2021330
8	2786700	1063610	2181640	4901980	1354270	4878600	3622900	6182180	1541690	3949740
9	4121700	1424540	3230510	7537520	1849760	7738950	5695680	10317600	2193660	6016730
10	5456700	1695070	4234070	10491100	2253050	10988400	7935530	15298000	2759300	8244870
11	6791700	1888380	5184770	13813900	2574640	14668500	10336900	21236400	3240500	10651400
12	8126700	2017080	6079360	17556300	2824700	18824400	12899000	28265800	3642420	13252100
13	9461700	2092250	6916780	21770500	3012640	23505900	15623300	36539100	3971480	16062700
14	10796700	2124750	7697210	26512700	3146990	28767300	18512700	46231100	4234480	19098900
15	12131700	2122250	8421620	31843800	3235390	34668200	21571200	57539700	4438180	22377400
16	13466700	2092360	9091410	37830300	3284640	41273500	24803500	70689200	4589060	25915400
17	14801700	2041260	9708320	44545000	3300770	48654400	28214800	85933200	4693230	29731000
18	16136700	1974130	10274300	52067900	3289090	56888700	31810800	103558000	4756360	33843300
19	17471700	1895260	10791300	60486800	3254270	66061300	35597600	123885000	4783640	38272600
20	18806700	1808200	11261500	69898000	3200390	76264900	39581800	147279000	4779850	43040200
21	20141700	1715680	11687000	80407500	3131000	87601000	43770300	174150000	4749300	48168700
22	21476700	1620610	12069900	92131900	3049220	100180000	48170400	204957000	4695890	53682200
23	22811700	1524330	12412400	105199000	2957700	114123000	52789500	240219000	4623130	59605900
24	24146700	1428530	12716400	119750000	2858780	129560000	57635700	280519000	4534160	65966700
25	25481700	1334380	12984000	135939000	2754440	146637000	62717100	326512000	4431790	72793300

Using formula (9) compute expected profitability of every project (Table 6).We obtained that only 2nd, 4th, 8th and 10th projects are profitable during first 25 years.

However graphical analysis has shown us that we should throw away 2nd project if we need a long-term project.

Table 4. Profit of every project (yearly).

year	Number of investment project									
	1	2	3	4	5	6	7	8	9	10
1	-656700	-784573	-734401	-533205	-746219	-595554	-678280	-608522	-772278	-592871
2	-440100	-428385	-438676	-428104	-433394	-440656	-444426	-445787	-432885	-437174
3	-223500	-180197	-197714	-256905	-193749	-242098	-216828	-238967	-184363	-242286
4	-6900	-16469,6	-10118	12449	-15802	6945	-2678	11162	-13102,9	2215
5	209700	95767,6	155810	355814	118829	314156	227161	341319	119587,9	284792
6	426300	178969	328468	728211	224235	688167	508498	807565	242464	588314
7	642900	236106	502281	1143690	303373	1138670	841003	1437670	351308	918340
8	1335000	462392	1075990	2380030	596997	2508970	1888450	3377740	730959	1928410
9	1335000	360930	1048870	2635540	495490	2860350	2072780	4135420	651970	2066990
10	1335000	270530	1003560	2953580	403290	3249450	2239850	4980400	565640	2228140
11	1335000	193310	950700	3322800	321590	3680100	2401370	5938400	481200	2406530
12	1335000	128700	894590	3742400	250060	4155900	2562100	7029400	401920	2600700
13	1335000	75170	837420	4214200	187940	4681500	2724300	8273300	329060	2810600
14	1335000	32500	780430	4742200	134350	5261400	2889400	9692000	263000	3036200
15	1335000	-2500	724410	5331100	88400	5900900	3058500	11308600	203700	3278500
16	1335000	-29890	669790	5986500	49250	6605300	3232300	13149500	150880	3538000
17	1335000	-51100	616910	6714700	16130	7380900	3411300	15244000	104170	3815600
18	1335000	-67130	565980	7522900	-11680	8234300	3596000	17624800	63130	4112300
19	1335000	-78870	517000	8418900	-34820	9172600	3786800	20327000	27280	4429300
20	1335000	-87060	470200	9411200	-53880	10203600	3984200	23394000	-3790	4767600
21	1335000	-92520	425500	10509500	-69390	11336100	4188500	26871000	-30550	5128500
22	1335000	-95070	382900	11724400	-81780	12579000	4400100	30807000	-53410	5513500
23	1335000	-96280	342500	13067100	-91520	13943000	4619100	35262000	-72760	5923700
24	1335000	-95800	304000	14551000	-98920	15437000	4846200	40300000	-88970	6360800
25	1335000	-94150	267600	16189000	-104340	17077000	5081400	45993000	-102370	6826600

Example 2.

