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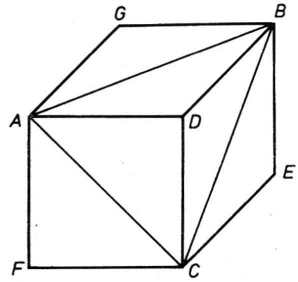
ON THE MATHEMATICAL WORKS OF KOCHAŃSKI

Adam A. Kochański is an unusual personality in the Polish science of the 17th century. He largely participated in the scientific activities in Europe at that time through his vast correspondence with the greatest minds of that epoque such as Leibniz, Hevelius, Wallis and others as well as through his active cooperation with the most renowned Leipzig scientific journal of the 17th century, that is, *Acta Eruditorum.* The scientific publications of Kochański that were placed in this journal since its origin mainly dealt with astronomical observations, mechanical constructions but, above all, with mathematics.

He published three works dealing with pure mathematics, their subject matter being up-to-date in the world of mathematics at that time.

The first work was published in *Acta Eruditorum* in 1682 and entitled *Solutio theorematum ab Illustri Viro in Actis huius Anni Mense Januario pag. 28 propositiorum.* The work is a reply to the question posed by an anonymous author in the very journal. Both Kochański's and Anonym's works were replies to the problem posed by S. F. Hartmann and partly solved by him in his work *la Duplation du triangle isogone et des autres figures rectilignes demontrée d'une nouvelle manière* (1668). Anonym's theorem reads as follows: "In every rectangular pyramid the sum of second powers of three edges coming out of the vertex of right angles equales the second power of the diameter of the described sphere". The presented theorem was educed as a conclusion from the following theorem: "In every rectangular pyramid the sum of second powers of three edges coming out of the vertex of right angles equals the second power of the diagonal of the whole parallelepiped formed by a completion of this pyramid". Kochański's proof is extremely simple from the point of view of modern terminology and denotations: $AB^2 = AD^2+DB^2$ because the angle D is right, the angle B in the triangle ABE is right hence: $AE^2 = AB^2+BE^2$ but $BE = DC$ hence the thesis of the theorem: $AE^2 = AD^2+DB^2+DC^2$.

In this work Kochański proved additionally the theorem that, recorded contemporarily, runs as follows: "If in the semicircle ADC with the diameter AC, two sections AD and DC are drawn, the perpendicular line DE is drawn from D to AC and similarly the perpendicular line EF is from E to AD, then the proportion AF:AE = = AD:AC is true".



The author completed the theorem about the proportional sections with a corollary that includes this problem when dealing with a three-dimensional space. Moreover, the theoretical problem suggested to Kochański the idea of constructing a device for building sections that remain in continuous proportion. He worked out two kinds of this device. The similar constructions were carried out by Descartes although for another purpose namely, in order to solve the algebraic equations by a graphic method.

Kochański's first work published in *Acta Eruditorum* presents the author's great mathematical maturity. He did not solve the problem that was interesting for him only from one point of view but he was interested in the theoretical and constructional aspects of the problem and in a generalization of the theorem.

The best known work of Kochański *Observationes cyclometricae ad facilitandam praxin accomodates,* the work which evoked much discussion throughout the following centuries was published in *Acta Eruditorum* in 1685. Kochański provided a long-awaited solution to the problem set by the ancient mathematics. The problem concerned the rectification of the circle, that is, the constructional determination of a section of the length equal to the perimeter of the circle by means of compasses and a scale.

The theorems that dealt with insolvability of this problem were presented and proved much later in the works of Lagrange, Ruffini, Gauss, Abel and Galois. Nevertheless, a search for the solutions throughout the centuries gave effects in the aproximate geometrical constructions that determined an irrational number π with some accuracy. Kochański's clever construction allowed acquiring the value $\pi = 3{,}141533338705093$.

Kochański leads tangents BG and DH equal to the radius AC to the semicircle BCD and connects points G and H. He describes an arc to the intersection with the semicircle BCD in points E and F by means of the radius CA from the point C. The secant AEI is led through E thus determining the section of the tangent BI. HL equal to BD is led, the section IL is equal to, roughly speaking, the length of the semicircle BCD. The calculus proof is based on Pythagorean theorem. $AB = 1$, $\mathrm{tg}\,30° = BI = 0{,}577350269189626$ (original record is an expression of this number by means of the shift of one place to the right in relation to 1), IG=

$= 0{,}422649730810373$, $KH+HL = KL = 2{,}422649730810373$, $IK^2+KL^2 = 9{,}86923\ldots$ and it equals IL^2, hence $IL = \pi$ that is expressed above.

