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A REINTERPRETATION OF ARISTOTLE'S SYLLOGISTIC

Contemporary intensive studies in the history of logic, inspired by H. Scholz's *Geschichte der Logik* (1931), are connected, as it is well known, with the investigations of J. Łukasiewicz in the field of ancient logic. After his pioneering work on Stoic logic *Zur Geschichte der* Aussagenlogik (1935), which fully rehabilitated the remarkable achievements of Chrysippos and other Stoic logicians, till then misunderstood, neglected or even unrightly denounced, Łukasiewicz focussed his attention on the logical system of Aristotle. Already in his lecture on Aristotle's syllogistic, held on June 9, 1939 [1], and later on in his famous monograph *Aristotle's Syllogistic from the Standpoint of Modern Formal Logic* (1951), he presented in a systematic manner a quite new insight into its essence and structure, radically differing from all older interpretations advocated from the traditional point of view in the last century, especially by C. Prantl and H. Maier. [2] The results, obtained by Łukasiewicz in his historical analyses or rather reconstructions of ancient logic, together with his methodological approach to the study of its history, widely based on theoretical conceptions and technical means elaborated in recent times only, determined all further research in this direction.

In spite of some doubts as to the appropriateness of analysing and interpreting Aristotle's logic from the standpoint of contemporary logic, raised by historians of philosophy or classical scholars, Łukasiewicz's methodology was, in principle, admitted as a useful tool by the majority of historians of logic. At the same time some of them,

[1] *O sylogistyce Arystotelesa,* in: J. Słupecki (ed.) J. Łukasiewicz, *Z zagadnień logiki i filozofii, Pisma wybrane,* Warszawa 1961, pp. 220–227.

[2] C. Prantl, *Geschichte der Logik im Abendlande,* Bd. I, Berlin 1855; H. Maier, *Die Syllogistik des Aristoteles,* Bd. I–II, Leipzig 1896–1900.

especially G. Patzig [3], accept his interpretation with some modifications or corrections. Others, mainly J. Corcoran and T. J. Smiley, strongly oppose his conclusions and suggest a very different explanation of Aristotle's ideas concerned with the nature of the syllogism and the deductive structure of his logical system.[4]

It is the aim of my paper [5] to evaluate these modern interpretations and to submit such a reconstruction of Aristotle's assertoric syllogistic that would be, as far as possible, supported by the text of the *Organon.* Before going into a detailed analysis of some basic topics related to the problem in question, I shall explicitly elucidate some methodological assumptions which will serve as a clue to my own approach.

First of all, I take for granted that the known text of the *Organon,* as arranged rather late by Andronikos, does not represent an authentic account of Aristotle's logic, elaborated by himself into a certain system, but rather expresses the endeavour of his followers to systematize his notes and drafts of his lectures on logic. This editorial work could not conceal the natural evolution of Aristotle's ideas, the modifications of his views, changes in the logical terminology nor the ripening of the logical core of his very broadly oriented examinations. His logical investigations were quite inevitably related to various ontological, linguistic and methodological problems and were realized on different levels of abstractions with the obvious tendency to increase the formal aspects of his initial, unformal analysis. In elaborating his assertoric syllogistic, anticipated in a rudimentary form in the *Topics* by his theory of presyllogistic arguments, Aristotle had to solve many serious problems: the delimitation of the logical and methodological nature of a syllogistic argument, the differentiation of valid and invalid moods, the delineation of criteria for deciding whether a given syllogism is valid, etc.

Secondly, Aristotle consciously utilizes in his investigations a dialectical analysis of all unsolved problems from various aspects, adducing the pros and cons for its most plausible solution. This approach affected the choice of topics discussed and the manner of their examination, as well as their interpretations without any attempt to later explain the reasons for a shift of his conceptions, without any correction of his previous views.

[3] G. Patzig, *Aristotle's Theory of the Syllogism. A Logico-philosophical Study of Book A of the Prior Analytics,* Dordrecht 1968 (Engl. transl. of *Die Aristotelische Syllogistik,* Göttingen 1959).

[4] J. Corcoran, *A Mathematical Model of Aristotle's Syllogistic,* "Archiv für Geschichte der Philosophie" 55 (1973), pp. 191–219; *Aristotle's Natural Deduction System,* in: J. Corcoran (ed.), *Ancient Logic and its Modern Interpretations,* Dordrecht—Boston 1974, pp. 85–131; T. J. Smiley, *What is a syllogism?*; "Journal of Philosophical Logic" 2 (1973), pp. 136–154.