Now consider a case with the influence of environment on the investment project. Let it be given with the complete set of pairwise independent events $\{A_1, A_2, ..., A_6,\}$ with probabilities $P(A_k), \mathrm{k} = \overline{1,6}$. We have 3 companies offering their investments. Suppose the amount of capital inflows of every investor is equal regardless of the conditions.

Table 5. Coefficients of profitability for every project (yearly).

year	Number of investment project									
	1	2	3	4	5	6	7	8	9	10
1										
2	-0,32983	-0,45399	-0,40268	-0,19711	-0,41921	-0,26009	-0,34478	-0,26743	-0,43947	-0,26262
3	-0,49216	-0,57936	-0,54929	-0,3999	-0,55295	-0,4506	-0,51212	-0,46394	-0,57411	-0,44579
4	-0,96913	-0,9086	-0,94883	-1,04846	-0,91844	-1,02869	-0,98765	-1,04671	-0,92893	-1,00914
5	-31,3913	-6,81481	-16,3993	27,58173	-8,51987	44,23485	-85,8249	29,57866	-10,1268	127,5743
6	1,032904	0,868784	1,108132	1,046606	0,887039	1,190526	1,238492	1,366012	1,027496	1,065767
7	0,508093	0,319256	0,529163	0,570548	0,352924	0,654642	0,653896	0,780253	0,448908	0,560969
8	1,076528	0,958409	1,142207	1,08101	0,967865	1,203422	1,245474	1,349454	1,080678	1,099887
9	0	-0,21943	-0,0252	0,107356	-0,17003	0,14005	0,097609	0,224316	-0,10806	0,071862
10	0	-0,25046	-0,0432	0,120674	-0,18608	0,136032	0,080602	0,204327	-0,13241	0,077964
11	0	-0,28544	-0,05267	0,125008	-0,20258	0,13253	0,072112	0,192354	-0,14928	0,080062
12	0	-0,33423	-0,05902	0,126279	-0,22243	0,12929	0,066933	0,18372	-0,16475	0,080685
13	0	-0,41593	-0,06391	0,126069	-0,24842	0,126471	0,063307	0,176957	-0,18128	0,080709
14	0	-0,56765	-0,06805	0,125291	-0,28514	0,123871	0,060603	0,171479	-0,20075	0,080268
15	0	-1,07692	-0,07178	0,124183	-0,34202	0,121546	0,058524	0,166797	-0,22548	0,079804
16	0	10,956	-0,0754	0,122939	-0,44287	0,119372	0,056825	0,162788	-0,2593	0,079152
17	0	0,709602	-0,07895	0,12164	-0,67249	0,117421	0,055379	0,159284	-0,30958	0,078462
18	0	0,313699	-0,08256	0,120363	-1,72412	0,115623	0,054144	0,156179	-0,39397	0,07776
19	0	0,174885	-0,08654	0,119103	1,981164	0,11395	0,053059	0,153318	-0,56788	0,077086
20	0	0,103842	-0,09052	0,117866	0,547387	0,1124	0,052128	0,150883	-1,13893	0,076378
21	0	0,062715	-0,09507	0,116701	0,287862	0,11099	0,051278	0,148628	7,060686	0,075698
22	0	0,027562	-0,10012	0,1156	0,178556	0,109641	0,050519	0,146478	0,748282	0,075071
23	0	0,012727	-0,10551	0,114522	0,1191	0,108435	0,049772	0,14461	0,362292	0,074399
24	0	-0,00499	-0,11241	0,11356	0,080857	0,107151	0,049165	0,142873	0,222787	0,073788
25	0	-0,01722	-0,11974	0,11257	0,054792	0,106238	0,048533	0,141266	0,150613	0,07323

Table 6. Expected profitability the project.

Number of investment project									
1	2	3	4	5	6	7	8	9	10
-1,2226	0,103138	-0,67405	1,226566	-0,37796	1,903003	-3,34044	1,364902	-0,19197	5,195829

We know indexes $\varepsilon_i(A_k)$ and $\gamma_i(A_k)$, $i=\overline{1,3}$, $k=\overline{1,6}$ predicted by the projects.

