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DETECTING SHOCKS IN THE ECONOMIC DEVELOPMENT DYNAMICS OF SELECTED COUNTRIES

Anna Janiga-Ćmiel, Ph.D.

University of Economics in Katowice
Faculty of Management
Department of Mathematics
1 Maja 50, 40-287 Katowice
e-mail: anna.janiga-cmiel@ue.katowice.pl

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Abstract

The paper examines the development of the Polish economy as well as the economies of selected countries in the period from 2001 to 2012. For that purpose, models based on the GDP growth in particular countries were built. A comparative analysis of the development of economies in the countries concerned (the United Kingdom, Belgium, Denmark, France, Poland, the Netherlands), based on a specially built full-factor multivariate GARCH model, is presented. The theory of the construction of a full-factor multivariate GARCH model and its estimation method are discussed. In the paper, a multivariate GARCH model where the covariance matrix is always positive, definite and the number of parameters is relatively small compared to other multivariate models is proposed. The causality of the impact that economies exert on one another is examined and the occurrence of the contagion effect is verified by means of the Forbes and Rigobon test.

Keywords: GARCH model, contagion effect, economic development.

JEL classification: C01, C32, C51.

Introduction

Economic development depends on a great number of factors. Each economy is shaped by country-specific factors as well as those related to the economic development of the neighboring countries. The relationships between these factors can be precisely characterised by means of dynamic correlation matrices and econometric models. Each model is an approximate description of the correlations taking place within its framework and is an image of the examined reality. This is confirmed by the fact that: "Every cycle theory specifies a different selection and interpretation of historical events, which is of great significance to prior findings, by means of the non-positivist methodology, valid theories enabling accurate interpretation of the reality. There is, therefore, no irrefutable historical evidence, none that would prove that a theory is right or wrong. As a result, we should be very careful and hold out modest hopes for the empirical confirmation of a theory. We have to, at most, be satisfied with the development of a logically coherent theory, possibly error-free in the chain of logical arguments, based on the fundamental principles of human action. With such a theory, one can see how well it fits with historical events and if it allows interpretation of real-life cases in a more general, more balanced and correct way than other alternative theories." [1]Models should include the expected value of operational processes, variances of these processes, their mutual covariances and a broad presentation of the dynamics of random components as well as the interrelations between random components of neighboring economies. Over a long period of time, economic development is characterised by unpredictable increases and decreases in indicators causing changes in the development dynamics, which focus on irregular fluctuations and cycles of varied length. This results in unpredictable changes in the variance in subsequent years. Heteroscedasticity prevents the use of linear models based on the classical method of least squares for the description of the dynamics. In order to describe such complicated processes, one can employ an autoregressive GARCH model or, in special cases, some of its more complex modifications. Researchers have proposed several possible modifications and extensions of the GARCH model[2] (Bollerslev, Chou and Kroner, Bera and Higgins, Engle and Nelson, Gourieroux, Osiewalski and Pipień, Tsay, Bauwens, Laurent and Rombouts, Weron, Brzeszczyński and Kelm, Doman, Fiszeder). One of the most important applications of these models is volatility modelling and forecasting. Multivariate GARCH models allow for description of both variable variances and covariances which describe the interrelations between the phenomena under examination. It is believed that the single equation GARCH models do not explain the causes of variation, while the multivariate GARCH models allow additional analyses of some of the causes. GARCH models are used to study the variability of a conditional variance and a conditional covariance. As a result, it is possible to detect shocks and their positive or negative impact on other populations. Appropriate econometric tools enable global scenarios of stagnation to be prevented, countries to be protected against possible economic slowdown and experience to be gained.

The aim of the studies discussed in the paper is an analysis of the interrelations between the economies of Poland and selected EU countries. The multivariate GARCH models showing mutual relative interrelations within the dynamics of empirical distributions, particularly within the dynamics of the expected values and variances, will be presented. The results will be statistically verified, which will allow for an analysis concerning the occurrence of the contagion effect.

