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EUROPEAN MATHEMATICS DURING THE EVOLUTIONARY PERIOD OF EARLY CAPITALISTIC CONDITIONS (15th AND 16th CENTURIES)

During the 15th and 16th centuries, the economic development of Western and Central Europe was characterized by a stepwise growth of commercial capital, principally in consequence of original accumulation. This process, differentiated both geographically and in time, was followed within the feudal society by the development of capitalistic methods of production: the transition from production by guilds to cottage industry and, later on, to the development of manufactures.

In this process, the aim of the forming new class was to become politically emancipated by means of developing the economy. The necessity imposed upon the new *bourgeoisie* of expanding trade and, later on, of improving their means of production and transport turned their interest in an ever increasing degree to making use of natural science and mathematics. The revival and fructification of ancient knowledge called the "Renaissance" was, therefore, the result of changed social conditions and not, by any means, their cause.

It seems worth investigating in detail, how the mathematical heritage taken over from antiquity has been resumed under social conditions of the 15th and 16th centuries, how it was first assimilated and adopted under new aspects, then rejected and again continued in an improved manner. Speaking more generally how, in the period of an altered orientation of objective social conditions offered to mathematics, the mathematical knowledge, accumulated under entirely different social conditions, was being "worked over."

Evidently the short time granted to speakers suffices barely for putting forth very few essential moments on this subject. We therefore must concentrate, as far as the period of early European capitalism is concerned, upon those social domains of mathematical orientation from which impelling forces towards reorientation have issued. These were:

1. The stepwise and extraordinary increase in money circulation led to a multitude of mathematical problems: book-keeping, widening of the complement of numbers, conversion of wide varieties of currencies and of different units of measure and weight, rates of interest and compound interest.

2. Navigation required new knowledge for high-sea sailing and naval construction work; hence demands were raised for nautical and astronomic data, that is, for spherical geometry and the computation of centres of gravity of vessels. In addition there were to be solved hydraulic problems of inland waterways involving the construction of canals, sluizes and river regulation.

3. The emergence and development of gunnery made armourers and ordnance men face a number of new ballistic problems; these problems had to be solved with highest exertion in view of the fact, that each shot was extremely expensive and that therefore sighst had to be taken very carefully. From these requirements heavy demands resulted as to geometry and trigonometry.

4. Astronomy, with its close relations to research on scientific cosmology and to the replacement of the geocentric by the heliocentric universe, to astrology and to the computation of the almanacs, brought, combined with the demands made by military and civil surveying, a multitude of incentives towards trigonometry. As a result of this, the necessity arose of improving and simplifying trigonometric calculations.

5. Architecture was confronted with difficult problems, especially in military building, such as defilading a fortress, that is, now to locate it including its bulwarks, ramparts, corners, etc. with due consideration of land forms, so that none of its parts, should be exposed to shelling by the besieging enemy's guns. Moreover, because of the increasing piercing power of shells, it became necessary to construct fortifications at greater depths, to build casemates. Hence the need of picturing three-dimensional objects in a plane which, in turn, developed fundamentals of descriptive geometry.

6. The practice of structural and descriptive art involved a variety of mathematical factors. Prominent buildings, statues and paintings, if meant to comply with the revided ideals of antique beauty, had to be designed in accordance with canonical rules, that is, their individual parts had to show definite proportions, like those called the "golden cut" (*der goldene Schnitt*). Consideration of linear perspective on paintings, an essential accomplishment of the art during the Renaissance, led to the determination of the vanishing point and the vanishing line.

It was within these domains that the demands of social life made on mathematics during the early capitalistic period in Europe were concentrated; and within this range the mathematical heritage of ancient times had to be assimilated, if mathematics of the ancient slave-holding society had already prepared the corresponding material. The antiquity became outdistanced or new mathematical disciplines developed, wherever the new evolution required new forms of thinking that had not advanced to maturity in the time of the antique.

Beginning with the 11th century, while still in contact with the Islamic World and the Byzantine Empire, parts of antique mathematics had reached Europe and had become part of the *quadrivium* taught at the universities of feudal Europe. These studies referred mainly to the content of Euclid's planimetrical works, to elementary numerals, to what was called planisphere courses, and to Ptolemy's *Almagest*, as well as to mathematical data from Plato and Aristotle. Practical mathematics were limited to some details of surveying, to *computus*, and to figuring by means of the abacus. It was not until the 13th century that an independent resumption and interpretation of traditional knowledge began, for instance by Leonardo Fibonacci from Pisa (ca. 1180—1250). And while in the 13th century there was taken up the translation of some of the masterpieces of antique mathematics—such as the works of Euclid by Johannes Campanus (ca. 1260), of Aristotle, Proclus, Archimedes, Heron and Ptolemy by Wilhelm von Moerbeke (d. 1286)—initially these achievements were practically futile, because commonly they were beyond comprehension, despite the fact that mathematics occupied a consolidated position within the structure of Scholastics.