Preliminary data are shown in the table:

Table 7.

к	1	2	3	4	5	6
$P(A_k)$	0,1	0,2	0,15	0,2	0,1	0,25
$\varepsilon_1(A_k)$	0,45	0,3	0,4	0,42	0,37	0,5
$\gamma_1(A_k)$	0,89	0,92	0,97	0,8	0,76	0,95
$\varepsilon_2(A_k)$	0,5	0,43	0,44	0,5	0,28	0,43
$\gamma_2(A_k)$	0,9	0,92	0,97	0,9	0,84	0,9
$\varepsilon_3(A_k)$	0,35	0,45	0,54	0,3	0,48	0,33
$\gamma_3(A_k)$	0,9	0,92	0,97	0,8	0,94	0,87

Let $N_i^0 = 1500000$ monetary units, $i = \overline{1,3}$. Similarly to the case when there is no outer influence suppose that the evolution function for the first investment project is known and given by:

$$N_1(t, A_k) = \begin{cases} N_1^0(1 + \varepsilon_1(A_k)t - \gamma_1(A_k)t + \dfrac{(\varepsilon_1(A_k)t)^2}{2}), t \le t_0 \\ N_1^0(-1 + \gamma_1(A_k)t) + N_1^0(2 + \varepsilon_1(A_k)t_0 - 2\gamma_1(A_k)t_0 + \dfrac{(\varepsilon_1(A_k)t_0)^2}{2}), t \ge t_0 \end{cases}$$

where $k = \overline{1,6}$ and $t_0 = 7$.

We are using *Mathematica* for computations and to build plots of functions. Find the averaged solution for the first project:

$$\overline{N_1}(t) = \begin{cases} 1500 - 721.5t + 130.29t^2, t \le 7 \\ -6532.29 + 1338t, t \ge 7 \end{cases}$$

The plot of this function is Fig. 11.

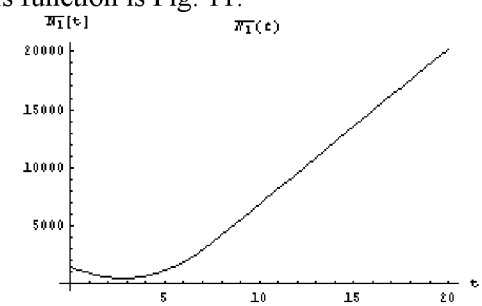

Fig. 11. Evolution in time of $\overline{N_1}(t)$

Substitute $N_1(t, A_k)$ into the solution for second and third project and find their averagings:

$$\overline{N_2}(t) = \begin{cases} 18\,e^{0.04t}(t-4)(t-3.13) + 13.5e^{0.13t}(t-11.91)(t-1.87) + \\ +150\,e^{-0.13t}(1-0.39t+0.07t^2)^{1.12} + 300\,e^{0.03t}(1-0.38t+0.09t^2)^{1.125} + \\ +150\,e^{0.05t}(1-0.44t+0.1t^2)^{1.01} + 375\,e^{-0.04t}(1-0.45t+0.125t^2)^{0.95}, t \le 7 \\ 150\,e^{-0.13t}(-3.7+.8t)^{1.12} + 300\,e^{0.03}(-3+0.8t)^{1.125} + \\ +150\,e^{0.05t}(-3.35+0.89t)^{1.01} + 300\,e^{0.13}(-7.6+0.92t) + \\ +375\,e^{-0.04t}(-2.7+0.95t)^{0.95} + 225\,e^{0.04t}(-5.86+0.97t)^2, t \ge 7 \end{cases}$$

The plot of this function looks like:

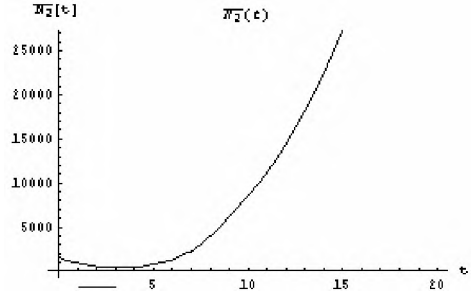

Fig. 12. Evolution in time of $\overline{N_2}(t)$

$$\overline{N_3}(t)=\begin{cases}18e^{0.14t}(t-4)(t-3.125)+13.5e^{0.15t}(t-11.92)(t-1.87)+\\+150e^{0.02t}(1-0.39t+0.07t^2)^{1.24}+300e^{-0.12t}(1-0.38t+0.09t^2)+\\+150e^{-0.1t}(1-0.44t+0.1t^2)^{1.01}+375e^{-0.13t}(1-0.45t+0.125t^2)^{0.92}, t\le 7\\150e^{0.02t}(-3.7+0.76t)^{1.24}+300e^{-0.12t}(0.8t-2.94)+\\150e^{-0.11t}(-3.35+0.89t)^{1.01}+300e^{0.15t}(0.92t-7.575)+\\+375e^{-0.13t}(0.95t-2.675)^{0.92}+225e^{0.14t}(0.97t-5.86), t\ge 7\end{cases}$$