1. Shocks

Economies can develop in a stable way – then the process is orderly and controlled. Relationships are interdependent, there are statistically significant changes in neither correlation coefficients nor trends over time, and the development dynamics can be seen as relatively constant. This kind of economic development in a particular country and in the neighbouring countries is uncommon. Frequently, unpredicted phenomena occur which significantly affect the economic development of the country concerned as well as the development of the neighbouring countries. From a statistical point of view, such a moment is characterised by a sharp change in trend directions, alternate increases and declines in all correlation coefficients between factors specific to economic development.

A shock in economic development can be defined as a situation where there is a clash between economic phenomena causing rapid changes in the trends which determine its direction. It is a moment when the memory of an economic development process is being obliterated. Components of economic development may become divergent, which is a cause and source of an economic crisis. The World Bank[3] proposes three definitions of contagion:

1. Contagion is the cross-country transmission of shocks or the general cross-country spillover effects.
2. Contagion is the transmission of shocks to other countries or the cross-country correlation, beyond any fundamental link among the countries and beyond common shocks.
3. Contagion occurs when cross-country correlations increase during “crisis times” relative to correlations during “tranquil times.”

The occurrence of a contagion effect can be analysed by means of various methods, both mathematical and econometric[4], the simplest one is an analysis of correlation coefficients between series of variables being tested which yields important information on correlation coefficients over time. In the literature, one can find, inter alia, research based on the significance of correlation coefficient increase during crisis times, or examination of the significance of correlation coefficient growth taking into account the fundamental link. Boyer, Gibson and Loretan[5], or Forbes and Rigobon believe that testing the consistency of correlation calculated for different periods of time may give misleading results due to possible changes in the variance. Forbes and Rigobon proposed an adjustment formula of a correlation coefficient which takes into account the occurrence of variance changes and introduced an adjusted correlation coefficient. Corsetti, Pericoli, Sbracia suggest using a correlation coefficient that involves a global factor to test a contagion effect. Klaassen introduces testing the consistency of conditional correlations based on residuals from the single equation AR-GARCH models. Analyses can also be carried out taking into account the multivariate GARCH models. Chesnay and Jondeau studied unconditional correlations based on the MS-GARCH model. Analyses based on the single-equation GARCH models were limited to modifying the equation for the conditional variance and introducing an additional explanatory variable in the form of delayed squares (or absolute values) of return rates or (residuals of models describing the internal structure) indices from other markets.

2. Modelling of the economic development process

In order to show economic development we can employ an econometric model, at least a two-equation one, where one equation features the dynamics of expected values and the other – the dynamics of the variance. It can be one of ARIMA (p, d, q) models, specific to the description of the variance dynamics of the GARCH model[6]:

$$y_t = \sum_{t=0}^{p} \alpha_i y_{t-i} + \sum_{t=1}^{d} \beta_i (y_{t-i} - y_{t-i-1}) + \sum_{i=1}^{q} \gamma_i \varepsilon_{t-i} \quad (1)$$

Alternatively, the expected value model:

$$y_t = \mu + \varepsilon_t \quad (2)$$

The conditional distribution of the multivariate variable Y_t is consistent with the normal distribution, the condition being the values of variable Y_t from the previous periods.

$$Y_t \left| Y_{y-1}, Y_{y-2}, \ldots, Y_{t-p} \right. \sim N(0, \Sigma_t) \quad (3)$$

The variable under consideration is an *N*-dimensional random variable, wherein *N* is the number of countries included. An ARIMA model is used to describe the dynamics of the expected value of economic development indicators which are characterized by the GDP for selected countries. In further analysis the variance is denoted by h_{it}, where *i* is the number of the country and *t* the number of the period:

$$h_{it} = \sum_{k=1}^{t} w_{ik} w_{tk} \sigma_{i,t}^2 \quad (4)$$

The residual variance model is written as follows:

$$\sigma_{i,t}^2 = \alpha_i + b_i y_{i,t-1}^2 + g_i \sigma_{i,t-1}^2, \; i = 1, \ldots, N, \; t = 1, \ldots, T \quad (5)$$

