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Organon 7, 285-289

1970

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MEASURE, PROPORTION AND MATHEMATICAL STRUCTURE OF GALILEO'S MECHANICS

The subject of measure and proportion as an object of scientific analysis in Galileo's works is closely connected with the broadest and most general problems of the history of the 17th-century science. When did the classical science arise? What concepts and notions designated its origin? In this respect the concepts of uniform motion of a body left to its own resources and of uniformly accelerated motion of a freely falling body, on the one hand, and methods of experimental and quantitative mathematical investigation of nature, on the other, are usually regarded as the most characteristic. The synthesis of the concepts mentioned is most typical of the 17th century. These components of the new science did exist earlier, but the synthesis changed the character of both altogether. The concepts of uniform and uniform-difform motion were used by the Paris nominalists. In the 17th century and, first of all, in the *Discorsi* (partly in the *Dialogo* already) they became quantitative characteristics of motion and made the quantitative mathematical approach to it possible in principle. Bacon announced the decisive part of experimental science, experiment had much success with Gilbert and a long story in the 15th and 16th centuries. But only with Galileo it became a quantitative experiment answering not only the question "is it so?", "is it not so?" and "what's the cause of this or that phenomenon?" but most often the question of "equal—unequal?", "equal to what?", "how much is this?" With Galileo mathematics had not yet become an apparatus of science but all the necessary prerequisites for it had been created.

We are going to discuss some peculiarities of the mathematical structure of Galileo's mechanics in connection with this fundamental problem of the history of the origin of classical science.

The measurement, and consequently, the very notion of measure was not only connected with the aim of the most important experiments of Galileo. It was also connected with his basic idea. For Aristotle and his followers to define motion was to reply to the question: is a body in its natural place (in case of "natural" motion) or had it been pushed forward (in case of "forced motion)? Galileo defines motion by its velocity which characterized with Aristotle not the motion itself but rather the form of a moving body and the medium in which it was moving. That is why the measurement of velocity is of fundamental importance to Galileo in the study of motion. In his *Dialogo* he speaks mostly about the unchangeable absolute velocity, about the uniform motion of cosmic bodies participating in the daily motion of the Earth. The velocity of uniform motion becomes an object of the analyses. The question here is the proportionality of the way covered and time passed. This concept of direct (and inverse) proportionality was familiar to Galileo's predecessors. But Galileo introduced a principally new thing as compared with all his forerunners; in fact Galileo was the first to introduce quite distinctly (and opposing the new notion to the traditional definitions) the concept of the proportionality for all values of time. In the *Discorsi* there is a definition of uniform motion (distances covered in any equal periods of time are equal) and the following addition to it: "Visam est addere veteri difinitioni (quae simpliciter appelat motum aequabilem dum temporibus aequalibus transiuntur spatia) particulam quibuscunque, hoc est omnibus temporibus aequalibus: fieri enim potest, ut temporibus aliquibus aequalibus mobile pertransiat spatia aequalia, dum tamen spacia transacta in partibus eorundem temporum minoribus, licet aequalibus aequalia non sint" (G. G., VIII, 191).

In such a definition of uniform motion a more general concept of accelerated motion is anticipated. One can say, to use modern terms, that the momentaneous constant velocity is introduced in connection with the possibility of changing velocity.

As soon as a certain change of velocity comes into play, a uniform velocity becomes the object of changes, the object undergoing changes, i.e. the object of analysis. This signifies a great advance: before Galileo the object of search for active causes had been the change of co-ordinates. With Galileo velocity became such an object, that is to say, proportions of time and space.

Galileo does not seek a physical cause of the change of velocity, but neither does he dismiss the concept of such a cause for uniform motion as it was done in the *Dialogo*. He discovers the law determining the change of velocity and the change of the way. The velocity of a freely falling body is in proportion to the time, the way to the square of time. This square-law immediately enlarges the number of physically valid abstract formulas of proportionality. Before Galileo such were the direct and inverse proportionalities only. Neither Galileo nor Newton or the mechanicists of the 18th century introduced the concept of acceleration—it appeared as late as the beginning of the 19th century. The velocity is the object of changes for Galileo. The law of such a change under uniformly accelerated motion is the law of a change which has not yet gained the form of costancy of a certain proportion. The proportionality of force and acceleration became the fundamental thesis in post-Newtonian classical mechanics. This constancy of proportion or, in other words, constancy of mass seemed to be an absolute truth until Einstein. Here an unalterable proportion becomes an unalterable measure—a dynamic variable.

The ratio of the covered space and passed time has already become an independent quantity—constant (as in the *Dialogo*) or variable (as in the *Discorsi*). One can say that velocity has become with Galileo already measure (before Galileo only space and time but not their proportion were measured), and acceleration still remained proportion and had not yet become an independent local predicate characterizing a body. That is why Galileo spoke of an unalterable proportion in case of uniform motion but did not speak of it in case of uniformly accelerated motion. We discover two basic ideas of proportionality in Galileo's mechanics:

1) A moving body left to its own resources covers portions of space which are in proportion to time.

