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THE IMPACT OF MACROECONOMIC FACTORS ON RESIDENTIAL PROPERTY PRICE INDICES IN EUROPE

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Abstract

This paper aims to determine the influence of selected variables on residential property price indices for the European countries, with particular attention paid to Italy and Poland, using a rough set theory and an approach that uses a committee of artificial neural networks. Additionally, the overall analysis for each European country is presented.
Quarterly time series data constituted the material for testing and empirical results. The developed models show that the economic and financial situation of European countries affects residential property markets. Residential property markets are connected, despite the fact that they are situated in different parts of Europe.
The economic and financial crisis of countries has variable influence on prices of real estate. The results also suggest that methodology based on the rough set theory and a committee of artificial neural networks has the ability to learn, generalize, and converge the residential property prices index.

Keywords: residential property prices indices, impact of macroeconomic factors, neural networks, rough set theory.

JEL classification: C15, R21, R31.

Introduction

In the recent years, European real estate markets have been subjected to difficult times. They are under a strong influence of crisis factors from the financial sector which are integrated in the real estate market system. Property markets in Europe respond differently to a crisis situation (Figure 1). A group of countries with economic problems encompassing the financial sector and fundamentally affecting the property market has entered a crisis phase. In another group of countries with fewer economic and financial problems the real estate market has been undergoing stagnation. Some European property markets have also proven to be insusceptible to the processes of crisis and stagnation.

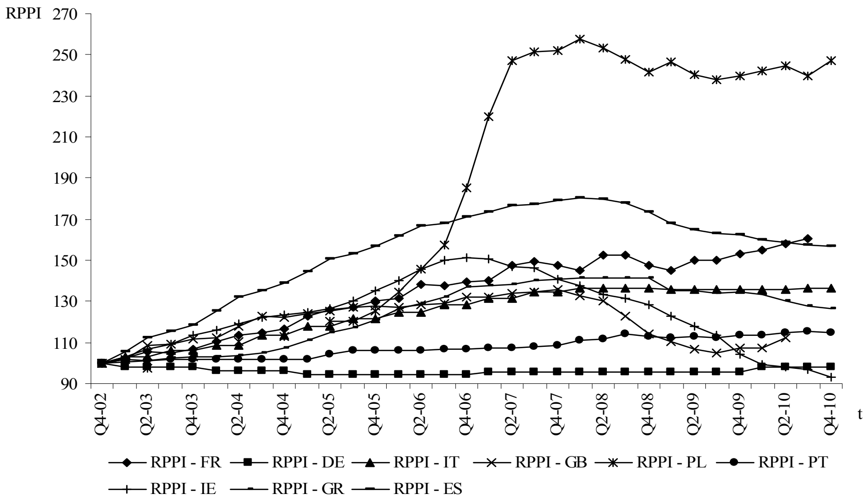

Fig. 1. Residential Property Prices Index for all dwellings in country (selected European countries), (per square m; index Q4-02 = 100)

Source: BIS.

In each of the designated groups, the processes occurring in real estate are subject to different impulses, depending on the financial and economic situation of a given country over the recent years. Thus, a group of these countries constitutes a good example for the analysis of relationships that occur between economic processes and processes of the real estate market[1].

The paper presents models of residential property prices indices for selected European countries. The purpose of the models presented in the study is to indicate whether the economic and financial situation of a given group or country is reflected in the overall economic situation of the real estate market. If such an influence is observed, a further purpose is to determine whether it is possible to indicate in which cases the effect is large, in which average, and in which only slight. We also hope to establish whether the (observed) relationships between the economic and financial situation of a country and the overall economic situation of its real estate market are correlated with the intensity of the crisis in a given group of countries or country. These questions are extremely important from the perspective of predicting the impact of a country's economic crises on the overall economic situation of its real estate market.

1. Macroeconomics and the economic crisis in the residential property market

1.1. Macroeconomics and the property market

The housing market is defined as one where housing services are allocated by the mechanism of supply and demand. One of the characteristics of the housing market that differs from the goods and services markets is the inelasticity of housing supply. Housing services are one of the most expensive household expenditures. Changing housing prices have been of concern to both individuals and governments in that they influence the socio-economic conditions and have a further impact on the national economic conditions. Expectations of capital gains from housing investments affect housing prices by increasing the demand for housing, which in turn causes high volatility in housing prices[2]. This leads to an increase in housing prices since the supply of housing cannot adjust in the short run. The housing market can be influenced by macroeconomic variables, spatial differences, characteristics of community structure, and environmental amenities[3].

Real estate assets are heterogeneous, that is, their characteristics vary. Researchers and practitioners have found that hundreds of factors might affect prices in various situations. Interaction effects and non-linear relationships between prices and hedonic variables complicate the issues[4]. This is why people interested in the prices of particular real estate assets consult valuers who collect and interpret recent sales evidence in order to arrive at a price estimate based on interpretation of differences between real estates[5].

Residential housing is an important aspect of the quality of life in any community. Therefore, the appropriate valuation of specific characteristics of a residential house is in order. To achieve this objective, empirical researchers often specify hedonic price functions or hedonic models[6].

An accurate prediction on the house price is important to prospective homeowners, developers, investors, appraisers, tax assessors, and other real estate market participants, such as, mortgage lenders and insurers[7]. Traditional house price prediction is based on cost and sale price comparison, lacking an accepted standard and certification process. Therefore, the availability of a house price prediction model would help fill an important information gap and improve the efficiency of the real estate market[8]. A broad overview of the studies conducted on housing prices can be found in the works of Limsombunchai et al.[9]

In the euro area, house prices have exhibited strong growth in many countries over the last decades. Such growth has been only partly related to movements in "dividends", in the form of housing rents, as measured in consumer price statistics.

1.2. Real estate markets in economic crisis

The turn from boom to bust and to crisis started with troubles in the sub-prime mortgage market in the US in early 2007. Initially, these troubles were thought to remain well contained. But in the summer of 2007, the sub-prime crisis turned into a wider US housing market downturn and financial crisis that spilled over into Europe. Following the Lehmann default in September 2008, the financial crisis deepened and after a severe economic downturn coupled with expansionary fiscal programmes, a sovereign debt crisis emerged in a number of European countries in early 2010[10].

Developments in the housing market have become an increasingly important element in the information set monitored by Central Banks. One of the lessons of the recent global crisis was that excessive asset price inflation, originating in the financial and in the real estate sector, needs to be kept under constant scrutiny for its potential disruptive impact on financial stability. As to the real estate market, in order to gauge the extent of misalignment of prices from fundamentals, good quality statistics on prices as well as on returns (i.e., rents) are essential[11].

Although the recent financial crisis has had a global impact, the channels of these effects vary across countries, reflecting not only differences in the structure of housing and mortgage markets, but also heterogeneity in the structures and linkages of macroeconomies[12]. Between 1999 and 2006, real house prices jumped in many OECD nations, rose more modestly in several other countries, and were flat in three (Japan, Germany, and Switzerland). Furthermore, while losses on nonprime mortgages have damaged the capital positions of financial firms across the globe, those losses disproportionately reflect mortgage defaults in the U.S. where an easing of mortgage credit standards played a role in the later generation of loan defaults.