Conditions and restrictions on the parameters of the model are as follows:

$$\alpha_i > 0, \; b_i \geq 0, \; g_i \geq 0, \; i = 1, \ldots, N \quad (6)$$

This model requires construction of a conditional covariance matrix H_t, binding matrix Σ_t of the residual variance with matrix *W*, which represents relationships between economic development in the selected countries. The matrix is defined as a normalised lower triangular matrix with respect to the main diagonal:

$$W = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ \omega_{21} & 1 & \ddots & 0 & 0 \\ \omega_{31} & \omega_{32} & 1 & & \vdots \\ \vdots & \ddots & \ddots & & 0 \\ \omega_{N1} & \omega_{N2} & \cdots & & 1 \end{bmatrix} \quad (7)$$

Σ_t is used to denote a matrix of unconditional residual variances, which is a diagonal matrix with variances on the main diagonal.

$$\Sigma_t = \begin{bmatrix} \sigma_{1t}^2 & & 0 \\ & \ddots & \\ 0 & & \sigma_{Nt}^2 \end{bmatrix} \quad (8)$$

In addition, matrix H_t is similar to matrix Σ_t, where respectively[7]:

$$H_t = W \Sigma_t W^T = W \Sigma_t^{\frac{1}{2}} \Sigma^{\frac{1}{2}}{}_t W^T = \left(W \Sigma_t^{\frac{1}{2}} \right) \left(W \Sigma_t^{\frac{1}{2}} \right)^T = LL^T \quad (9)$$

Matrix H_t is a square matrix of degree N, where N is the number of economies compared. This matrix is symmetric, we can find $\frac{N(N+1)}{2}$ items h_{ij} for $i \geq j$ (the lower triangle along with the main diagonal). In order to determine the elements of matrix W for the issue under consideration, a VECH model has to be built.

$$w_t = vech(H_t) = (h_{11,t}, h_{21,t}, h_{22,t}, h_{31,t}, h_{32,t}, h_{33,t}, ..., h_{N1,t}, h_{N2,t} h_{N3,t}, ..., h_{NN,t}) \tag{10}$$

$$H_t = W\Sigma_t W^T = \begin{bmatrix} h_{11,t} & h_{12,t} & h_{13,t} & \cdots & h_{1N,t} \\ h_{21,t} & h_{22,t} & h_{23,t} & \cdots & h_{2N,t} \\ h_{31,t} & h_{32,t} & h_{33,t} & \cdots & h_{3N,t} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ h_{N1,t} & h_{N2,t} & h_{N3,t} & \cdots & h_{NN,t} \end{bmatrix} = \begin{bmatrix} \sigma_{1,t}^2 & \omega_{21}\sigma_{1,t}^2 & \omega_{31}\sigma_{1,t}^2 & \cdots & \omega_{N1}\sigma_{1,t}^2 \\ \omega_{21}\sigma_{1,t}^2 & \Sigma_{i=1}^2 \omega_{2i}^2\sigma_{i,t}^2 & \Sigma_{i=1}^2 \omega_{2i}\omega_{3i}\sigma_{i,t}^2 & \cdots & \Sigma_{i=1}^2 \omega_{2i}\omega_{Ni}\sigma_{i,t}^2 \\ \omega_{31}\sigma_{1,t}^2 & \Sigma_{i=1}^2 \omega_{3i}\omega_{2i}\sigma_{i,t}^2 & \Sigma_{i=1}^3 \omega_{3i}^2\sigma_{i,t}^2 & \cdots & \\ \vdots & \vdots & \vdots & \ddots & \\ \omega_{N1}\sigma_{1,t}^2 & \Sigma_{i=1}^2 \omega_{Ni}\omega_{2i}\sigma_{i,t}^2 & \Sigma_{i=1}^3 \omega_{Ni}\omega_{3i}\sigma_{i,t}^2 & \cdots & \Sigma_{i=1}^N \omega_{Ni}^2\sigma_{i,t}^2 \end{bmatrix} \tag{11}$$