[5] Comp. also my paper *What is the Nature of Aristotle's Syllogisms?*, "Acta Universitatis Carolinae, Phil.-hist." 4 (1977), pp. 11–28.

For both these reasons, any presentation of his logic as a complete, fully developed system, unified in all details, which has been in vain attempted by many generations of logicians for many centuries, necessarily leads to a one-sided elucidation of his ideas or to textual desinterpretation, especially when it seems that some passages are incompatible with the main line of the suggested explanation. It is, hence, unwarranted to attempt just one interpretation that would enable us to work out a uniform, unproblematic solution. In contradistinction to this traditional approach, adopted in modern interpretation as well and even strengthened by the contemporary conception of completeness, I stress the "open" and "unfinished" nature of his logical investigations. I do not, therefore, blame Aristotle for making this or that mistake. Neither do I make an attempt to conceal what seems to appear as contradictory or faulty conceptions, nor do I accuse him of inconsistencies or unexplained changes of his standpoint, but I consider any such discrepancy as a fact for which we can find various, often very diversified reasons. This critical attitude to his lasting heritage cannot depreciate his ingenious results in the field of logic. In fact, it corresponds more faithfully to the truly Aristotelian tradition than any sophisticated systematization of his doctrines. At the same time, it helps us to follow the evolution of his conceptions, the diachronical picture of his logical ideas synchronically petrified by the editor of the *Organon.*

Thirdly, it can be easily shown that all interpretations of Aristotle's logic have always been determined by the knowledge of logic attained in the epoch in question as well as by the then adopted philosophy of logic. From the traditional point of view the essence of an Aristotelian syllogism is rooted in the methodological function of the middle term. Contemporary interpretations concentrate their attention on purely formal aspects of his syllogistic. In both cases, of course, more apparently in modern views, such an approach is burdened by an ahistorical attitude which neglects the fundamental difference between a historical analysis attempting to reproduce the initial state in the development of logic, and its reconstruction based on conceptions which were elaborated under different conditions many centuries later.

For these reasons I consider it very important to draw a clear distinction between what Aristotle explicitly says and what he implicitly presupposes in his procedures. Similarly, I differentiate between what we can prove in respect to the adopted interpretation without any doubts and what we can only hypothetically suppose that he intended or might have intended to say. At the same time, I am very well aware that the desirable aim to reproduce Aristotle's own views authentically cannot be realized. Nevertheless, we always have to attempt such an interpretation that seems to be approximately appropriate for the level of knowledge of the man who laid the foundations of logic and gradually developed its first deductive system. When criticising the transfer of modern conceptions into a historical context which leads then rather to a modern study in logic based on an ancient text, than to the evaluation of the text itself, I object neither against the use of such conceptions nor against the application of modern technical means, provided that they are conceived only as methodological tools, but not as a goal of such "pseudo-historical" investigations, and so far as their utilization is in agreement with philological reasons and theoretical analyses.

Let me now, after these preliminary remarks, lay down the core of the contemporary controversion about the nature of Aristotle's syllogistic.

Under the philosophical assumption that it is not true to think "that logic is the science of the laws of thought" (1951, 12), which is obviously influenced by Łukasiewicz's desire to avoid the conception of psychologism in logic, he draws a rather surprising conclusion that "the logic of Aristotle was conceived as a theory of special relations, like a mathematical theory" (1951, 15).

From this viewpoint which, however, yields the counter-factual consequence that the syllogistic of Aristotle is not a logical theory, as it is otherwise commonly maintained, he further concludes that this special theory is an axiomative deductive system formulated in the object-language. According to his view, this axiomatic system contains—besides auxiliary expressions and formulae of an underlying, but not explicitly formulated more elementary logic, namely the propositional logic of Stoics, developed after Aristotle's death—four primitive relations and four axioms. The constants of this system are four relations between universal terms, namely "to belong to all", "to belong to none", "to belong to some" and "to not-belong to some", symbolized by medieval logicians by the letter *a*, *e*, *i* and *o* respectively (1951, 14). As axioms, at least, for the first, basic deductive system of Aristotle's syllogistic, Łukasiewicz assumes the mood *barbara* of the first figure, the mood *datisi* of the third figure, and two formulae which, however, Aristotle did not explicitly consider as archai of his system, viz. "*A* belongs to all *A*" and "*A* belongs to some *A*" (1951, 46).