The plot of this function looks like:

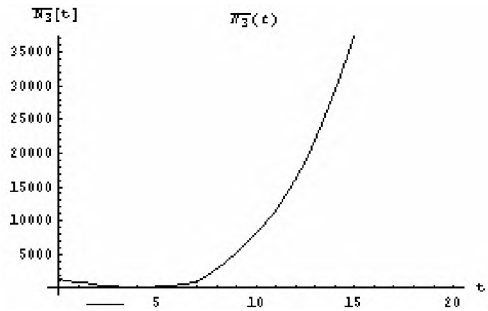

Fig. 13. Evolution in time of $\overline{N_3}(t)$

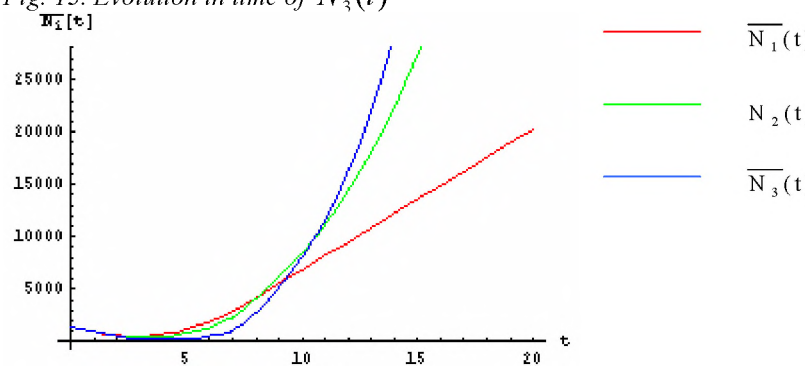

Fig. 14. Evolution in time of the averaged solutions

One can easily see that with such data, basing our decision upon the averaging, we should prefer the first project for short-term projects (to 10 years) and third one for the investment projects lasting over than 10 years.

Compute the index $\frac{\varepsilon_i(A_k)}{\gamma_i(A_k)}$ for $i=\overline{1;3}$, $k=\overline{1;6}$. Obtained results summarized in the table 8. One can see that in every state of the environment there exists most attractive project (in states 1 and 4 – second, in states 2, 3, 5 – third, in the state 6 – first).

Using formulas find the averagings for the coefficients of increase and expenses.

$$\overline{\varepsilon_1}=0.411,\ \overline{\varepsilon_2}=0.4375,\ \overline{\varepsilon_3}=0.3965,$$
$$\overline{\gamma_1}=0.892,\ \overline{\gamma_2}=0.9085,\ \overline{\gamma_3}=0.891.$$

Table 8.

к	1	2	3	4	5	6
$\varepsilon_1(A_k)/\gamma_1(A_k)$	0,506	0,326	0,412	0,525	0,487	0,526
$\varepsilon_2(A_k)/\gamma_2(A_k)$	0,556	0,467	0,454	0,556	0,333	0,478
$\varepsilon_3(A_k)/\gamma_3(A_k)$	0,389	0,489	0,557	0,375	0,511	0,379

Find the solutions of the averaged system (denote them as $\hat{N}_i(t)$):

$$\hat{N}_1(t)=\begin{cases}1500-721.5t+126.69t^2, t\le 7\\ -6708.65+1338t, t\ge 7\end{cases}$$

$$\hat{N}_2(t)=\begin{cases}1500e^{0.189t}(1-0.481t+0.0845t^2)^{1.0185}, t\le 7\\ 1500e^{0.189t}(-4.473+0.892t)^{1.0185}, t\ge 7\end{cases}$$

$$\hat{N}_3(t)=\begin{cases}1500e^{-0.014t}(1-0481t+0.0845t^2)^{0.999}, t\le 7\\ 1500e^{-0.014t}(-4.473+0.892t)^{0.999}, t\ge 7\end{cases}$$