The matrix forms a basis for the construction of a maximum likelihood function:

$$l(y|\theta) = (2\pi)^{-\frac{TN}{2}} \prod_{t=1}^{T} |H_t|^{-\frac{1}{2}} \exp[-\frac{1}{2}\sum_{t=1}^{T}(y_t - \mu)^T H_t^{-1}(y_t - \mu)] \tag{12}$$

The function can be transformed to the following form:

$$\begin{aligned} L(y|\theta) &= -\frac{TN}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\ln\left|W\Sigma_t W^T\right| - \frac{1}{2}\sum_{t=1}^{T}(y_t - \mu)^T \left(W\Sigma_t W^T\right)^{-1}(y_t - \mu) = \\ &= -\frac{TN}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\left[\sum_{i=1}^{N}\left[\ln(\sigma_{i,t}^2\right]\right] - \frac{1}{2}\sum_{t=1}^{T}\left[\sum_{i=1}^{N}\left[\frac{x_{i,t}^2}{\sigma_{i,t}^2}\right]\right] \end{aligned} \tag{13}$$

The maximization parameter θ is a vector of specified coordinates:

$$\theta = (\mu_1, \mu_2, \mu_3, ..., \mu_N, \alpha_1, \alpha_2, ..., \alpha_N, b, g, \omega_{21}, \omega_{31}, \omega_{32}, ..., \omega_{N1}, ..., \omega_{N,N})^T \tag{14}$$

In order to simplify the calculations we distinguish three vectors using the block nature of the matrix:

$$\theta_1 = (\mu_1, \mu_2, \mu_3, ..., \mu_N)^T \tag{15}$$

$$\theta_2 = (\alpha_1, \alpha_2, ..., \alpha_N, b_1, ..., b_N, g_1, ..., g_N)^T \tag{16}$$

$$\theta_3 = (\omega_{21}, \omega_{31}, \omega_{32}, ..., \omega_{N1}, ..., \omega_{N,N-1})^T \tag{17}$$

Due to parameter θ_1 we get:

$$\frac{\partial L_T}{\partial \theta_1} = \sum_{i=1}^{T}\left\{\sum_{i=1}^{N}\left[\frac{1}{2\sigma_{i,t}^2}\frac{\partial \sigma_{i,t}^2}{\partial \theta_1}\left(\frac{x_{i,t}^2}{\sigma_{i,t}^2}-1\right)-\left(\frac{x_{i,t}}{\sigma_{i,t}^2}\frac{\partial x_{i,t}}{\partial \theta_1}\right)\right]\right\} \tag{18}$$

The solution of the matrix equation $\frac{\partial L_T}{\partial \theta_1} = 0$ provides a basis for determining the expected GDP values across the selected countries. Due to parameter θ_2 we get:

$$\frac{\partial L_T}{\partial \theta_2} = \sum_{i=1}^{T}\left\{\sum_{i=1}^{N}\left[\frac{1}{2\sigma_{i,t}^2}\left(\frac{x_{i,t}^2}{\sigma_{i,t}^2}-1\right)\frac{\partial \sigma_{i,t}^2}{\partial \theta_2}\right]\right\} \tag{19}$$