2. The influence of factor analysis on the real estate market

2.1. Data

The data set used in the presented study includes quarterly time series for two European countries: *Italy* (Annex 1) and *Poland* (Annex 2). The data is from the Q4-2002 – Q4-2010 period. Residential Property Prices Indices (RPPIs) for all dwellings in the country per square m (Q4-2000 index = 100) and data concerning 14 variables describing the economic situation of a given country were collected for each of the countries. In some cases, appropriate changes and transformations of variables were made in order to make the data uniform. Table 1 summarizes the variables applied in this study. The basic descriptive statistics for RPPI (includes a suffix denoting the country: IT – *Italy*; PL – *Poland*) have been compiled in Table 2.

Table 1. Definition of variables

Variable	Name	Definition
Dependent	RPPI[1]	Residential Property Prices Index for all dwellings in country, per square m (index Q4-2002 = 100)
Independent	GDP[2]	Gross Domestic Product – expenditure approach. Growth rate compared to previous quarter, seasonally adjusted
	UE[2]	Harmonized unemployment rate: all persons. Level
	MEI[2]	Long-term interest rates, per cent per annum quarterly
	CPI[2]	Consumer Prices Index – all items. Percentage change from previous period quarterly
	HICP_H[3]	HICP - Housing, water, electricity, gas and other fuels. Quarterly rate of change. Neither seasonally nor working day adjusted
	HICP_AR[3]	Actual Rentals for housing. Quarterly rate of change. Neither seasonally nor working day adjusted
	HICP_M[3]	Maintenance and repair of the dwelling. Quarterly rate of change. Neither seasonally nor working day adjusted
	HICP_HS[3]	Housing Services. Quarterly rate of change. Neither seasonally nor working day adjusted
	ANNI[*4]	Adjusted Net National Income (annual % growth)
	FCE[*4]	Final Consumption Expenditure (annual % growth)
	HFCE[*4]	Household Final Consumption Expenditure (annual % growth)
	PG[*4]	Population Growth (annual %)
	GGD[4]	General Government Debt. Quarterly rate of change (index Q4-2002 = 1)
	LTL[4]	Long Term Loans. Quarterly rate of change (index Q4-2002 = 1)

[*] Annual data adopted in each quarter at an annual level, no data available for the year 2010.

1 – BIS (www.bis.org). 2 – OECD (http://stats.oecd.org). 3 – European Commission (Eurostat) and European Central Bank calculations based on Eurostat data (http://epp.eurostat.ec.europa.eu). 4 – IMF (www.imfstatistics.org).

Source: own calculations.

Table 2. Descriptive statistics for all RPPI

Variable	No. of cases	Average	St. dev.	Min.	Max.	Skewness	Kurtosis
RPPI-PL	28	193.63	62.20	97.27	257.73	–0.46	–1.70
GDP	33	1.12	0.59	–0.40	2.21	–0.38	–0.12
UE	33	13.34	4.98	6.97	20.17	0.15	–1.76
MEI	33	5.83	0.60	4.72	7.25	0.53	0.35
CPI	33	0.66	0.61	–0.52	2.01	0.43	–0.18
HICP_H	33	4.72	1.79	2.70	9.23	1.23	0.80
HICP_AR	33	4.11	1.31	2.53	6.47	0.45	–1.28
HICP_M	33	3.20	3.83	–0.33	12.17	1.25	0.35
HICP_HS	33	4.21	1.87	2.17	8.20	1.02	–0.09
ANNI	29	4.55	1.92	1.22	7.10	–0.14	–1.66
FCE	29	3.86	1.47	1.94	6.32	0.36	–1.11
HFCE	29	3.74	1.52	1.89	5.92	0.01	–1.67
PG	29	–0.03	0.05	–0.07	0.06	1.25	0.05
GGD	33	1.04	0.09	0.89	1.36	1.50	4.70
LTL	33	1.00	0.09	0.76	1.25	0.43	2.45
RPPI-IT	33	124.67	12.44	100.00	136.56	–0.66	–1.08
GDP	33	0.02	0.79	–3.01	1.05	–2.42	6.94
UE	33	7.54	0.84	6.00	8.73	–0.43	–1.21
MEI	33	4.22	0.37	3.39	4.90	–0.44	–0.13
CPI	33	0.50	0.33	–0.44	1.14	–0.71	1.48
HICP_H	33	3.27	2.49	–2.23	8.17	–0.34	–0.04
HICP_AR	33	2.56	0.49	1.60	3.60	0.27	–0.25
HICP_M	33	2.81	0.60	1.73	4.10	0.30	–0.33
HICP_HS	33	3.01	0.53	2.10	4.37	1.03	1.06
ANNI	29	0.04	1.67	–3.55	1.75	–1.25	0.72
FCE	29	0.63	0.91	–1.16	1.39	–1.16	–0.30
HFCE	29	0.39	1.08	–1.74	1.25	–1.14	–0.31
PG	29	0.73	0.14	0.31	0.99	–0.35	1.87
GGD	33	1.02	0.06	0.91	1.13	0.19	–0.44
LTL	33	0.89	0.33	0.03	2.15	1.09	6.66

Source: own calculation.

2.2. Committee of Artificial Neural Networks (CANN)

The opportunity to use Artificial Neural Networks (ANN) to estimate price indices has been investigated in the recent years. For example, Borst reported the use of ANNs on data sets of family residences in New England[13]. Tay and Ho examined sets in Singapore of 833 residential apartment properties and tested them against a 222 case set of similar apartment properties[14]. Do and Grudnitiski used data from a multiple listing service in California, while Evans worked with residential housing in the United Kingdom[15]. Recent works have been published by Borst, Borst and McCluskey, McCluskey et al., Rossini and Worzala[16]. Rossini's research was based on data from South Australia and demonstrated that the results from Artificial Neural Networks could, under certain circumstances, potentially produce results superior to more traditional econometric models.

Nguyen and Cripps compared the predictive performance of ANNs and multiple regression analysis (MRA) for single family housing sales[17]. They concluded that ANNs outperform MRA when a moderate to large data sample size is used. Despite the contribution of these models, there is little or no evidence to suggest the existence of models that could predict changes in the value of real estate over time. Therefore, there is a need for practical and automated models that would help achieve this important goal.

Curry et al. examined the potential of a neural network approach for the analysis of hedonic regressions, in which price is dependent on quality characteristics. They viewed neural network modeling as a useful means of specification testing, and hence their results imply some support for a linear formulation as an adequate approximation[18].

It is well known that a committee of predictors can improve prediction accuracy[19]. The process of creating a committee of neural networks is a difficult matter. The reduction of errors in the network committees should be combined, in particular, with the reduction of variance obtained by averaging a number of partial solutions. This suggests that a network that is meant to constitute a part of a committee cannot be selected in a way that would optimize the ratio of variance and bias[20]. The choice should rely on selecting a network with a relatively low value of bias because the variance itself can be reduced by averaging the results of individual networks within the committee[21]. In certain practical situations a phenomenon based on the fact that certain networks which make up a committee have better predictive ability than others is observed. In such cases, one can adopt a strategy which involves the creation of a committee using networks with assigned weights corresponding to their predictive ability.

Committees of artificial neural networks have been applied to the real estate market by Wisniewski[22]. In these works, it was proven that the application of committees of neural networks significantly contributes to lowering property value modeling errors in comparison to MRA, ADL, and ARIMA models. The study was conducted for various property markets, including the residential market.