Applying the method of sequential approximations with the surplus and insufficiency Kochański created a table from which one can determine π of an approximate value 3,1415926535897932384626433.

Kochański was deeply interested in the problems dealing with the theory of numbers, especially with their magic, as their properties were treated then. Problems connected with the magic of numbers had already been known in ancient India and China. Their revival took place in Europe in the 17th century in the works of the mathematicians interested in the properties of whole numbers in general.

The methods of forming magic squares (introduced into Europe by Moscopulos in the 15th century) were applied. The work of Claude Gaspar Bachet de Méziriac from the year 1613 entitled *Problèmes plaisans et délectables qui se font par les nombres* contained a method of forming the magic squares of even order. Pierre Fermat, Bernard Frénicle de Bessy, Athanasy Kircher and others also dealt with these problems.

In 1686 Kochański published his work entitled *Considerationes quaedam circa quadrata et cubos magicos, nec non aliquot problemata omnibus Arithmophillis ad investigandum proposita* in *Acta Eruditorum.* It was a work the purpose of which was to stir an interest in magic squares of constant difference. So far the squares of the constant sum had been formed mainly and exclusively on the basis of the method described by Kircher in *Arithmologia sive de abditis Numerorum mysteriis* (Rome, 1665). The famous method of Kircher (precisely speaking that of Bachet de Méziriac) of forming magic squares with a side equal 4 and the constant sum of numbers in each row, column and along diagonals became the starting point for forming new "sum" squares (Fig. 1—a, b, c, d) by Kochański through his discovery

Fig. 1

a)

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

b)

12	6	7	9
1	15	14	4
13	3	2	16
8	10	11	5

c)

16	13	3	2	
4	1	15	14	4
9	12	6	7	9
5	8	10	11	5
	13	3	2	16

d)

6	9	12	7	
3	16	13	2	3
15	4	1	14	15
10	5	8	11	10
	9	12	7	6

of their representations. The squares of the same type but of the order 5 are presented in examples in Fig. 2, 3, 4. Mainly the property of arithmetic sequences was utilized in

11	24	7	20	3
4	12	25	8	16
17	5	13	21	9
10	18	1	14	22
23	6	19	2	15

Fig. 2

12	21	20	9	3
1	15	24	18	7
10	4	13	22	16
19	8	2	11	25
23	17	6	5	14

Fig. 3

15	4	7	16	23
24	12	1	8	20
17	21	13	5	9
6	18	25	14	2
3	10	19	22	11

Fig. 4

For the first time in the works dealing with this subject, Kochański wrote about the magic differential squares. For the author a magic differential square of the order 4 was such a setting of numbers 1-16 in a square that the difference of the sum of numbers in even places and the difference of the sum of remaining numbers in the odd places along with the successive increasing setting were constant in every row, column and along diagonals. For instance Fig. 5 and 6:

1	6	13	2
10	5	14	9
7	12	11	16
8	3	4	15

Fig. 5

11	24	9	16	3
4	12	25	8	20
19	5	13	21	7
6	18	1	14	22
23	10	17	2	15

Fig. 6

for the first row of the square 5: 1, 2, 6, 13 is $(2+13)-(1+6) = 8$. Similarly for the differential square of the order 5 Kochański demanded the difference of the sum of the numbers in the odd places and of the sum of the numbers in the even places to be constant in all rows, columns and along the diagonals. It can be noticed (Kochański does not provide a method of forming) that a square of an ascribed property can be formed while shifting into the square the remaining numbers on the enterier to the places distant of 5 (Fig. 7). In this

1
10 2
11 9 3
20 12 8 4
21 19 13 7 5
22 18 14 6
23 17 15
24 16
25

Fig. 7

way square 6 was formed. Kochański presents further squares of this type with the imposed in advance sum or difference presenting, *e.g.*, the year of their forming as an interesting result of the previous method. For instance, the date of the edition of the work in *Acta Eruditorum* is a "magic" sum for the square 8:

423	421	418	424
428	414	417	427
416	426	429	415
419	425	422	420

Fig. 8

For the first time, Kochański proposed forming of magic cubes. A magic cube is such a setting of the successive natural numbers from 1 to n^3 in the cube with the edge equal n, that a sum of numbers in each column, row and along four diagonals of the cube could be equal $(1+n^3)\, n^3 : 2n^2 = (1+n^3)\, n{:}2$. The examples of this kind of cubes were not published in his work but Kochański proposed them to others especially in case of $n = 6$, 10, 14 and with the additional property of "magic" in regard to the difference. The magic cubes were formed later by Sauveur in his work *Construction des Quarrés magiques* (Paris, 1710), Hugel (1859), Sheffler (1882) and others. Kochański's work on magic squares considered against the historical background of this problem shows its great originality, its own unique contribution to the development of the problem that was particularly complicated for Kochański's contemporaries and the mathematicians of two following centuries. However, he did not introduce the general methods of forming his magic squares but that could only be expected at the time when the regularities and general principles of the basis of the theory of numbers and the theory of the group of transpositions were discovered. In the history of this problem, Kochański is the only one who formed the differential squares and the first who generalized the formation of magic figures against the three-dimensional space.

In the 17th century and later, the correspondence was a form of the exchange of experiences and information apart from scarce publications in the then existing scientific journals. Thus Kochański's correspondence with many scientists, especially with Leibniz, is essential for the history of mathematics. A special attention should be paid to the problem of the "universal language", the so-called "universal characteristic" of Leibniz which was discussed by Kochański in his letters (*e.g.* letter to Leibniz dated on May 30th, 1692). Leibniz was working then on forming a universal language for transferring a human thought by means of a symbolic calculus similar to the algebraic operations. Kochański's version was connected more with the linguistic side of the problem as he aimed at some uniformity of European languages not only in spelling but also in sound reproduction.

The correspondence with Leibniz is also the only source enabling us to determine the bulk of Kochański's mathematical investigations. The part of his work remaining in manuscripts was burnt (1944) during the fire of Warsaw together with the materials of Samuel Dickstein who was preparing not only their penetrating evaluation but also edition. In 1934, on the 2nd Congress of the Mathematicians of the Slavonic Countries in Prague Dickstein informed the participants about his undertakings.

Kochański's mathematical interests were various: a construction of an arithmetic machine and, connected with it, making of mathematical tables, a calculation of the side of the polygon inscribed in the circle with a given radius and, at last, the analytic calculus of Leibniz. He succeeded in finding out the sides of the polygons up to the hundredth angle although it was not a general method and did not include 83, 89 and 97th angles (at that time he did not know the works giving more general results, *e.g.* those of Viète of 1574). At high expenditure of work he constructed the mathematical tables of sines and tangents for the first and last degree of a quadrant of the circle, in every second interval when the radius equals 10^{10}. He also succeeded in constructing the tables of squares and cubes with their logarithmus assuming 10^{10} as a side of the first square. Kochański worked on mathematical tables and simultaneously he attempted at constructing the arithmetic machine that could multiply and divide. The work became more satisfactory in the years 1692-1698 when in his correspondence with Leibniz the inspiring discussion of his arithmetic machine brought newer and newer solutions and improvements to Kochański. He proposed a construction of the machine on the basis of Neper's rods, with which a cylinder or tenangled prism would be inlaid to make possible carrying out suitable operations. However, this method was unsatisfactory in regard to more complicated calculi. The discussed results of Kochański were never published as separate works although they undoubtedly deserved it according to Leibniz's opinion. Through his correspondence with Leibniz Kochański became familiar with Leibniz's analytic calculus that he admired but he had not enough time to devote himself to it thoroughly.

The vast correspondence of Kochański with many scientists representing different scientific fields shows his activity in the scientific life, in solving the actual problems of mechanics, astronomy and mathematics. His opinion was appreciated and his advice was eagerly awaited in difficulties of the theoretical or experimental character.

Through his correspondence and the vast knowledge Kochański was one of few scientists representing Poland in the European science of that time.