Solving the equation $\frac{\partial L_T}{\partial \theta_2} = 0$ we get numerical evaluation of the parameters of the GDP variation models in particular countries. Then, for parameter θ_3 we get:

$$\frac{\partial L_T}{\partial \theta_3} = \sum_{i=1}^{T}\left\{\sum_{i=1}^{N}\left[\frac{1}{2\sigma_{i,t}^2}\frac{\partial \sigma_{i,t}^2}{\partial \theta_3}\left(\frac{x_{i,t}^2}{\sigma_{i,t}^2}-1\right)-\left(\frac{x_{i,t}}{\sigma_{i,t}^2}\frac{\partial x_{i,t}}{\partial \theta_3}\right)\right]\right\} \tag{20}$$

At this stage, by solving the equation $\frac{\partial L_T}{\partial \theta_3}$, we calculate variance coefficients that are extensions of the VECH H_t matrix elements. In the process under consideration, a maximum likelihood method should be adopted to evaluate the estimation of the model parameters. Based on the model, the delay of correlation coefficients between random components is determined, which provides a basis for a study aimed at detecting the presence of a contagion effect. The study consists of two separate analyses: one examines the model of the expected GDP values, taking into account one delay in the explanatory variables, while the other one looks at the dynamics of the variance.

3. The study of correlation coefficients

Correlation coefficients between the time series of economic development in particular countries should be fixed and invariant over time. However, the variance of the economic development process frequently changes over a period of time. In the case of dramatic variance changes, one can identify periods of low and high variance, which is referred to as heteroscedasticity[8]. The low variance is denoted by σ_l^2, while the high one – by σ_h^2. For the period of low variation we have correlation coefficient ζ_l, whereas correlation coefficient ζ_h is related to high variation.

Next, an adjusted correlation coefficient is introduced, where the correction factor is a relative gain in the variation, represented by formula (21) proposed by Forbes and Rigobon[9].

$$\delta = \frac{\sigma_h^2}{\sigma_l^2} - 1 \tag{21}$$

ζ_h denotes a correlation coefficient calculated for the period when the variance was assumed as high σ^2_h. The adjusted correlation coefficient ζ_S is given by (22) and covers the whole period, where ζ_h is a coefficient for the entire period before adjusting.

$$\varsigma_s = \frac{\zeta_h}{\sqrt{1 + \delta(1 - \zeta_h^2)}} \tag{22}$$

Where respectively: ζ_l – a correlation coefficient for the period of low variance ζ_S – an adjusted correlation coefficient, ζ_h – a coefficient of correlation for the period of high variance. As for correction factor δ, it should be stated that it is non-negative and the value of null indicates that in the economic development of the compared countries a shock phenomenon did not occur. If, however, its value is significantly bigger than null, shock phenomena definitely occurred. Now, it is necessary to verify the relevance of the difference in coefficient ζ_S and coefficient ζ_l before the contagion period, i.e. during the period of low variance. We put forward the following hypotheses:

$$\begin{aligned} H_0 &: \zeta_s = \zeta_l \\ H_1 &: \zeta_s > \zeta_l \end{aligned} \tag{23}$$

According to Forbes and Rigobon, the equality of adjusted correlation coefficient ζ_S with correlation coefficient ζ_l, prior to the moment of contagion, has to be verified. No grounds for the rejection of hypothesis H_0 means that there was no contagion. However, the rejection of H_0 and acceptance of H_1 shows that a moment of contagion was detected in the economic development of a particular country due to unexpected changes which took place in the economic development of another country. This is a point corresponding to the change in volatility of the variance.

4. An empirical example

In order to conduct analyses for selected countries (the United Kingdom, Belgium, Denmark, France, Poland, the Netherlands) empirical data were prepared based on the data on annual GDP[10] published by the Central Statistical Office and Eurostat. The analysis presents a comparison of Poland's economy with the economies of selected countries because they enjoy stable development and economic growth. The GDP[11] ratio is a fundamental determinant of changes in economies and is also a factor affecting economic fluctuations. The analysis covers the years 2001–2012. The next phase of the study began[12] with determining the equations of the GDP expected values model (2). An ARIMA (1, 1, 1) model was used to estimate the appropriate model. It has been already shown that the random component tends to be autocorrelated. It is also heteroscedastic, there are periods of aggregated variance. Due to the limited length of the paper, the models are not presented. The matrix of residuals ε_t of this model is as follows:

Table 1. The matrix of residuals ε_t

Belgium	France	The United Kingdom	The Netherlands	Denmark	Poland
ε_{1t}	ε_{2t}	ε_{3t}	ε_{4t}	ε_{5t}	ε_{6t}
2.25	3.33	1.83	0.97	2.84	–1.19
0.45	–2.32	1.84	–2.61	–1.16	–0.19
–1.75	–0.05	2.85	–0.94	1.42	0.74
–0.35	0.60	0.87	1.06	–1.87	–1.40
–1.55	–1.40	–1.13	0.23	–0.44	–0.40
–1.95	–0.09	–3.15	1.23	–2.58	0.53
–0.35	–1.09	–3.18	2.81	–0.29	0.38
1.65	1.57	–1.22	–0.03	–1.58	3.24
1.05	–0.74	0.77	–0.19	3.56	–0.12
0.25	0.91	–1.23	0.23	–0.16	–1.26
0.25	–0.74	1.75	–2.77	0.27	–0.33

Source: based on own research.

In the course of further analysis, due to the requirement of stationarity, time series y_{it} are released from the trend and considered only with respect to other residual series ε_{it}. For the presented matrix of random components, matrices of correlation coefficients are determined. The results are presented in Table 2.

Table 2. The matrix of correlation coefficients

	Belgium	France	UK	The Netherlands	Denmark	Poland
Belgium	1					
France	0.109787	1				
UK	0.266297	0.080956	1			
The Netherlands	–0.103270	0.345870	–0.172920	1		
Denmark	0.116934	0.220104	0.105816	–0.09578	1	
Poland	0.041236	–0.002500	–0.263000	–0.03073	–0.2479	1

Source: based on own research.

We can see that the correlation coefficients of random components in the above matrix are not significantly different from zero. This is proved by the comparison of absolute values with the critical value $r^{*}= 0.6$, determined for the relevance level of 0.05. The values of the correlation coefficient do not allow for rejecting hypothesis H_0, which concerns the independent development of particular economies. This means that the economies of the countries under consideration develop independently of one another. In the presented formula of the maximum likelihood function, matrix H_t plays a significant role, in particular its changes from period to period. In order to simplify the calculations, we take advantage of the block nature of a VECH matrix and consider it in three separate blocks θ_1, θ_2, θ_3, setting a sequence of vectors which contain estimates divided according to their nature into groups of model parameter estimates (2), (5), (4), μ , α , w .

Table 3. The values of vector θ_1 estimates

Belgium	Denmark	France	UK	The Netherlands	Poland
119.9	125.0	109.9	116.5	131.3	55.3

Source: based on own research.

Vector θ_2 (16) shows the estimates of the model parameters (5) arranged in columns into groups of parameters: α , b, g. The numerical evaluations of vector θ_2 coordinates are as follows:

Table 4. Vector θ_2 grouped according to the respective columns

	α_i	b_i	g_i
Belgium	1.310	0.140	0.350
France	12.610	0.001	0.152
UK	7.990	0.010	0.100
Poland	0.540	0.480	0.300
The Netherlands	31.910	0.002	0.360
Denmark	19.770	0.001	0.260

Source: based on own research.

Analysing the parameter estimates at the delayed values of the GDP and the delayed unconditional variances, we can see that all coefficients are positive, which indicates their stimulative impact on the current economic development variance. In addition, at the delayed values of the GDP from the previous period the coefficients are strictly lower than at the economic development variances. This means that in the examined countries (except Poland) past events have little impact on current fluctuations in economic development. However, in the case of Polish economy a reversed situation can be observed over the last twelve years. At the delayed GDP the coefficient value is higher than at the coefficient of current fluctuations. Another vector θ_3 contains the elements of a normalised lower triangular matrix W. They are as follows: $\theta_3 = (1;\ 0.618;\ 1; 8.896; .....; -0.294; 1)$.