In this study, committees of artificial neural networks consisting of single networks of MLP and RBF type were used. A feed-forward MLP with one hidden layer was employed in the study. A multilayer perceptron (MLP) is a feedforward artificial neural network model that maps sets of input data onto a set of appropriate output. An MLP consists of multiple layers of nodes in a directed graph, with each layer fully connected to the next one. With the exception of input nodes, each node is a neuron (or processing element) with a nonlinear activation function (Bishop 1995). Neural networks (MLP) are tools of nonlinear analysis which usually use iterative algorithms. The most recommended MLP learning algorithms are the BFGS (Broyden-Fletcher-Goldfarb-Shanno) and the SCG (Scaled Conjugate Gradient)[23]. Although these algorithms are much better than previously used algorithms, such as the Backpropagation Method, they have high demands for computer memory and speed calculations. Overall, however, they require less iterations in the learning process because they are rapidly converging with a more advanced exploration mechanism.

A radial basis function network (RBF) was used in the study in order to introduce solutions other than those obtained by applying MLP models to the neural networks. A radial basis function network is an artificial neural network that uses radial basis functions as activation functions. It constitutes a linear combination of radial basis functions. Such networks typically have three layers: an input layer, a hidden layer with a non-linear RBF activation function, and a linear output layer.

RBF networks with linear activation functions are learnt in two stages. In the first, radial functions are to be arranged using only input variables from the data. In the second stage, the network is fixed with the weights connecting the radial functions of the output neurons. In the case of a linear activation function, it is only necessary to simply reverse the matrix at the output, which is more accurate and does not require iteration.

The linear learning process, however, applies only if the error function constitutes a sum of squares and the activation function of output neurons is linear. If these conditions are not met, the activation function is more complicated; one must return to the iterative algorithms, such as RBFT (Red Baron Flight Training), in order to determine the weights of the hidden layer and finish the process of neural network learning of the RBF type.

The process of creating responses in the interpreter module of a committee of neural networks, that is the means of determining a result for a whole committee of neural networks, can be carried out using different methods. The property value for a given observation (i = 1, ..., N, where N is the number of observations used in network processes) can be obtained by using methods of calculating the arithmetic mean of the outputs of individual networks in the committee – SRA[24]:

$$SRA_i = \frac{1}{M}\sum_{k=1}^{M} y_k \tag{1}$$

where: y_k is the value calculated by the k-th network for the i-th observation (RPPI), $k = 1, ..., M$, M – the number of networks in the committee.

Error functions used in neural network learning should give some measure of the differences between the prediction and actual value, at a given point of the input variable space. It is, therefore, natural to use the sum of squares of differences (SOS) as an error function:

$$E_{SOS} = \sum_{i=1}^{n} \left(x_i - y_i\right)^2 \tag{2}$$

where n is the number of cases (input-output pairs) used for learning, y_i is the prediction of a network (network output), and x_i is a "actual" value (output according to data) for the *i*-th case. The bigger the differences the greater the error and the more corrections required by a network.

Training, testing, and validation of CANNs was carried out under the following assumptions[25]:

a) Dependent variable: RPPI – *x* (*x* – suffix appropriate for the given country).
b) Independent variables: 14 – as shown in Table 1 (plus suffix appropriate for the given country)
c) Number of ANNs tested in each CANN: 500.
d) Number of retained networks: 50.
e) Types of networks/learning method: RBF/RBFT, MLP3/BFGS.
f) Activation function: linear, logistic, hyperbolic tangent, exponential function (negative), and gauss (RGB).
g) Number of hidden neurons: MLP – minimum – 5 (1/3 Inputs), maximum – 15 (1/2 Inputs + Outputs) + square root of the number of patterns in the training file); RBF – minimum – 7 (1/2 Inputs), maximum – 15 (number of Inputs).
h) Number of cases used in network processes: 33. Missing cases were replaced with the average at input.
i) Weight reduction was applied to the hidden and output layers.
j) Criterion for selecting the retained networks: the correlation coefficient (R_{cy}).
k) Division of the data set into trials: learning (L), testing (T), and validation (V): share of individual tests: L – 70%, T – 15%, V – 15%. Number of samples: 500.
l) Subsampling method: bootstrap – generate multiple subsamples of the original dataset based upon sampling with replacement.
m) Operationalization of variables – minimax (0.1).
n) Error function for the learning, validation, and testing of networks: E_{SOS}.
o) Quality of learning, validation, and testing of networks: the correlation coefficient (R_{cy}).

p) Formation of a committee: SRA – arithmetic mean from the outputs of individual networks.
q) The quality of CANN: the mean absolute percentage difference between the real x and predicted $\hat{y}$ (MAPD), the mean absolute error between the real and predicted $\hat{y}$ (MAE), the root mean square error (RMSE), R^2, the correlation coefficient (R_{cy}) for learning (L), testing (T) and validation (V) sets.

The STATISTICA auto design function of artificial neural networks was used during the construction of CANNm. In the case of each data set (2 sets – 2 countries), the process of finding the top 50 ANNs was launched once. Guidelines assumed in section four were applied over the course of the study. The test results are summarized in Table 3 (CANN models).

Table 3. Results of modeling RPPI for CANN models

Predictor	MAPD	MAE	RMSE	R^2	R_{cy} U	R_{cy} T	R_{cy} V	d^*
CANN – IT	0.013	1.537	2.219	0.975	0.949	0.993	0.980	–
OLS** – IT	0.014	1.755	2.228	0.967	–	–	–	2.521
CANN – PL	0.016	2.614	3.628	0.997	0.994	0.992	0.999	–
OLS** – PL	0.024	3.705	4.764	0.995	–	–	–	1.483

* *Durbin-Watson (D-W) coefficient.* The bounds at a 95% level of significance are $d_{L,0.025} = 0.43$, $d_{U,0.025} = 2.72$. Thus, there is no negative autocorrelation between observations and no evidence to suggest positive autocorrelation.

** *Models estimated by ordinary least squares using all the data.*

Source: own calculations.

In OLS models most of the variables were not found to be statistically significant at a significance level of 0.05. The significance of variables increases only slightly at 0.1. In the case of Italy, the following the variables were shown to be significant (at a level of 0.1): UE, HICP_H, HICP_AR, HICP_HS, ANNI, FCE, HFCE, in Poland: GDP, UE, HICP_H, FCE and HFCE. The mean absolute percentage differences (MAPD) between the real and predicted $\hat{y}$ values range from 1.3% (OLS – IT) to 2.4% (OLS – PL), with mean absolute error (MAE) values ranging from 1.755 (OLS – IT) to 3.70 (OLS – PL), root mean square error (RMSE) from 2.228 (OLS – IT) to 4.764 (OLS – PL), and R^2 from 0.967 (OLS – IT) to 0.995 (OLS – PL). The test results are summarized in Table 3 (OLS models).

In CANN models, the mean absolute percentage differences (MAPD) between the real and predicted $\hat{y}$ values range from 1.3% (OLS – IT) to 1.6% (OLS – PL), with mean absolute error (MAE) values ranging from 1.537 (OLS – IT) to 2.614 (OLS – PL), root mean square error (RMSE) from 2.219 (OLS – IT) to 3.628 (OLS – PL), and R^2 from 0.975 (OLS – IT) to

0.997 (OLS – PL). The test results are summarized in Table 3 (CANN models). The resulting coefficients indicate that the CANN models were well matched with the data.