2) A body undergoing "natural" movement, i.e. freely, covers portions of space which are in proportion to square of time.

The first idea is characterized with converting proportion into measure: the proportion of space and time becomes an independent variable—an object of causally explained alterations. The second idea had not yet acquired this character: the proportion of space and square of time had not so far been converted into a certain measure—acceleration. Galileo does not examine the alterations of this acceleration. That is why acceleration as a subject of changes does not figure in Galileo's mechanics.

Later on different kinds of functional dependence lost their specific physical sense and were used in an abstract way. But with Galileo the mathematical categories figure as definitions of motion. In this concrete form we can clearly see the connection between measure and proportion. Preservation of a certain quantity of a certain measure (of velocity—explained in the *Dialogo*, of acceleration—implied in the *Discorsi*) is formulated as an immediate result of preservation of proportionality expressing a physical law.

A physical law is expressed in a one-to-one correspondence of the two sets: the set of positions of a body and the set of instants. Since motion is continuous, we have one-to-one correspondence of the two infinite sets. The finite trajectory of a particle becomes an infinite set of points and the finite time of motion becomes an infinite set of instants. Laws of mechanics give us a one-to-one correspondence between the elements of these sets.

Obscure as they may appear, these internal mathematical and logical peculiarities of Galileo's dynamics enabled him to express some very profound ideas about infinite sets.

Galileo devoted many pages in the *Discorsi* to infinity (infinity as a result of an infinite division of a finite quantity). One of the most important ideas here is the assertion of the impossibility to apply concepts discovered in the study of finite sets to infinite ones.

Simplicio pays attention in the *Discorsi* to a paradoxical feature of segments composed of an infinite number of points: one infinite set can be bigger than another. Salvaiti's answer is: "Queste son quelle difficoltà che derivano dal discorrer che noi facciamo, col nostro intellecto finito intorno a gl'infiniti, dandogli quelli attributi che noi diamo alle cose finite e terminate..." (G. G., VIII, 77–78).

Galileo did not know that when the notions "bigger" and "smaller" are generalized and the notion of *Mächtigkeit* introduced, the problem of comparison of infinite numbers can be solved not only in the negative (impossibility to apply the logic of finite numbers) but also in the positive. But he saw the beginning of the way leading later in the works of Cantor to the positive solution. Salviati gives an extremely interesting example of such idea. The set square is smaller than the set of all numbers (not all the numbers are squares), but is equal to the set of roots (each square has its root), and the set of roots is equal to the set of all numbers (each number can be a root). Here it can be seen that Galileo does not give a positive solution of the problem but sees the initial way to such a solution. He passes from vain searches for an actual infinity as a calculated innumerable set, which had been fairly frequent before Galileo, to the comparison of sets according to the one-to-one correspondence of their elements ("each square has its root", "each number can be a root").

Coming back to Galileo's dynamics we discover physical prototypes of a similar trend. A physical variable is measure, which had not broken away with proportion. Measure itself became an independent object of analysis (velocity) or, explicit, stays within limits of proportion (acceleration). But in all the cases one deals with the comparison of sets: instants and positions, instants and velocities.

The ultimate physical meaning of Galileo's ideas about infinity is the fact that they preserve the "birthmarks" of their origin and had not so far become so abstract as they did later, in the explanation of one more characteristic point. We find out in Galileo's ideas some very vague conjecture about infinitesimal quantity as a variable—about "un terzo medio termine, che à il rispondere ad ogni segnato numero." In other words, it is a quantity which can be equal to any part of the whole, to its infinitesimal part as well.

Proportion is an early pseudonym to the later simple functional dependence, expressing the principal absolute trustworthiness of scientific statements about the position of a particle in this or that point at the given moment. This trustworthiness is the basis of classical notions about nature. With Galileo mathematics is not only an instrument of science but also a criterion of its trustworthiness.

In the wide-known lines of the *Dialogo* devoted to the extensive and intensive trustworthiness of scientific knowledge Galileo says: "L'intellecto umano ne intende alcune cosi perfettamente, e ne ha cosi assoluta certezza, quanto se n'abbia l'istessa natura; e tali sono le scienze matematiche pure, cioè la geometria e l'aritmetica" (G. G., VII, 128–129). Mathematical cognition, Galileo proceeds, "arriva a comprendere la necessità, sopra la quale non par che possa esser sicudezza maggiore."

Necessity is a physical law; it is expressed in the form of constancy of some proportions and measures while other ones do alter. Thus, the concepts of Galileo mentioned above engendered, to some extent, the idea of a univocal determination of all the processes in nature.