Another specific feature of Łukasiewicz's conception is his interpretation of the Aristotelian syllogism itself. Apparently under the influence of his work on ancient propositional logic, he holds for granted that an Aristotelian syllogism—in contradistinction to the formulation of syllogisms in traditional logic—is not a rule of inference or an argument-scheme, but a logical law, a logical thesis (1951, 55). This view seems to be supported by the formulation of syllogisms with variable terms, *e.g.* *A, B, C* in the first figure, in the systematic exposition in *Prior Analytics,* viz. "If (ei) *A* is predicated of all *B* and (kai) *B* of all *C*, then it is necessary that *A* is predicated of all *C*" (*Anal. pr.* I c. 4 p. 25b 37–39). Łukasiewicz (1951, 78) interprets this formulation of the mood *barbara* as a (material) implication containing in its antecedent the conjunction of both premises and having as consequent the conclusion of this syllogism, namely

$$CKAabAbcAac.$$

From this explanation, which is based on the same interpretation of the expression "ei" as in Stoic logic, it however follows that Aristotle does not deal in his syllogistics with arguments, *i.e.* assertoric or modal syllogisms, but only with compound propositions, similarly as in any system of propositional logic.

Adopting this point of view Łukasiewicz was forced to criticise, I think illegitimately, Aristotle's indirect proof of the mood *baroco,* and similarly in the case of the mood *bocardo* (comp. *Anal. pr.* I c. 5 p. 27a 36ff; c. 6 p. 28b 17ff) as "neither sufficient nor as a proof by reductio ad impossibile" (1951, 55).

As far as I know, with the only exception of the standpoint maintained by B. S. Gryaznov [6], Łukasiewicz's interpretation of the Aristotelian syllogism, and consequently his reconstruction of Aristotle's syllogistic, in their orthodox version, are quite generally rejected today. I have stressed already in a paper published many years ago [7] that to conceive of an Aristotelian syllogism as an implication does not hold; neither in all cases nor in respect to its role from the methodological point of view. Even in *Prior Analytics (e.g.* I, c. 14 p. 32b 38f; c. 15 p. 34a 34ff; c. 6 p. 36a 8ff), Aristotle expresses the syllogism without the connective "ei" and in the *Posterior Analytics (e.g.* I c. 6 p. 75a 8ff, c. 12 p. 78a 15ff; c. 13 p. 78b 24ff) he uses the formulation with "ara" separating, thus, the conclusion from the premises in an inference or inference-scheme as it is usual in traditional logic. For this discrepancy I have suggested the following explanation. Though Aristotle did not

[6] B. S. Gryaznov, *On the Historical Interpretation of Aristotle's Analytics,* "Organon" 11 (1975), pp. 193–203. Comp. *e.g.* the following statements: "[Łukasiewicz] has managed to formalize Aristotelian syllogistics in a most adequate manner" (p. 195), "Aristotelian syllogistics [...] is not a theory of proofs, but a theory of propositions [...] Aristotle himself asserted that syllogism is not an inference rule, but a proposition" (p. 197).

[7] K. Berka, *K formulaci sylogismu u Aristotela,* "Filosofický časopis" IV (1956), pp. 365–373.

need to make the distinction between "implication" and "inference", since he does not deal with propositional logic, he nevertheless differentiates between the expression and function of the syllogism. In passages where he analyses its formal properties, he formulates—from the modern point of view—the syllogism as a logical thesis. Taking, however, into account its application in his apodeictics, his syllogism functions as an inference.

A similar explanation was suggested by W. Kneale and M. Kneale.[8] Even Patzig—who otherwise attempts to develop Łukasiewicz's conception—changes in the preface to the second edition of his monograph (1968, XV) his older standpoint that Aristotle's logic is rather a logic of propositions than a logic of rules and claims that the connective "ei" is more adequately explicated in the meaning of Lorenzen's "logical implication".

