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THE COPERNICAN HELIOCENTRIC MODEL OF THE UNIVERSE AND THE DEVELOPMENT OF CELESTIAL MECHANICS

Contemporary celestial mechanics is a branch of physics, and deals with the motion of celestial bodies, as governed by the universal gravity forces. The language it uses is mathematics with its main divisions: geometry, analitics, and algebra. The development of this science was closely related to progress in both theoretical knowledge, and experimental techniques. The problems of celestial mechanics exerted decisive influence on the development of mathematics and physics.

In its pre-Copernican form celestial mechanics presupposed the earth to be a round, motionless body in the centre of the universe. Within a general frame of this geocentric theory two systems developed: homocentric — proposed by Plato, Eudoxus of Cnidos, and Aristotle; and epicyclic — advanced by Appolounius and Hipparchus, and given its final form by Ptolemy.

The homocentric system assumed that the phenomena of the planet motion and the motion of concentric spheres were closely related. Rotational axis of each sphere had definite relationship with the next external sphere. Observations of planet movements served to determine the velocity of rotation and the location of the axes. Fixed stars were located on the last external sphere which moved around the axis of the world reproducing the twenty-four hour cycle of celestial sphere. The entire system contained 27 spheres.

In the epicyclic system the planets were assumed to describe a circle (epicycle), the centre of which described a larger circle (deferent). The earth was located eccentrically in relation to the center of deferent , and the angular velocity of the center an epicycle was constant in relation to equant center (Fig. 1). The moon and the sun circled the earth directly on deferents. In order to achieve better agreement of this theory with empirical observations the number of epicycles was increased.

Pre-Copernican theories had been accepted for fifteen centuries exerting strong influence on philosophy, literature, and science. Their complicated nature, however, made impossible a quantitative explanation of the ever increasing amount of observations. And then, our great compatriote Nicholas Copernicus (1473-1543), introduced heliocentric model of the universe in which the sun was the center of the world and of the planetary system. Planets rotated around the sun in circular orbits. That moment marked the beginning of modern astronomy. Heliocentric theory was consequently modified by Giordano Bruno. For him the sun was just one of the stars, and our solar system only one of the infinite number of systems in the universe.

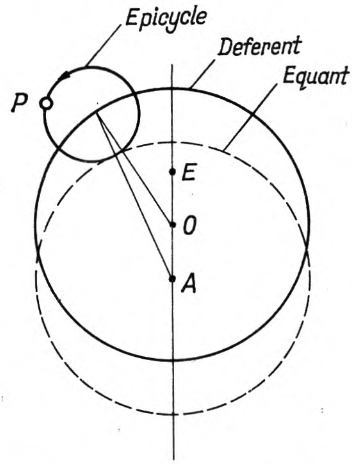

Figure 1. Epicycle: E—the earth, O—the center of deferent, A—the center of equant AO=OE

The beginning of the 17th century brought further development of astronomy. On the basis of observations made by Tycho Brahe (1546-1601), Johannes Kepler formulated three laws governing the motion of planets.

(I) The planets move about the sun in ellipses, at one focus of which the sun is situated.

(II) The radius vector joining each planet with the sun describes equal areas in equal times.

(III) The cubes of the mean distances of the planets from the sun are proportional to the squares of their times of revolution about the sun.

Kepler's laws were a strong argument in favor of Copernican theory. They showed how simple it was to describe the motion of planets with the sun taken as a reference point. These laws, however, had no theoretical interpretation being purely empirical in nature. The dynamic meaning was given to Kepler's laws by Newton (1642-1727) who formulated basic mechanics as a deductive system. Interpretation of these laws in the light of Newtonian mechanics is as follows:

According to the law of universal gravitation and to Newton's second law, the motion of a planet in polar coordinates system can be described by following equations:

$$m(\ddot{r}-r\dot{\varphi}^2)=P_r,$$

$$2\dot{r}\dot{\varphi}+r\ddot{\varphi}=0,$$

$$P_r=-k\frac{Mm}{r^2}, \tag{1}$$

where k is the gravitation constant, M is the mass of the sun, m is the mass of a planet and r is a distance from the sun to the planet.
From the second equation of equation set (1) it follows that:

$$r^2\dot{\varphi}=\text{const.} \tag{2}$$

which proves Kepler's second law.

From transformation of $(\ddot{r}-r\dot{\varphi}^2)$ component of the first equation of (1) we obtain:

$$\ddot{r}-r\dot{\varphi}^2=\left[\frac{d}{d\varphi}\left(\frac{dr}{d\varphi}\frac{d\varphi}{dt}\right)-r\frac{d\varphi}{dt}\right]\frac{d\varphi}{dt}. \tag{3}$$

Since from (2)

$$\frac{d\varphi}{dt}=\frac{c}{r^2}, \tag{4}$$

therefore

$$\ddot{r}-r\dot{\varphi}^2=\left[\frac{d}{d\varphi}\left(\frac{dr}{d\varphi}\frac{c}{r^2}\right)-\frac{c}{r}\right]\frac{c}{r^2}, \tag{5}$$

thus

$$c^2\left[\frac{d^2}{d\varphi^2}\left(\frac{1}{r}\right)+\frac{1}{r}\right]=\alpha. \tag{6}$$

Solving differential equation (6) we obtain a function:

$$\frac{1}{r}=\frac{\alpha}{c^2}\left[1+A\cos(\varphi+\varepsilon)\right] \tag{7}$$

Rearranging

$$\mathrm{r}=\frac{c^2}{\alpha[1+A\cos(\varphi+\varepsilon)]} \tag{8}$$

where A and ε are integration constants.

It can be seen that planets follow conical curves with foci at the center of force origin. Of course, for the planetary system these curves are ellipses and thus Kepler's second law is proved.