In the case of almost all parameters, OLS models performed worse than CANNs. Based on the conducted analyses it can, therefore, be assumed that CANN models enable better modeling of RPPI than OLS models. They allow for the searching of a large variability space contained within the data, obtaining the necessary accuracy, and an adequate level of generalization. All these factors support the application of CANNs in modeling RPPI.

Variable sensitivity analysis relies on calculating the residual sum of squares in a situation where the tested predictor has been removed from the network. Quotients (for the full model in relation to the model with the removed predictor) are given, with the actual predictors sorted according to their importance to a given neuron network.

2.3. Model of rough set theory

Procedures of determining the effect of macroeconomic factors on residential property price indices in Poland and Italy assume the application of the rough set theory. The method based on the rough set theory accounts for the specific features of real estate data. Developed by a Polish information technology engineer, Professor Zdzisław Pawlak, the theory is used to test imprecision, vagueness, and uncertainty in the process of data analysis.

Although the rough set theory is a relatively young approach, it has a growing number of scientific applications[26]. It has been used in medicine, pharmacology, economics, banking, chemistry, sociology, acoustics, linguistics, general engineering, neuroengineering and machine diagnostics. Several studies published in the recent years discuss rough set theory applications in geography and spatial planning[27].

The applied method for determining the effect that real estate factors have on the decision taken based on the rough set theory poses an alternative to statistical analyses employed in real estate market surveys.

The analyzed set of information has been presented as a decision-making chart, in which the macroeconomic factors are conditional factors (14 variables), while the residential property prices index is a decision-making factor represented by 9 levels (due to 9 years included in the analysis). The research is based on detailed data from Italy and Poland (see Table 1). The example of Poland is used to demonstrate the procedure.

The procedure assumes that the values of each factor were grouped according to the indiscernibility relation, based on the rough set theory[28]. For the purpose of analyzing this highly specific set of real estate data and different scales of factor assessment, the classical rough set theory was expanded to include the value tolerance relation formula. The formula, developed and discussed by Stefanowski and Tsoukias[29], was applied in real market analyses by d'Amato and Renigier-Biłozor[30].

The classical rough set theory is based on the indiscernibility relation as a crisp equivalence relation, i.e., two real estate sites will be indiscernible only if characterized by similar variables. When the rough set theory is expanded to include the value tolerance relation, the upper and lower approximation of the data set can be determined at different levels of the indiscernibility relation. The above relation can be expressed by the following formula:

$$R_j(x, y) = \frac{\max\left(0, \min\left(c_j(x), c_j(y)\right) + k - \max\left(c_j(x), c_j(y)\right)\right)}{k} \tag{3}$$

where:

$R_j(x, y)$ – relation between sets with membership function [0.1],

$c_j(x)$, $c_j(y)$ – variable of the analyzed real estate,

k – coefficient of standard mean in the variable set of a given real estate.

The formula is used to compare two data sets with results falling within the 0–1 range marking the level of the indiscernibility relation. If the *k* coefficient from the above formula is the standard mean of the analyzed variables, similarity (indiscernibility) matrices for the *k* coefficient are determined separately for each factor. A sample matrix of the GDP factor for Poland is presented in Table 4.

Table 4. Value tolerance relation matrix of the GDP factor – for Poland

GDP	1	2	3	4	5	6	7	8	9	...	33
1	1	0.74	0.14	0.51	0.72	0	0.67	0.36	0.56		0.99
2	0.74	1	0	0.26	0.47	0	0.41	0.61	0.31		0.74
3	0.14	0	1	0.63	0.42	0.85	0.47	0	0.58		0.15
4	0.51	0.26	0.63	1	0.79	0.48	0.85	0	0.95		0.52
5	0.72	0.47	0.42	0.79	1	0.27	0.94	0.08	0.84		0.73
6	0	0	0.85	0.48	0.27	1	0.33	0	0.43		0.00
7	0.67	0.41	0.47	0.85	0.94	0.33	1	0.02	0.90		0.68
8	0.36	0.61	0	0	0.08	0	0.02	1	0		0.35
9	0.56	0.31	0.58	0.95	0.84	0.43	0.90	0	1		0.57
...											
33	0.99	0.74	0.15	0.52	0.73	0.00	0.68	0.35	0.57		1

Source: own study.

In the next step of the variables validity analysis, the results generated by the above matrices are added up and the sum matrix is determined based on the following assumption:

$$R_j(x,p) = \max\left(\sum_{j=1}^{n} R_j(x,p)\right) \tag{4}$$

where R_j is the value tolerance relation, x is the factor of the analyzed real estate, p is the factor relating to the conditional part of the investigated decision rule, and n is the number of real estate factors in the conditional part of the decision rule. A sample sum matrix is presented in Table 5.

Table 5. Matrix of sums of real estate values determined based on the value tolerance relation matrix of each conditional factors – 14 variables for Poland

Sum	1	2	3	4	5	6	7	8	9	...	33
1	**14.00**	**11.00**	10.34	9.82	9.80	9.49	8.91	9.15	9.02		8.96
2	**11.00**	**14.00**	**12.23**	**11.71**	**12.08**	9.99	8.65	9.67	9.12		9.81
3	10.34	**12.23**	**14.00**	**12.29**	**11.96**	10.50	8.59	8.69	9.03		9.04
4	9.82	**11.71**	**12.29**	**14.00**	**12.20**	9.94	8.94	8.09	9.02		9.30
5	9.80	**12.08**	**11.96**	**12.20**	**14.00**	10.23	9.15	8.65	10.05		10.36
6	9.49	9.99	10.50	9.94	10.23	14.00	10.73	11.23	11.46		8.88
7	8.91	8.65	8.59	8.94	9.15	10.73	14.00	10.85	11.41		8.13
8	9.15	9.67	8.69	8.09	8.65	11.23	10.85	14.00	12.11		8.21
9	9.02	9.12	9.03	9.02	10.05	11.46	11.41	12.11	14.00		8.89
...											
33	8.96	9.81	9.04	9.30	10.36	8.88	8.13	8.21	8.89		14.00

Source: own study.

In the next step of the discussed procedure, abstract classes were determined for each indiscernibility relation at a corresponding level of similarity between real estate sites. A similarity level of 80% was adopted, based on the specific features of the real estate market, the number of analyzed sites and the diversified method of encoding various factors. The above implies that if 14 dependent variables are applied in the analysis, indiscernible (similar) sites will be ones in which the sum, resulting from value tolerance relation matrices (formula 4), is 11 or higher (14 × 80% = 11). Those sums are indicated in bold in Table 5.

Abstract (indiscernibility) classes were then established for dependent variables as follows:

I) – 1,2; II) – 1, 2, 3, 4, 5; III) – 2, 3, 4, 5; IV) – 2, 3, 4, 5; V) – 2, 3, 4, 5...; XXXIII)…

Next, the coverage and accuracy of approximation were determined for sets from the family of decisional factors. For this purpose, the entire information set was divided into 9 decision sub-groups (corresponding to 9 years) for which coverage indicators were determined:

I (2002 year) – 1; II (2003 year) – 2, 3, 4, 5; III (2004 year) – 6, 7, 8, 9; IV (2005 year) – 10, 11, 12, 13; V (2006 year) – 14, 15, 16, 17; VI (2007 year) – 18, 19, 20, 21; VII (2008 year) – 22, 23, 24, 25; VIII (2009 year) – 26, 27, 28, 29; IX (2010 year) – 30, 31, 32, 33.