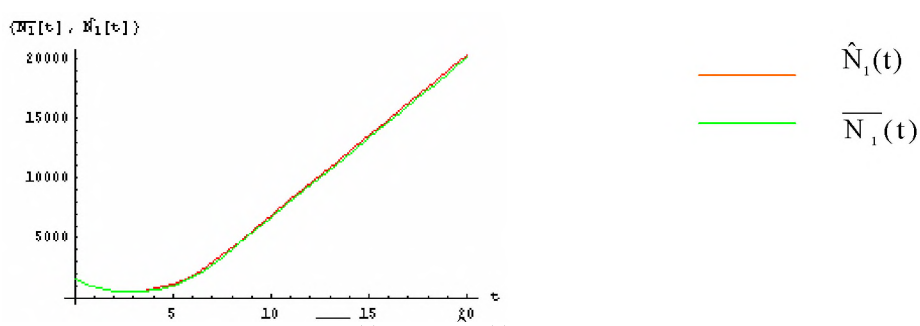

Fig. 15. Evolution in time of $\overline{N}_1(t)$ *and* $\hat{N}_1(t)$

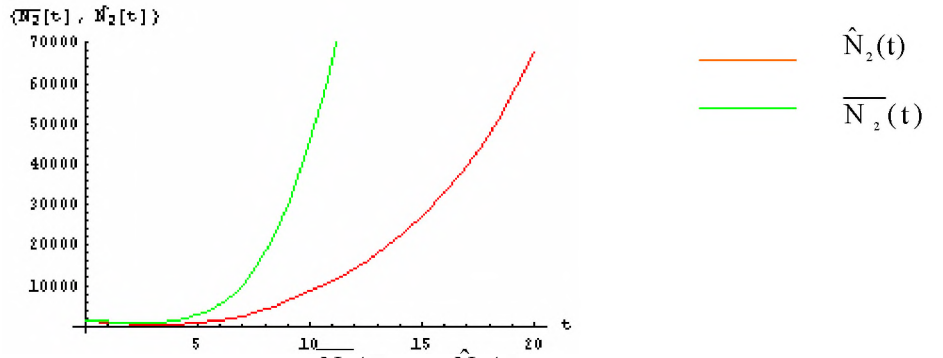

Fig. 16. Evolution in time of $\overline{N}_2(t)$ *and* $\hat{N}_2(t)$

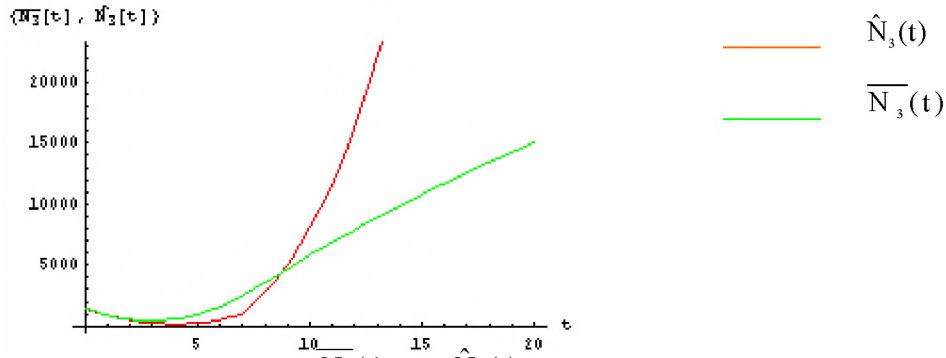

Fig. 17. Evolution in time of $\overline{N}_3(t)$ *and* $\hat{N}_3(t)$

It is easy to see that the averaged solutions converge to the solutions of the averaged system only for the first project; for other projects this converges needs additional conditions on the coefficients of increase and expenses.

SUMMARY

The article presents certain model of choice of investition projects by firm in deterministic and stochastic approach. It is based on system of differential equation. The solution gives tool in decision process. Two numerical implementation of model are given.

MODEL WYBORU PROJEKTÓW INWESTYCYJNYCH

STRESZCZENIE

W artykule przedstawiono pewien model wyboru projektów inwestycyjnych przez firmę w wariancie deterministycznym i stochastycznym. Właściwym modelem tworzy układ równań różniczkowych zwyczajnych. Jego rozwiązanie pozwala na przyjęcie

odpowiedniej decyzji. Zaprezentowano dwa warianty implementacji numerycznej modelu.

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