Remarkable in the transition from the late Scholastics to the Renaissance is the sharp break that occurred not so much as to time, but rather in the shift in the social domain. During a process lasting for entire generations this domain, from which was supposed to originate the impulse towards the future evolution of mathematics, moved away from the universities to practice. There was a wide field of operation for mathematics in practical life; in this manner social evolution cleared the road to the critical perusal of the more advanced parts of ancient mathematics. It was only when mathematics, derived from practical application, had reached some sort of maturity, that it became responsive to Diophantes and to the complex parts of the work of Apollonius, Archimedes and other scientists. The great advancement of mathematics during the end of the 16th and the beginning of the 17th centuries resulted thus from the masterpieces of ancient mathematics being joined to a new conception of mathematics put in relief during the previous period.

This step-by-step reception and recast of the ancient mathematical heritage might be illustrated by a multitude of examples, proving it to be distinctly typical; however, for the sake of brevity, only a few facts shall be called to mind here.

Towards the end of the 16th century, the altercation between the abacists and the algorithmics came to a close with a victory of the latter, the disciples of calculating by the use of Indian-Arabic numbers. However, it became necessary to develop in an appropriate way the methods of written calculation—a task to be accomplished without recourse to antiquity. The multitude of mathematical textbooks published in the 16th century—beginning with Widmann and Adam Ries up to the Cossist writings—picture faithfully this slow and difficult procedure. And with M. Stifel's excellent *Arithmetica integra*, written in 1544, a new stage in the appropriation of antiquity was reached. Using the Cossist method, Stifel succeeded in making available Euclid's difficult Volume X of the *Elements*, the part dealing with irrationals of type $\sqrt{a+\sqrt{b}}$ after the ancient method of geometrical algebra. In this manner, implicitly inspired by Euclid Stifel managed to surpass Euclid and to study irrationals of the type $\sqrt[n]{a+\sqrt[n]{b}}$.

Also in the middle of the 16th century and in a similar manner the antique doctrine on equations was mastered. The transition to the algebraic solution of quadratic, cubic and biquadratic equations was accomplished by a group of Italian mathematicians: Tartaglia, A. M. Fior, and several others, who abandoned the antique method of applying plane geometry. And it was Cardano who, in 1545 in his *Ars magna*, gave a summary of antique achievements considered from the viewpoint of the new treatment of equations.

Quite as typical as the above progress was the turn in the methodical transformation of the antique, based, as far as substance is concerned, on ancient tradition, that is seen in the elaboration of algebra by Fr. Vieta (1540—1603). Vieta's *Logistica speciosa*, his splendid contribution, represents an attempt of reconstructing the proper method suggested by Diophantes whose doctrine had meanwhile become commonly acceptable. However, Vieta achieved much more than expanding the disposition of algebra during the antique; he attained a degree of perfection in numerical methods, unknown in the antique.

These few examples might easily be supplemented by mentioning the discovery of logarithms, by pointing out the interrelation between the sexagesimal and the decimal system, by indicating the history of tradition in compiling and improving astronomical tables, by calling attention to the transformation of the ancient trigonometry of secants into a sine geometry, accompanied by the evolution of the numerical methods involved, or by the changes arrived at in the evaluation of the doctrine of conical sections. In the present paper it was merely intended to exhibit the most fundamental principles.

In general outlines, the evolution of mathematics attained in the 15th and 16th centuries may be summed up as follows: as the result of the demands made by social life, mathematics advanced in three principal directions: improvement of the calculation technique, algebraization of methods of computation, and development of trigonometry into a homogeneous system.

With the rise of early capitalism in Europe mathematics attained, compared with the antique, a fundamentally novel social position. Mathematics had ceased to be merely a science in Plato's sense, and both directly and indirectly—also in view of its ties with natural sciences—the possibilities of mathematics in consideration of production had found recognition. In certain fields, the mathematical heritage taken over from the antique—particularly geometry and trigonometry—constituted sound mathematical foundations, whereas with regard to algebraic evolution it stimulated incentives. On the other hand, the new social requirements demanded the command of multifarious numerical methods, a domain in which ancient tradition was by no means prepared to satisfy the needs set off by the Renaissance. Undoubtedly, the emphasis put on numerical methods by the Islamitic mathematicians has considerably aided the acquisition of mathematical knowledge in antiquity as well as supported the development of European mathematics during the Renaissance.

As far as necessary, traditional knowledge was absorbed in the new perspectives envisaged by social requirements. It was only later, in the 17th century, that other suggestive topics initiated in antique mathematics were resumed and further developed, such as the fructification of the difficult parts of Archimedes' teachings dealing with researches on problems of quadrature and rectification. Also much later, in the 17th century, when methods suggested by Apollonius and studies of conics were resumed, elements of the geometry of co-ordinates were taken over from the antique and expanded. The new social prospects opened to mathematics and constantly improved during the Renaissance cleared the road for these parts of the antique mathematics to effective operation in the future and to the release of essential impulses for the evolution of modern mathematics of variable quantities.