Due to interest in the sensitivity analysis of changes in the price index, the overall level of approximation quality was calculated for the entire data set. Consequently, the general **quality of approximation** of family F in the space of approximation S relative to a set of C was conducted in accordance with the following formula:

$$\gamma_{\widetilde{C}}(F) = \frac{card\left(\underline{POS_{\widetilde{C}}(F)}\right)}{card(U)} \tag{5}$$

At the assumed level of 80%, the similarity level of the 14 conditional factors was equal to 0.81. To estimate the validity of each variable in the price index the entire procedure was repeated from the moment of calculating matrix sums, by excluding every successive factor and observing the changes induced by coverage indicators in the set of analyzed data.

Subsequent stages of the procedure are identical to those which have been performed above for all sites. The procedure finishes with an estimation of coverage results following the removal of every successive factor – Table 6.

Table 6. Approximation of classification of sets for Poland

Poland	All factors	Following the removal of the successive factor													
		GDP	UE	MEI	CPI	HICP_H	HICP_AR	HICP_M	HICP_HS	ANNI	FCE	HFCE	PG	GGD	LTL
Quality of aproximation	0.81	0.61	0.54	0.79	0.51	0.73	0.76	0.57	0.70	0.48	0.70	0.73	0.42	0.79	0.79

Source: own study.

3. Results

Results of the sensitivity analysis of independent variables for CANN models are presented in Annex 3. The following independent variables were shown to be the most significant in the CANN models (the first six variables were subjected to analysis):

A) in RPPI-IT models:

1) FCE, 2) UE, 3) HFCE, 4) HICP_AR, 5) HICP_H, 6) ANNI;

B) in RPPI-PL models:

1) UE, 2) FCE, 3) HFCE, 4) HICP_HS, 5) PG, 6) HICP_H.

Other variables occupy successive positions in the sensitivity analysis. The remaining variables, i.e., GDP, CPI, GGD and LTL, were last in the carried out sensitivity analysis.

The results of the sensitivity analysis of independent variables for RST models are presented in Annex 4. The following independent variables were shown to be the most significant in the RST models (the first six variables were subjected to analysis):

A) in RPPI-IT models:

1 – GDP, 2 – HFCE, 3 – ANNI, 4 – HICP_H, 5 – FCE, 6 – CPI;

B) in RPPI-PL models:

1 – PG, 2 – ANNI, 3 – CPI, 4 – UE, 5 – HICP_M, 6 – GDP.

Other variables occupy successive positions in the sensitivity analysis. The remaining variables, i.e., MEI, GGD and LTL, were last in the carried out sensitivity analysis.

As we can see from the analysis, some variables recur despite the application of two totally different methods?

In the case of Italy these are:

a) FCE – the final consumption expenditure variable,

b) HFCE – the household final consumption expenditure variable,

c) HICP_H – housing, water, electricity, gas and other fuels,

d) ANNI – the net national income variable.

In the case of Poland these are:

a) UE – the increase in the unemployment variable,

b) PG – the population growth variable.

The authors consider these variables to be the most important, that is, to have the most influence on the real estate market.

Conclusions

The significant variables for Poland, i.e., unemployment rate and population growth, are characteristic of developing countries and constitute a very important factor influencing the demand for real estate, and thus its. After accession to the EU prices of properties increased significantly. This change in property prices was most likely caused by the inflow of foreign capital and an increase in Poland's attractiveness as an EU member. Moreover, the Polish government had introduced a number of programs and initiatives supporting the residential real estate sector. The above mentioned variables are directly related to the fact that young married couples with children entering the labor market are mainly responsible for generating demand for real estate.

In developed countries, such as Italy, basic needs have already been met so demand is generated for the purpose of prestige (higher standards of living) and investment. This thesis is confirmed by the variables that where shown to be most important in the presented analysis, i.e.: consumption expenditure, household consumpion expenditure and housing expenses. The national income factor is currently also highly dependent on individual consumption.

The analysis of results of both methods implies that government debt and long term loans have the least influence on property prices. These variables were shown to be irrelevant, both in Poland and Italy, since prior 2011 very little attention was paid to them by countries, investors, and public opinion alike. It can be assumed that the information regarding debt (not the condition of the economy) and the behavior of speculative real estate is influenced by the crisis.

Annex 1. Data set – Italy

	RPPI-IT	GDP-IT	UE-IT	MEI-IT	CPI-IT	HICP_H-IT	HICP_AR-IT	HICP_M-IT	HICP_HS-IT	ANNI-IT	FCE-IT	HFCE-IT	PG-IT	GGD_IT	LTL-IT
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Q4-2002	100.00	0.19	8.47	4.68	0.67	1.23	2.60	2.87	2.93	0.45	0.73	0.20	0.31	1.00	1.00
Q1-2003	103.30	–0.22	8.73	4.24	0.81	3.27	2.70	2.77	3.00	0.22	1.25	0.99	0.78	1.08	1.24
Q2-2003	103.30	–0.40	8.47	4.16	0.72	4.03	2.90	2.90	2.97	0.22	1.25	0.99	0.78	1.12	0.86
Q3-2003	106.24	0.34	8.40	4.33	0.52	3.33	3.10	2.77	3.13	0.22	1.25	0.99	0.78	1.04	0.99
Q4-2003	106.24	0.39	8.20	4.45	0.46	2.93	2.90	2.47	2.73	0.22	1.25	0.99	0.78	1.01	0.77
Q1-2004	108.76	0.58	8.20	4.28	0.57	1.73	3.00	2.23	2.87	1.75	1.17	0.79	0.99	0.98	0.85
Q2-2004	108.76	0.36	8.10	4.46	0.75	1.33	3.00	2.33	3.03	1.75	1.17	0.79	0.99	1.02	0.84
Q3-2004	113.59	0.35	7.87	4.32	0.43	2.07	2.80	2.80	3.07	1.75	1.17	0.79	0.99	1.01	0.95
Q4-2004	113.59	–0.16	7.93	3.97	0.21	2.97	2.80	2.97	3.07	1.75	1.17	0.79	0.99	1.06	1.10
Q1-2005	117.71	–0.00	7.83	3.74	0.51	3.70	2.60	3.20	2.90	–0.04	1.39	1.16	0.74	1.03	0.91
Q2-2005	117.71	0.59	7.77	3.54	0.69	4.80	2.30	3.37	2.77	–0.04	1.39	1.16	0.74	1.00	2.15
Q3-2005	121.65	0.26	7.67	3.39	0.60	5.63	2.20	2.93	2.70	–0.04	1.39	1.16	0.74	1.00	1.02
Q4-2005	121.65	0.34	7.50	3.55	0.34	5.93	2.00	2.67	2.57	–0.04	1.39	1.16	0.74	0.97	0.86
Q1-2006	124.71	0.64	7.23	3.72	0.49	5.87	2.30	2.33	2.73	1.26	1.06	1.25	0.57	1.10	0.91
Q2-2006	124.71	0.61	6.90	4.27	0.78	6.13	2.30	2.43	2.60	1.26	1.06	1.25	0.57	1.09	1.12
Q3-2006	128.62	0.41	6.60	4.17	0.54	6.00	2.30	2.57	2.53	1.26	1.06	1.25	0.57	0.96	0.89

