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Introduction

In general, one of the fundamental objectives of measurement is to determine the value of the measured quantity. We obtain the information necessary for verification, detection and comparison of different phenomena and processes by measurement. Application of different measurements, measuring instruments and methods of measurement enabling to track at the same time a few of dependent variables thanks to sensors may be seen in today's dynamic development of new technologies, microelectronics and computer technology. After reading the measured quantity, it is necessary to transform obtained data and information by using the A/D converters for measuring systems that can monitor and record measured data, subsequently evaluate data and display them by using the programs of your computer. It is convenient to use gauging systems that enable to connect different measuring sensors [Patel: 2012]. For example, among the advantages of the universal measurement system QuantumX MX840 are universal inputs, modular system and flexible use, support TEDS (Transducer Data Sheet Electronic) and high transfer rate [Hoffmann 1989].

One of the possible methods of determination of stress in construction points is to experimentally measure the strain of components in the actual operating conditions another one is to experimentally determine the strain at models. The components of stress are the most often determined by base of physical equations, which express the relationship between stress and the strain. Presented contribution describes the use of the measuring apparatus for the measurement of strain of the circular and circular hollow rod by load torsion moment. In the paper the maximum values of the stress obtained analytically and by deriving from measured deformations are compared. The assembled measuring apparatus should serve for the activation of students within the teaching of subject Experimental methods and technical diagnostics.

Torsion loading

Machine tools and shaping machine, but also parts of the space structures are often loading by torsion. Shafts with circular and annulus section have special statuses among the bodies of different shapes. Moreover they have found the widest application in practice [Trebuňa 2000]. Twisting is characterized by a single non-zero internal variable that is torque. According to the basic knowledge of linear theory of elasticity and strength, it can be said that the individual cross-sections of shafts or rods are loaded only by torsion load, which is given by following equation:

$$\tau(r) = \frac{M_k}{J_p} r, \quad (1)$$

where: $\tau(r)$ = shear stress at a distance r from neutral axis at the given cross-section (MPa), M_k = the size of torque at the given cross-section (N·mm), J_p = Polar Moment of Inertia of an Area (mm^4), r = radial distance from the neutral axis (mm).

Maximum shear stress τ_{max} exits in external fiber of the cross-section (Fig. 1):

$$\tau_{max} = \frac{M_k}{W_k}, \quad (2)$$

where: W_k = section modulus in torsion, $W_k = \frac{J_p}{\frac{D}{2}}$ (mm^3), D = diameter of the circular section (mm).

Section modulus in torsion could by expressed as:

of circular section: $W_k = \frac{\pi D^3}{16}$, (3)

of hollow circular section: $W_k = \frac{\pi (D^4 - d^4)}{16\, D}$, (4)

where D = outside diameter of the cross-section (mm), d = inside diameter of the cross-section (mm).

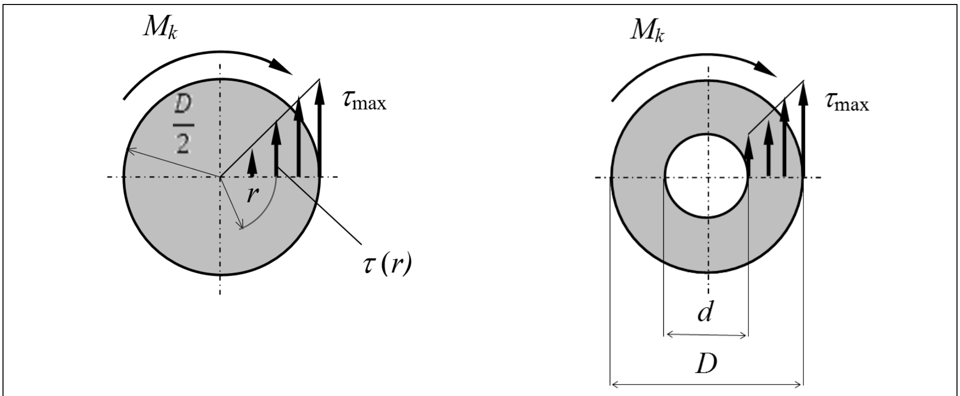


Fig. 1. Distribution of shear stress

According to the pure shear properties, we could declare following:

In torsion, a brittle material will break along planes whose coincide with maximum tension, that is, along 45° to the axis of rod. In these planes are generated the principal normal stresses. The values of the principal normal stresses are equal to values of the maximum shear stress. The shear stress is zero in planes of principal normal stresses. A planes, which have incline to the axis rod at an angle 45°

are strained by tensile. The planes which are right-angled to them are strained by compressive. This is used when measuring strain by tensometric bridge.