The matrix determines the similarity scale of matrices H_t and Σ_t. The table below shows the elements of matrix W:

Table 5. Matrix *W*

	Belgium	Denmark	France	UK	The Netherlands	Poland
Belgium	1					
Denmark	0.618	1				
France	0.896	0.597	1			
UK	0.480	–0.008	0.451	1		
The Netherlands	–0.043	0.159	0.322	–0.031	1	
Poland	–0.568	–0.099	–0.634	–0.913	–0.294	1

Source: based on own research.

The vectors of parameter estimates θ_1, θ_2, θ_3, lead to the construction of a generalised covariance matrix H_t. The matrix is used to illustrate the relationships between the unconditional and conditional variances.

Table 6. Matrix H_t for $t = 0$, the initial state

	Belgium	Denmark	France	UK	The Netherlands	Poland
Belgium	2.08	1.86	1.99	0.56	–0.14	–0.54
Denmark	1.86	5.80	3.59	0.49	0.59	–0.62
France	1.99	3.59	5.40	1.18	1.48	–1.32
UK	0.56	0.49	1.18	5.16	–0.15	–3.92
The Netherlands	–0.14	0.59	1.48	–0.15	3.79	–0.28
Poland	–0.54	–0.62	–1.32	–3.92	–0.28	4.89

Source: based on own research.

However, for $t = 1, 2, ..., T$ we get:

$$H_t = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0.618 & 1 & 0 & 0 & 0 & 0 \\ 0.896 & 0.597 & 1 & 0 & 0 & 0 \\ 0.480 & -0.008 & 0.451 & 1 & 0 & 0 \\ -0.043 & 0.159 & 0.322 & -0.031 & 1 & 0 \\ -0.568 & -0.099 & -0.634 & -0.913 & -0.294 & 1 \end{bmatrix} \begin{bmatrix} \sigma_{1t}^2 & 0 & \cdots & 0 \\ 0 & \sigma_{2t}^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_{6t}^2 \end{bmatrix} \begin{bmatrix} 1 & 0.618 & 0.960 & 0.480 & -0.043 & -0.568 \\ 0 & 1 & 0.597 & -0.008 & 0.159 & -0.099 \\ 0 & 0 & 1 & 0.451 & 0.322 & -0.634 \\ 0 & 0 & 0 & 1 & -0.031 & -0.913 \\ 0 & 0 & 0 & 0 & 1 & -0.294 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

The model captures particular variations over time. In matrix H_t, there are both positive and negative elements. Positive estimates are of significant importance as they indicate a harmonised development of the economies. On the other hand, in the case of the parameters concerning the relationships between Poland's economic development and that of the selected countries all coordinates are negative, which means that economic development in Poland and in the selected countries is not harmonised. If matrix H_t contains negative coefficients, we can expect critical economic situations, which are a source of the contagion effect.

5. The contagion effect test

Since critical situations tend to occur in the economic development of the selected countries, it is necessary to examine the existence of the mutual contagion effect among particular economies. First, the values of correlation coefficients of the economic development in Poland and in the selected countries are determined, taking into account the delay periods of t back. Due to the limited length of the paper, the table with the defined values of the coefficients is not presented. Then, the value of adjusted coefficient ζ_S is determined and compared with the value of correlation coefficient ζ_l in the period prior to the test, in accordance with the procedure outlined earlier. For Poland we get:

Table 7. The values of coefficient ζ_S

ζ_S PB	ζ_S PD	ζ_S PF	ζ_S PUK	ζ_S PH
0.413581	0.237794	0.343178	0.00726	0.02241

Source: based on own research.