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Q4-2006	128.62	1.05	6.40	4.03	0.00	4.70	2.70	2.60	2.70	1.26	1.06	1.25	0.57	1.01	0.91
Q1-2007	131.49	0.16	6.13	4.24	0.41	4.13	2.50	3.30	3.00	1.27	1.02	1.08	0.73	1.00	0.95
Q2-2007	131.49	0.08	6.00	4.54	0.64	2.47	2.30	3.00	3.67	1.27	1.02	1.08	0.73	1.03	0.89
Q3-2007	134.41	0.21	6.20	4.64	0.58	1.50	2.20	3.13	4.20	1.27	1.02	1.08	0.73	1.04	0.89
Q4-2007	134.41	–0.41	6.30	4.53	0.70	2.77	2.00	3.47	4.17	1.27	1.02	1.08	0.73	1.01	0.84
Q1-2008	136.29	0.41	6.47	4.38	1.10	4.20	2.00	3.17	4.37	–3.55	–0.37	–0.76	0.77	1.13	0.79
Q2-2008	136.29	–0.65	6.80	4.78	1.14	6.80	2.30	2.93	3.73	–3.55	–0.37	–0.76	0.77	1.01	0.82
Q3-2008	136.56	–1.13	6.80	4.90	0.98	8.17	2.60	3.97	3.47	–3.55	–0.37	–0.76	0.77	0.92	0.74
Q4-2008	136.56	–2.04	6.93	4.66	–0.44	6.73	3.00	4.10	3.43	–3.55	–0.37	–0.76	0.77	0.98	0.57
Q1-2009	136.02	–3.01	7.37	4.54	–0.19	3.87	3.60	3.87	2.77	–0.76	–1.16	–1.74	0.65	1.02	0.52
Q2-2009	136.02	–0.32	7.57	4.46	0.51	0.47	3.60	3.63	2.97	–0.76	–1.16	–1.74	0.65	1.09	1.27
Q3-2009	135.74	0.39	8.03	4.19	0.24	–1.93	3.30	2.33	2.97	–0.76	–1.16	–1.74	0.65	1.09	1.16
Q4-2009	135.74	0.03	8.30	4.05	0.10	–2.23	2.90	1.97	2.90	–0.76	–1.16	–1.74	0.65	0.95	0.46
Q1-2010	135.74	0.52	8.37	4.02	0.43	–1.20	2.37	1.73	2.77	–0.76	–1.16	–1.74	0.65	0.99	0.90
Q2-2010	135.74	0.53	8.47	4.03	0.62	0.77	2.00	1.87	2.43	–0.76	–1.16	–1.74	0.65	0.91	0.43
Q3-2010	136.29	0.32	8.37	3.90	0.45	2.43	1.80	2.07	2.20	–0.76	–1.16	–1.74	0.65	1.12	0.81
Q4-2010	136.29	0.12	8.50	4.20	0.26	3.23	1.60	2.10	2.10	–0.76	–1.16	–1.74	0.65	0.95	0.03

Source: own study.

Annex 2. Data set – Poland

	RPPI-PL	GDP-PL	UE-PL	MEI-PL	CPI-PL	HICP_H-PL	HICP_AR-PL	HICP_M-PL	HICP_HS-PL	ANNI-PL	FCE-PL	HFCE-PL	PG-PL	GGD_PL	LTL-PL
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Q4-2002	100.00	0.81	20.17	5.98	0.31	4.80	4.27	0.70	4.27	1.22	3.00	3.44	–0.05	1.00	1.00
Q1-2003	98.64	0.52	19.80	5.61	0.56	3.87	4.43	0.63	3.87	2.48	2.52	1.89	–0.07	1.07	1.00
Q2-2003	97.27	1.76	19.47	5.19	0.37	3.40	4.33	0.27	3.63	2.48	2.52	1.89	–0.07	1.02	1.04
Q3-2003	100.19	1.35	19.77	5.64	–0.52	2.87	4.23	0.27	3.60	2.48	2.52	1.89	–0.07	0.99	0.98
Q4-2003	103.10	1.11	19.63	6.67	0.99	3.23	4.33	0.27	3.57	2.48	2.52	1.89	–0.07	1.17	1.10
Q1-2004	106.66	1.93	19.87	6.71	0.74	2.90	3.00	0.63	2.67	3.13	3.82	4.15	–0.06	1.04	0.93
Q2-2004	110.22	1.18	19.27	7.20	2.01	4.23	2.77	8.07	2.47	3.13	3.82	4.15	–0.06	1.05	0.99
Q3-2004	111.64	0.09	18.63	7.25	0.63	4.37	2.80	11.83	2.63	3.13	3.82	4.15	–0.06	1.03	0.97
Q4-2004	113.06	1.29	18.27	6.42	0.86	4.33	2.83	12.17	2.80	3.13	3.82	4.15	–0.06	1.15	1.06
Q1-2005	116.59	0.70	18.23	5.75	0.21	5.10	3.03	11.80	2.97	6.03	2.86	2.05	–0.04	0.97	0.76
Q2-2005	120.12	0.54	18.27	5.25	0.59	3.60	3.07	4.23	2.90	6.03	2.86	2.05	–0.04	1.05	0.88

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Q3-2005	120.44	1.68	17.70	4.72	–0.12	3.13	3.07	0.27	2.90	6.03	2.86	2.05	–0.04	1.00	0.92
Q4-2005	125.52	1.12	16.87	5.15	0.53	3.33	3.27	–0.27	2.97	6.03	2.86	2.05	–0.04	1.00	1.01
Q1-2006	134.46	1.66	15.57	4.84	–0.03	3.50	2.67	–0.33	2.17	5.40	5.24	5.00	–0.06	1.07	0.95
Q2-2006	145.53	1.63	14.37	5.28	0.90	4.77	2.53	0.07	2.40	5.40	5.24	5.00	–0.06	1.02	1.06
Q3-2006	157.52	1.97	13.30	5.55	0.23	4.60	3.43	0.83	2.87	5.40	5.24	5.00	–0.06	1.00	1.00
Q4-2006	185.55	1.38	12.27	5.25	0.23	4.57	4.03	2.03	3.40	5.40	5.24	5.00	–0.06	1.05	0.99
Q1-2007	220.06	1.80	10.83	5.18	0.65	4.50	5.37	3.10	4.63	7.10	4.51	4.91	–0.05	1.05	0.99
Q2-2007	246.82	1.67	9.83	5.36	1.25	4.33	6.00	7.03	5.23	7.10	4.51	4.91	–0.05	1.01	0.96
Q3-2007	251.58	1.27	9.30	5.66	–0.13	4.80	6.07	8.53	5.43	7.10	4.51	4.91	–0.05	1.02	1.06
Q4-2007	251.95	2.21	8.60	5.73	1.53	4.77	6.47	7.60	5.43	7.10	4.51	4.91	–0.05	1.08	0.98
Q1-2008	257.73	1.38	7.53	5.87	1.48	6.40	6.37	7.40	7.67	6.14	6.32	5.92	0.01	1.04	1.02
Q2-2008	253.13	0.72	7.27	6.17	1.33	7.17	6.13	4.33	8.10	6.14	6.32	5.92	0.01	1.04	1.00
Q3-2008	247.99	0.79	6.97	6.15	0.22	7.87	5.67	2.73	8.20	6.14	6.32	5.92	0.01	0.89	0.84
Q4-2008	241.79	–0.40	7.00	6.09	0.53	9.23	5.80	2.90	8.20	6.14	6.32	5.92	0.01	0.89	0.99
Q1-2009	246.39	0.41	7.50	5.88	1.33	9.13	5.27	2.30	6.83	2.38	1.94	2.31	0.06	0.94	0.97
Q2-2009	240.40	0.56	8.00	6.28	1.96	7.80	5.13	1.37	5.87	2.38	1.94	2.31	0.06	1.10	1.06
Q3-2009	238.10	0.38	8.50	6.15	0.15	6.47	4.73	1.07	5.47	2.38	1.94	2.31	0.06	1.19	1.25
Q4-2009	239.59	1.51	8.90	6.17	0.15	4.60	3.50	0.67	4.90	2.38	1.94	2.31	0.06	1.05	1.00
Q1-2010	241.89	0.62	9.67	5.98	0.93	2.87	2.93	0.53	3.10	2.38	1.94	2.31	0.06	1.01	0.95
Q2-2010	244.78	1.15	9.60	5.72	0.95	2.70	2.83	0.70	2.97	2.38	1.94	2.31	0.06	0.93	0.90
Q3-2010	239.43	1.23	9.63	5.65	0.00	2.93	2.70	0.77	2.50	2.38	1.94	2.31	0.06	1.36	1.24
Q4-2010	247.24	0.82	9.70	5.78	1.00	3.43	2.63	1.03	2.40	2.38	1.94	2.31	0.06	0.98	1.08