Today, in connection with this problem, I would like to modify my previous explanation by pointing out that the connective "ei" need not be interpreted in the same meaning as in Stoic propositional logic. Especially in the case of perfect syllogisms, this connective has to be understood in the meaning of derivability. Under this interpretation, the above mentioned symbolization of the mood *barbara* has to be expressed more aptly as follows

$$Aab,\ Abc \vdash Aac,$$

$$\text{or} \quad \frac{Aab,\ Abc}{Aac} \quad \text{respectively} \quad \frac{Aab \wedge Abc}{Aac.}$$

Assuming a very peculiar conception that mathematical logic is "a branch of applied mathematics which constructs and studies mathematical models in order to gain understanding of logical phenomena" (1974, 86), which is doubtlessly at least from a historical point of view, quite unjustified, J. Corcoran adopts an obviously anti-Łukasiewiczian point of view. In his very ambitious conception which, however, in many relevant cases either ignores or misinterprets passages which do not support his ahistorical reconstruction of Aristotle's logic, he tries to prove the following issues. Aristotle's syllogistic is an underlying logic which includes a natural deduction system and not an axiomatic theory, an axiomatic science. Aristotle's theory of deduction is logically sound in every detail. This system is complete in the sense that every semantically valid argument is deducible.[9] Aristotle's logic presupposes no other logical concepts, not even those of propositional logic.

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[8] W. Kneale—M. Kneale, *The Development of Logic*, Oxford 1962, pp. 80f, 370.
[9] Comp. similarly J. Corcoran, *Completeness of an Ancient Logic*, "The Journal of Symbolic Logic" 37 (1972), pp. 696–702.

In respect to the topic of my paper I have to discuss in a more detailed manner, first of all, how Corcoran explains the nature of the syllogism. According to his view (1974, 98ff), an Aristotelian syllogism is primarily a deductive argument, a deduction [10], including in addition to premises and conclusion a chain of reasoning showing that the conclusion really follows from the premises. It is, therefore, an inference rule from true premises to a true conclusion.

This interpretation is incompatible with many important issues. It neglects the difference between syllogism as analysed in *Prior Analytics* and as utilized in *Posterior Analytics*. It excludes from Aristotle's logic syllogisms with false premises. [11] It confuses the fundamental difference between syllogism and proof, the distinction between the validity of an argument and the truth of its components, between syllogisms as a special kind of forms of reasoning and various proof-methods as well as between "syllogistic proofs" used in Aristotle's theory of deduction and "scientific proofs" utilized in particular sciences.

To justify my criticism, let me just mention two passages from the *Organon* which evidently contradict Corcoran's point of view. In his definition of a perfect syllogism Aristotle says quite unambiguously that such a syllogism is a discourse "which needs nothing else than what has been stated to make plain what necessarily follows" (*Anal. pr.* I c. 1 p. 24b 22–24). In another connection Aristotle explicitly stresses that he considers as syllogisms—obviously in its basic meaning, since he admits as syllogism also the enthymema (*Anal. pr.* II c. 27 p. 70a 10) and the polysyllogism (*Anal. pr.* I c. 25 p. 41b 36nn)—only such arguments in which the "conclusion follows from two premises and not from more than two" (*Anal. pr.* I c. 25 p. 42a 32–34). It is, thus, quite clear that an Aristotelian syllogism does not include its proof and is not in itself a deduction in the sense of Corcoran's interpretation.

There is, of course, one important point in which I agree with Corcoran's criticism of Łukasiewicz's reconstruction. Taking into account what Aristotle explicitly says and how he, often only tacitly, proceeds in the "reductions" of imperfect moods, it is definitely more appropriate to conceive of his syllogistics as a system of natural deduction [12] than as an axiomatic system. This interpretation, which I shall now illustrate by a textually exemplified reconstruction of Aristotle's syllogistic, helps us to elucidate other problems as well, especially the controversial issue

[10] Analogously, Emiley treats "syllogisms as deductions", op. cit., p. 140.

[11] In respect to this problem I fully agree with the opposite view of G. Patzig. Comp. his paper *Aristotle and Syllogisms from False Premises*, "Mind" 68 (1959), pp. 186–192.

[12] J. Słupecki and L. Borkowski, *Elementy logiki matematycznej i teorii mnogości* (Elements of Mathematical Logic and Set Theory), Warszawa 1963, pp. 19f.