From (2) we can establish that:

$$cT = \pi ab \tag{9}$$

where a and b are the major and the minor planet ellipse semiaxes. Solving (9) for c and substituting it to (8) we have:

$$r = \frac{\pi^2 a^2 b^2}{T^2 a}\left[1 + A\cos(\varphi + \varepsilon)\right]^{-1}. \tag{10}$$

Let us note now that $r_{\max}$ appears as coefficient on the right hand side of (10)

$$a\left(1 + \sqrt{1 - \left(\frac{b}{a}\right)^2}\right) = \frac{\pi^2 a^2 b^2}{T^2 a} \cdot \frac{1}{1 - A}. \tag{11}$$

Since for any given ellipse b is proportional to a, therefore

$$T^2 = \beta a^3, \quad (\beta = \text{const.}), \tag{12}$$

which proves Kepler's third law because

$$a = \frac{1}{2}(r_{\max} + r_{\min}).$$

Scientific thought in its constant expansion is always deeply rooted in facts. The number of facts is steadily growing and with the passage of time these facts are arranged. Copernican model and Tycho Brahe's experiments were such facts for Kepler. Kepler's laws in turn served as such for Newton. The need for a theoretical interpretation of these laws was one of the stimuli which prompted the development of classical mechanics. In this sequence of events Copernicus deserves credit not only for the development of astronomy but also for the development of physics.

Further development of celestial mechanics paralleled closely the development of analitical mechanics, the fundamentals of which were worked out mainy by Lagrange, d'Alambert, Hamilton, Jacoby, Euler, and others. Lagrange's or Hamilton's equations are more advanced scientific tools, as compared to Newton's mostly because they do not change with transformation of coordinates. They describe motion in any given curvilinear system and allow for far reaching generalisations.

Let q^i be the curvilinear cooordinate, E — the kinetic energy of the system, and Q_i — the force acting on the ith coordinate. The equations of motion will appear now as:

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}^i}\right) - \frac{\partial E}{\partial q^i} = Q_i + \lambda_\mu \frac{\partial \Phi\mu}{\partial q^i}, \tag{13}$$

where $\Phi\mu(q^i, \ldots, q^N, t) = 0$ is the geometrical constraints equation, N is the number of degrees of freedom, and λ_μ is Langrange's multiplier.

Kinetic energy is a sum of three components:

$$E = E_0 + E_1 + E_2 \tag{14}$$

where

$$E_0 = a_0 (q^1, \ldots, q^N, t),$$

$$E_1 = a_j (q^1, \ldots, q^N, t) \dot{q}^j,$$

$$E_2 = \frac{1}{2} a_{jk} (q^1, \ldots, q^N, t) \dot{q}^j \dot{q}^k. \tag{14a}$$

By introducing Riemann's space defined by the metric tensor a_{jk} $(q^1, \ldots .. q^N, t)$, equation (13) can be reduced to:

$$\ddot{q}^i + \Gamma^i_{rs} \dot{q}^r \dot{q}^s = F^i, \tag{15}$$

where Γ^i_{rs} is Christoffel's symbol of the II kind,

$$F^i = \left[Q_j - \frac{d}{dt}\left(\frac{\partial E_1}{\partial \dot{q}^j}\right) + \frac{\partial}{\partial q^j}(E_0 + E_1) - \frac{\partial a_{jk}}{\partial t}\dot{q}^k + \lambda_\mu \frac{\partial \Phi\mu}{\partial q^j}\right] a^{ij},$$

and a^{ij} is the conjugated metric tensor.

Each of the N equations of (15) is a second order equation. In dealing with dynamic systems, encountered in astronomy, which are scleronomic and conservative, these equations take on special form. In such systems total energy of the system is constant:

$$T = E + V \tag{16}$$

where V is the potential energy.

For conservative systems another Riemann's space can be constructed, the local properties of which are described by the following metric tensor:

$$b_{ik} = a_{ik} (T - V). \tag{17}$$

Trajectories of motion in this new space are geodisic lines since the motion equation can be presented as:

$$\frac{d^2 q^i}{ds^2} + \Gamma^i_{mn} \frac{dq^m}{ds} \frac{dq^n}{ds} = 0, \tag{18}$$

where Γ^i_{mn} is expressed by the b_{ik} tensor components, and ds is an element of the arc ($ds = \sqrt{b_{ik}\, dq^i dq^k}$).

As can be seen, problem of n bodies can be reduced to finding geodisic line in Riemann's space called configuration space. These generalisations appear to have played an important part in the development of celestial mechanics. Analogies which appear here with the general theory of relativity need to be pointed out at this point because astronomy owes a great deal to this theory.

Equations (18) are — according to Einstein's views — natural generalisations of Newton's first law. Christoffel's symbols depend on gravitational field in which the motion of a material point (planet) is observed.

One of the more important achievements of the theory of relativity was explanation of Mercury's perihelion advance. Constant motion of Mercury's ellipse axis cannot be explained by Newton's theory. It turns out to be a result of the sun's gravitation field influence. Since Mercury's trajectory is the closest one to the sun, this effect is most pronounced there. Same effect was later observed for the Earth and Venus.

The case of Mercury's perihelion advance illustrates clearly how much better understanding of the universe phenomena we have gained from Copernican model; how much closer desciption of reality this model offers, as compared to Ptolemy's.

Since Ptolemy's time until today, celestial mechanics has been in the focus of attention of astronomers, physicists, and mathematicians. This resulted in the development of theory of dynamic systems, advance in observation techniques, and also in mathematics where discoveries were prompted by practical needs. Development of these sciences has influenced philosophical thinking of the last few centuries. Copernican model of the universe was the corner stone of this progress. Active will and contemplative mind in search for the truth gave rise to scientific thought that has shaped our view of the surroundig reality.