Next, we determine the value of ζ_l:

Table 8. The values of coefficient ζ_l

ζ_l PB	ζ_l PD	ζ_l PF	ζ_l PUK	ζ_l PH
0.617826	0.896437	0.479545	0.04315	0.56785

Source: based on own research.

Comparing the values in accordance with the procedure (23), we can see that in each case$|\zeta_S| < |\zeta_l|$, which means that in Polish economy the contagion effect does not occur, contagion does not spill over Polish economy from any of the economies concerned.

Conclusions

This paper is an analysis of the impact exerted by the economic development of selected countries on the development of the Polish economy. For this purpose, a full-factor multivariate GARCH model was presented. The results of the study conducted by means of the GARCH model indicate that the economies of the selected EU countries developed in a harmonised way. However, the study into the interrelationships between Poland's economic development and that of the selected EU economies shows that it was not harmonised. The model was used to show a tendency of processes towards critical phenomena, and next, by means of the Forbes and Rigobon test, the actual and adjusted correlation coefficients were determined and used to examine the occurrence of the contagion effect. It was shown that at no time was the Polish economic development affected by contagion transmitted from the selected countries. In the case of Poland, no periods of increased correlation between its economic development and that of the selected EU countries was found, which means that they did not have a negative impact on Poland's economic development.

Notes

1 Huerta de Soto (2009).
2 Fiszeder (2009).
3 Czech-Rogosz (2009).
4 Fiszeder, Razik (2003).
5 Fiszeder (2009).
6 Francq, Zakoian (2010).
7 Vrontos, Dellaportas, Politis (2003).
8 Hosking (1980).
9 Forbes K, Rigobon (2002).
10 Hellwig (1997).
11 Yamarone (2006).

References

Czech-Rogosz, J. (2009). *Koniunktura gospodarcza po składzie*. Warszawa: Wydawnictwo C.H. Beck.

Fiszeder, P. (2009). *Modele klasy GARCH w empirycznych badaniach finansowych*. Toruń: Wydawnictwo Naukowe Uniwersytetu Mikołaja Kopernika.

Fiszeder, P. & Razik, W. (2003). *Analiza efektu zarażania na przykładzie zależności pomiędzy indeksem WIG a indeksami wybranych rynków akcji na świecie*. *Ekonomia XXXIII*. Acta Universitatis Nicolai Copernici. Toruń: UMK.

Forbes, K. & Rigobon, R. (2002). No Contagion, Only Interdependence: Measuring Stock Market Co-movements. *The Journal of Finance*, 57, 2223-2261. DOI: 10.1111/0022-1082.00494.

Francq, C. & Zakoïan, J. (2010). *GARCH models: structure, statistical inference, and financial applications.* John Wiley & Sons.

Hellwig, Z. (1997). *Ekspansja gospodarcza Polski końca XX wieku*. Poznań: Wydawnictwo Wyższej Szkoły Bankowej.

Hosking, J. (1980). The Multivariate Portmanteau Statistic, *The Journal of American Statistical Association*, 75, 602–608. DOI: 10.1080/01621459.1980.10477520.

Huerta de Soto, J. (2009). *Pieniądz, kredyt bankowy i cykle koniunkturalne*. Warszawa: Instytut Ludwiga von Milesa.

Janiga-Ćmiel, A. (2013). *Analiza zależności przyczynowych rozwoju gospodarczego Polski i wybranych państw Unii Europejskiej*. *Studia Ekonomiczne – Zeszyty Naukowe 159*, 51–72. University of Economics in Katowice, Katowice: Wydawnictwo Uniwersytetu Ekonomicznego w Katowicach.

Vrontos, I.D., Dellaportas, P.& Politis, D.N. (2003). A full-factor Miltivariate GRACH model. *The Journal of Ecomometrics*, 6, 311–333. DOI: 10.1111/1368-423X.t01-1-00111.

Yamarone, R., (2006). *Wskaźniki ekonomiczne: przewodnik dla inwestora*. Gliwice: Wydawnictwo Helion.