Source: own study.

Annex 3. Sensitivity analysis for the independent variables in the CANN models

Predictor	Variables													
CANN-IT	FCE	UE	HFCE	HICP_AR	**HICP_H**	ANNI	MEI	PG	HICP_HS	GDP	LTL	HICP_M	CPI	GGD
% *	42.2	15.5	13.7	6.5	3.7	3.3	3.0	2.9	2.3	1.9	1.4	1.3	1.1	1.0
CANN-PL	UE	FCE	HFCE	HICP_HS	PG	HICP_H	ANNI	HICP_AR	HICP_M	MEI	GDP	CPI	GGD	LTL
% *	69.7	4.9	4.6	4.5	4.2	4.1	1.8	1.2	1.1	1.0	0.8	0.8	0.7	0.7

* % contribution of the variable.

Source: own study.

Annex 4. Sensitivity analysis for the independent variables in the RST models

Predictor	Variables													
RST-IT	GDP	**HFCE**	**ANNI**	**HICP_H**	**FCE**	CPI	HICP_AR	HICP_M	HICP_HS	PG	UE	MEI	GGD	LTL
%*	75	71	63	34	26	22	12	12	8	8	8	8	4	4
RST-PL	PG	ANNI	CPI	UE	HICP_M	GDP	HICP_HS	HICP_H	HFCE	FCE	HICP_AR	MEI	GGD	LTL
%*	48	41	33	33	30	25	14	10	10	6	6	3	3	3

* % significance of the variable.

Source: own study.

Notes

1 Wiśniewski (2011).

2 Selim (2009), pp. 2843–2852.

3 Kim, Park (2005), pp. 221–232.

4 Liu et al. (2006), pp. 1187–1191.

5 Wolverton, Senteza (2000), pp. 235–253.

6 Bao, Wan (2004), pp. 487–507; Bełej, Kulesza (2012), pp. 61–72; Fan et al. (2006), pp. 2301–2315; Filho, Bin (2005), pp. 93–114; Fletcher et al. (2004), pp. 189–200; Janssen et al. (2001), pp. 342–360; Stevenson (2004), pp. 136–153).

7 Frew, Jud (2003), pp. 77–86.

8 Calhoun (2003), pp. 31–41.

9 Limsombunchai et al. (2004), pp. 193–201.

10 Agnello, Schuknecht (2011).

11 See discussion by Visco to Goodhart (2005).

12 Wiśniewski (2011).

13 Borst (1991).

14 Tay, Ho (1992), pp. 525–540; (1994), pp. 5–25.

15 Do, Grudnitiski (1992), pp. 38–45; Evans (1993) pp. 195–204.

16 Borst (1995), pp. 5–15; Borst, McCluskey (1996); McCluskey et al. (1996), pp. 25–32); Rossini (1997); Worzala et al. (1995).

17 Nguyen, Cripps (2001), pp. 313–336.

18 Curry et al. (2002), pp. 951–969.

19 Kontrimasa, Verikasb (2011), pp. 443–448.

20 Jacobs (1997), pp. 369–383; Geman et al. (1992), pp. 1–58.

21 Bishop (1995).

22 Wiśniewski (2003), pp. 253–264, (2008).

23 Bishop (1995).

24 Wiśniewski (2008).

25 Wiśniewski (2011).

26 Including Deja (2000); Komorowski et al. (1999), pp. 3–98; Mrózek, Płonka (1999); *Rough Sets in Knowledge Discovery...* (1998); Pawlak (1997); Słowiński (1992).

27 Including d'Amato (2007), (2008); Kotkowski, Ratajczak (2002), pp. 35–44; Renigier-Biłozor, Biłozor (2007), (2009), pp. 103–107.

[28] Pawlak (1982), p. 341, (1991).

[29] Stefanowski, Tsoukias (2000); Stefanowski (2001).

[30] d'Amato, (2007), (2008); Renigier-Biłozor (2008), pp. 35–51, (2011), pp. 107–118).

References

Agnello, L. & Schuknecht, L. (2011). Booms and busts in housing markets: Determinants and implications. *J. Housing Econ.*, DOI: 10.1016/j.jhe.2011.04.001.

Bao, H.X.H. & Wan, A.T.K. (2004). On the use of spline smoothing in estimating hedonic housing price models: Empirical evidence using Hong Kong data. *Real Estate Economics*, 32 (3).

Bełej, M. & Kulesza, S. (2012). Modeling the real estate prices in Olsztyn under instability conditions. *Folia Oeconomica Stetinensia*, Vol. 11 (19).

Bishop, C. (1995). *Neural Networks for Pattern Recognition*. Oxford: University Press.

Borst, R.A. & McCluskey, W. (1996). *The Role of Artificial Neural Networks in the Mass Appraisal of Real Estate*. Paper presented to the Third European Real Estate Society Conference, Belfast, June 26–28.

Borst, R.A. (1991). Artificial Neural Networks: The Next Modelling/Calibration Technology for the Assessment Community? *Property Tax Journal*, IAAO, 10 (1), 69–94.

Borst, R.A. (1995). Artificial neural networks in mass appraisal. *Journal of Property Tax Assessment &Administration*, 1 (2), 5–15

Calhoun, C.A. (2003). Property Valuation Models and House Price Indexes for The Provinces of Thailand: 1992–2000. *Housing Finance International*, 17.

Curry, B., Morgan, P. & Silver, M. (2002). Neural networks and nonlinear statistical methods: An application to the modelling of pricequality relationships. *Computers & Operations Research*, 29.

d'Amato, M. (2007). Comparing Rough Set Theory with Multiple Regression Analysis as Automated Valuation Methodologies. *International Real Estate Review*, Vol. 2.

d'Amato, M. (2008). *Rough Set Theory as Property Valuation Methodology: The whole story.* In: T. Kauko, M. d'Amato (Eds.), *Mass Appraisal Methods. An international perspective for property valuers.* Oxford: Blackwell Publishing. RICS Research.