If we are able to measure principal strains, we will be able to calculate the principal stresses by the equation for Hooke's law for plane stress:

$$\sigma_1 = \frac{E}{1-\mu^2}(\varepsilon_1 + \mu\varepsilon_2),\ \sigma_2 = \frac{E}{1-\mu^2}(\varepsilon_2 + \mu\varepsilon_1), \quad (5)$$

where σ_1, σ_2 are principal normal stresses, μ = Poisson's ratio, $\varepsilon_1, \varepsilon_2$ are principal strains.

Considering case of the pure shear, $\varepsilon_2 = - \varepsilon_1$ ($\varepsilon_1 = \varepsilon$) and some mathematical modifications, we have: $\sigma_1 = +\frac{E\varepsilon}{1+\mu}, \quad \sigma_2 = -\frac{E\varepsilon}{1+\mu}$. (6)

Maximum shear stress can be calculated by equation:

$$\tau_{max} = |\sigma_{1,2}| = 2G\varepsilon. \quad (7)$$

Note: relationship between Young's Modulus (E) and shear modulus (G) was used: $G = \frac{E}{2(1+\mu)}$. (8)

Description of the measuring apparatus

To measure deformation of the rods with circular and hollow circular cross section measuring apparatus was constructed according to the scheme in Fig. 2. All strain-gauge configurations are based on the concept of a Wheatstone bridge [Šturcel 2002]. Measuring axes of two strains gauges are mutually perpendicular (The gauge factor K = 2.05). Strains gauges are linked in half-bridge and they are connected to the measurement system QuantumX MX840 ② which is controlled by computer ③. Program Catman Easy from firm HBM was used to acquire data, to manage the measurement system and to process the obtained data.

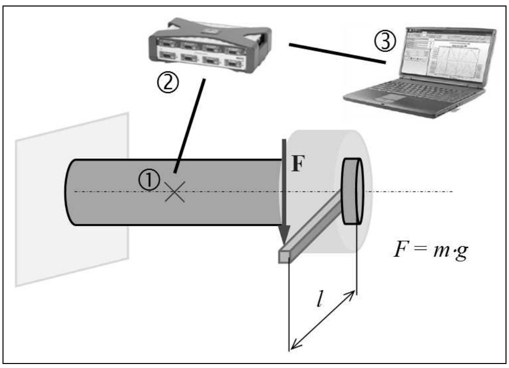


Fig. 2. The scheme of measurement apparatus

Figure 3 shows the rod itself, which was loaded by torque. Torque was created by solid bodies with given weight (m) acting on the arm (l). Material properties of testing rods were: G = 81 000 MPa, μ =0.29.

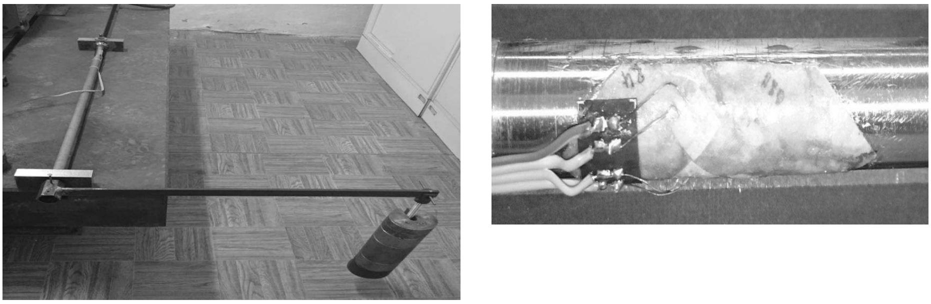


Fig. 3. Rod loaded by torsion and location of strain gauges

The dimensions and parameters of the rod with circular cross-section: diameter D = 16.18 mm, arm length l = 500 mm, modulus in torsion W_k = 831.6976 mm^3.

The dimensions and parameters of the rod with hollow circular cross-section: outside diameter D = 22.25 mm, inside diameter d = 12.55 mm, arm length l = 490 mm, modulus in torsion W_k = 1943.9031mm^3.

Comparison of stresses obtained experimentally and analytically

Value of torque M_k is constant over the length of the rod and is given by the product: $M_k = m \cdot g \cdot l$, (9)

where m = weight of bodies used as a weight, g = gravitational acceleration (g = 9.81 m·s^{-2}), l = length of arm.