Deja, R. (2000). *Zastosowanie teorii zbiorów przybliżonych w analizie konfliktów.* PhD thesis. Instytut Podstaw Informatyki Polskiej Akademii Nauk.

Do, A.Q. & Grudnitiski, G. (1992). *A Neural Network Approach to Residential Property Appraisal*. The Real Estate Appraiser.

Evans, A., James, H. & Collins, A. (1993). Artificial Neural Networks: an Application to Residential Valuation in the UK. *Journal of Property Valuation & Investment*, 11.

Fan, G., Ong, Z.S.E. & Koh, H.C. (2006). Determinants of house price: A decision tree approach. *Urban Studies*, 43 (12).

Filho, C.M. & Bin, O. (2005). Estimation of hedonic price functions via additive nonparametric regression. *Empirical Economics*, 30 (1).

Fletcher, M., Mangan, J. & Raeburn, E. (2004). Comparing hedonic models for estimating and forecasting house prices. *Property Management*, 22 (3).

Frew J., & Jud, G.D. (2003). Estimating The Value of Apartment Buildings. *The J. Real Estate Res*., 25.

Geman, S., Bienenstock, E. & Doursat, R. (1992). Neural Networks and the bias-variance dilemma. *Neural Computation* 4.

Goodhart, C. (2005). Beyond current policy frameworks. *Working Papers* 189. BIS.

Jacobs, R.A. (1997). Bias/Variance Analyses of Mixtures-of-Experts Architectures. *Neural Computation*, 9 (2).

Janssen, C., Söderberg, B. & Zhou, J. (2001). Robust estimation of hedonic models of price and income for investment property. *Journal of Property Investment & Finance*, 19 (4).

Kim, K. & Park, J. (2005). Segmentation of the housing market and its determinants: Seoul and its neighbouring new towns in Korea. *Australian Geographer*, 36 (2).

Komorowski, J., Pawlak, Z., Polkowski, L. & Skowron, A. (1999). *Rough sets: A tutorial.* In: S.K. Pal, A. Skowron (Eds.), *Rough fuzzy hybridization: A new trend in decision making*, Singapore: Springer-Verlag.

Kontrimasa, V. & Verikasb, A. (2011). The mass appraisal of the real estate by computational intelligence. *Applied Soft Computing*, 11.

Kotkowski, B. & Ratajczak, W. (2002). *Zbiory przybliżone w analizie danych geograficznych.* In: H. Rogacki (Ed.), *Możliwości i ograniczenia zastosowań metod badawczych w geografii społeczno-ekonomicznej i gospodarce przestrzennej*. Poznań: Bogucki Wydawnictwo Naukowe.

Limsombunchai, V., Gan, Ch. & Lee, M. (2004). House Price Prediction: Hedonic Price Model vs. Artificial Neural Network. *American Journal of Applied Sciences* 1 (3).

Liu, J.G., Zhang, X.L. & Wu W.P. (2006). Application of Fuzzy Neural Network for Real Estate Prediction. *Lecture Notes in Computer Science*. *NUMB* 3973.

McCluskey, W., Dyson, K., McFall, D. & Anand S. (1996). Mass Appraisal for Property Taxation: An Artificial Intelligence Approach. *Land Economics Review*, Vol. 2, No. 1.

Mrózek, A. & Płonka, L. (1999). *Analiza danych metoda zbiorów przybliżonych.* Warszawa: Akademicka Oficyna Wydawnicza PLJ.

Nguyen, N. & Cripps, A. (2001). Predicting housing value: A comparison of multiple regression analysis and artificial neural network. *Journal of Real Estate Research*, 3 (22).

Pawlak, Z. (1982). Rough sets. *International Journal of Information and Computer Science*, 11.

Pawlak, Z. (1991). *Rough sets: Theoretical aspects of reasoning abaut data.* Kluwer Academic Press.

Pawlak, Z. (1997). *Rough Sets and their Applications.* Seminar Department of Computing – Macquarie University.

Renigier-Biłozor, M. & Biłozor, A. (2007). *Application of the rough set theory and the fuzzy set theory in land management*. Londyn: Annual conference The European Real Estate Society – ERES.

Renigier-Biłozor, M. & Biłozor, A. (2009). *The significance of real estate attributes in the process of determining land function with the use of the rough set theory*. Olsztyn: Wyd. Studia i Materiały Towarzystwa Naukowego Nieruchomości.

Renigier-Biłozor, M. (2008). *Zastosowanie teorii zbiorów przybliżonych do masowej wyceny nieruchomości na małych rynkach* (Application of rough sets theory for mass valuation of real estate in small markets). ACTA SCIENTIARUM POLONORUM, Administratio Locorum 7 (3).

Renigier-Biłozor, M. (2011). Analysis of Real Estate Markets With the Use of the Rough set Theory. *Journal of the Polish Real Estate Scientific Society*, Vol. 19, No. 3.

Rossini, P.A. (1997). Application of Artificial Neural Networks to the Valuation of Residential Property 3rd Pacific Rim Real Estate Society Conference, New Zealand.

Rough Sets in Knowledge Discovery 2: Applications, Case Studies and Software Systems (1998). L. Polkowski, A. Skowron (Eds.). Physica-Verlag, Heidelberg.

Selim, H. (2009). Determinants of House Prices in Turkey: Hedonic regression versus artificial neural network. *Expert Systems with Applications*, 36.

Słowiński, R. (1992). *Intelligent Decision Support. Handbook of Applications and Advances of the Rough Sets Theory.* Dordrecht: Kluwer Academic Publishers.

Stefanowski, J. & Tsoukias, A. (2000). *Valued Tolerance and Deciosion Rules*. In: W. Ziarko, Y. Yao (Eds.). Proceedings of the RSCTC 2000 Conference, Banff.

Stefanowski, J. (2001). *Algorytmy indukcji reguł decyzyjnych w odkrywaniu wiedzy.* Habilitation thesis. Poznań: Wydawnictwo Politechniki Poznańskiej.

Stevenson, S. (2004). New Empirical Evidence on Heteroscedasticity in Hedonic Housing Models. *Journal of Housing Economics*, 13.

Tay, D. & Ho, D. (1992). Artificial Intelligence and the Mass Appraisal of Residential Apartment. *Journal of Property Valuation & Investment*, 10.

Tay, D. & Ho, D. (1994). *Intelligent Mass Appraisal*. Journal of Property Tax Assessment & Administration, Vol. 1, No. 1.

Wiśniewski, R. (2003). *Modular Neural Networks*. *Proceedings IV National Conference on Methods and Computer Systems in Research and Engineering Design*. Kraków: Wyd. AG-H, Politechnika Krakowska, Uniwersytet Jagielloński.

Wiśniewski, R. (2008). Multidimensional Forecasting of the Value of a Real Estate. *Dissertations and Monographs*, No. 124, UWM.

Wiśniewski, R. (2011). *Modeling of Residential Property Prices Index Using Committees of Artificial Neural Networks for PIGS, the European-G8, and Poland*. Economic Modelling (sent to the publisher).

Wolverton, M.L. & Senteza, J. (2000). Hedonic Estimates of Regional Constant Quality House Prices. *Journal of Real Estate Research*, 19 (3).

Worzala, E., Lenk, M. & Silva, A. (1995). An Exploration of Neural Networks and Its Application to Real Estate Valuation. *The Journal of Real Estate Research*, Vol. 10, No. 2.