The values of maximum shear stress of the circular cross section according to the size of the torque are recorded in the Table 1. $\tau_{max,1}$ represents values of the stress, which were analytically calculated according to equation (2). $\tau_{max,2}$ represents the size of stresses, which were calculated according to equation (7) from strains obtained by experimental measurement. To determine the percentage error of the measurement value of shear stress from analytical calculation has been considered as the true value. Establishing the percentage error allows the measuring apparatus to be used in the laboratory.

Table 1

Values of maximum shear stress of circular rod

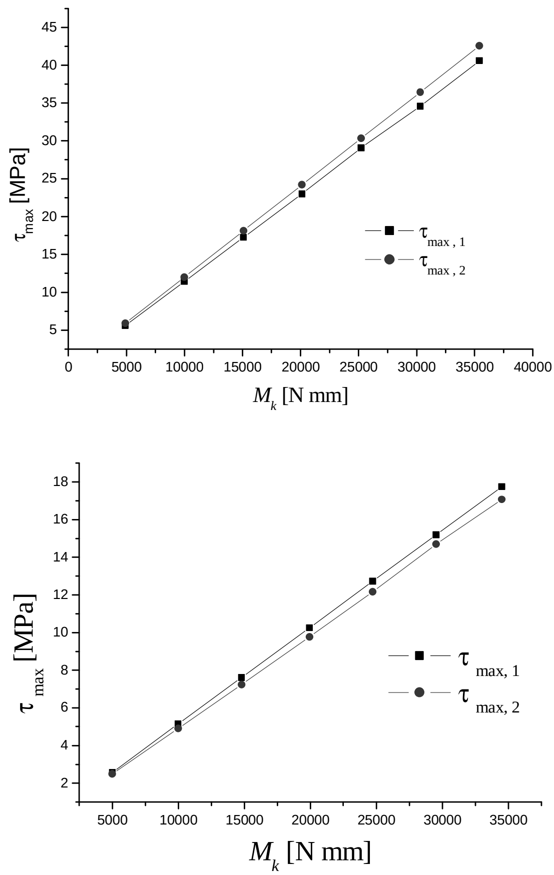
M [g]	M_k [N·mm]	ε [10^{-5} m·m^{-1}]	$\tau_{max,1}$ [N·mm^{-2}]	$\tau_{max,2}$ [N·mm^{-2}]	error [%]
998.83	4899.26	3.457	5.600	5.891	4.927
2037.63	9994.58	7.052	11.425	12.017	4.928
3076.57	15090.58	10.654	17.259	18.144	4.880
4105.02	20135.12	14.196	22.998	24.210	5.007
5143.96	25231.12	17.954	29.085	30.337	4.126
6178.91	30307.55	21.335	34.562	36.441	5.154
7217.62	35402.43	25.061	40.598	42.567	4.623

Table 2

Values of maximum shear stress of hollow circular rod

m [g]	M_k [N·mm]	ε [10^{-5} m·m^{-1}]	$\tau_{max,1}$ [N·mm^{-2}]	$\tau_{max,2}$ [N·mm^{-2}]	error [%]
1038.94	4994.081	1.5323	2.569	2.482	3.378
2077.74	9987.488	3.0209	5.138	4.894	4.749
3076.57	14788.764	4.4642	7.608	7.232	4.939
4145.13	19925.225	6.0310	10.250	9.770	4.682
5143.96	24726.501	7.5082	12.720	12.163	4.377
6141.96	29523.788	9.0694	15.188	14.692	3.262
7176.61	34498.689	10.535	17.747	17.067	3.833

Figure 4 shows experimentally and analytically obtained values of the maximum shear stresses.



a) the circular section

b) the hollow circular section

Fig. 4. Variations of shear stresses

Conclusion

The contribution deals with determining of stress from measured values of strains. Gauge strains, which were linked in half-bridge, and the universal measurement system QuantumX MX840 were used to experimentally obtain the strains. Rods with circular section and hollow circular section serve as tests samples. Students within the educational process will have the opportunity to acquaint with the universal measuring system and see the interconnectedness of theory and practical application.

Literature

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Abstract

The contribution is focused on determination of the maximum shear stress in rods with circular and annulus section. The values obtained analytically and derived from measured unit less of strains are compared. Measurement system used should contribute to teaching process by activating of students and also by linking theory with practice. Universal logger QuantumX MX840 was used in experimental measurements of unit less of strain.

Keywords: shear stress, circular rod, hollow circular rod, measurement system.