"how can the syllogisms themselves be proved—*quis demonstrabit demonstrationes ipsas?*".[13]

In his first, obviously oldest deductive system (*Anal. pr.* I c. 2, 4–6), Aristotle explicitly accepts as inference rules the four perfect moods of the first figure:

barbara	$AaB \wedge BaC \vdash AaC$ [14]
celarent	$AeB \wedge BaC \vdash AeC$
darii	$AaB \wedge BiC \vdash AiC$
ferio	$AeB \wedge BiC \vdash AeC.$

Implicitly, he assumes[15] the rules of contradiction (CONTRAD) holding for subject-predicate propositions of opposite quality and quantity:

$$AaB \vdash \neg(AoB),\quad AeB \vdash \neg(AiB),\quad AiB \vdash \neg(AeB),\quad AoB \vdash \neg(AiB)$$

and vice versa, and the rules of contrariety (CONTRAR) similarly holding for such propositions of the same quantity, but opposite quality

$$AaB \vdash \neg(AeB) \qquad AeB \vdash \neg(AaB).$$

Only intuitively he uses the rules for elimination and introduction of conjunction, which I symbolize as (EC) and (IC) respectively. These rules are explicitly or implicitly considered as primitive rules of inference: their validity is, therefore, evident without any proof.

Aristotle's system of natural deduction contains, further, three proof-methods: the proof by exposition (*ekthesis*), the direct or ostensive proof (DPR), and the indirect proof or proof by *reductio ad impossibile* (IPR). For proving the validity of all imperfect syllogisms, the direct and indirect proofs are sufficient. The proof by exposition is, in fact, necessary only in one case, namely for proving the validity of one conversion rule.

Besides these proof-methods and primary inference rules, Aristotle introduces as secondary rules of inference the rules of *conversio simplex* (CS) for *e*-propositions and *i*-propositions, and the rule of *conversio per accidens* for *a*-propositions. The rule of *conversio simplex* for *e*-pro-

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[13] Patzig, *Aristotle's Theory of the Syllogism*, p. 133.

[14] We are using a unified notation, based on Aristotle's symbolization of the first figure and on his "quasi-formalized" notation according to which the first variable denotes the predicate and the second the subject of the proposition in question (comp. *e.g. Anal. pr.* I c. 4 p. 25b 37–39), instead vice versa as in traditional logic. As already mentioned, Aristotle says "If *A* is predicated of all *B* and *B* of all *C*, then it is necessary that *A* is predicated of all *C*", whereas in traditional logic we find formulation "All *B* are *A*, all *C* are *B*, therefore all *C* are *A*". In our notation—in distinction to that adopted by Łukasiewicz—the letters *A*, *B*, *C* denote the syllogistic terms and the letters *a*, *e*, *i*, *o* indicate the quality and quantity of the subject-predicate proposition contained in the syllogistic moods.

[15] Comp. *De Interp.* c. 7 p. 17b 5ff.

positions is proved by *ekthesis,* and the other two conversion rules are proved indirectly.

The indirect proof of the conversion rule for *a*-propositions is expressed in a very condensed way:

"But if *A* belongs to all *B,* also *B* belongs to some *A*. For if it would belong to none, *A* will belong to no *B*. But it was supposed that it belongs to all". (*Anal. pr.* I c. 2 p. 25a 17–19).

In the system of natural deduction, this proof can be reproduced as follows:

$$AaB \rightarrow BiA$$ [16]

Proof:	(1)	AaB	supposition
(IPR)	(2)	$\neg(BiA)$	supposition of IPR
	(3)	BeA	(2) : CONTRAD
	(4)	AeB	(3) : CS
	(5)	$\neg(AaB)$	(4) : CONTRAR
		inconsistency	(1), (5)

By means of these primary and secondary rules the validity of imperfect syllogisms of the second and third figure is proved either indirectly (the moods *baroco* and *bocardo*) or directly (all other ones).

In order to demonstrate Aristotle's ostensive procedure I mention the "reduction" of the mood *festino* (2nd figure) —

"If *M* belong to no *N*, but to some *X*, then it is necessary that *N* should not belong to some *X*. For since the negative is convertible, *N* will belong to some *M*; but *M* was admitted to belong to some *X*; therefore *N* will not belong to some *X*. The conclusion is reached by means of the first figure" (*Anal. pr.* I c. 5 p. 27a 32–36)—which can be analysed as follows:

$$MeN \wedge MiX \rightarrow NoX$$

Proof:	(1)	$MeN \wedge MiX$	supposition
(DPR)	(2)	MeN	(1) : EC
	(3)	MiX	
	(4)	NeM	(2) : CS
	(5)	$NeM \wedge MiX$	(4), (3) : IC
		NoX	(5) : *ferio*

The direct proof of the mood *disamis* (3rd figure) with a reversed order of premises, namely—

[16] As it is common in systems of natural deduction, all not yet proved formulae are expressed in the form of implicative propositions. After being proved they serve or can serve as secondary rules, and are then formulated similarly as primitive rules, *i.e.* in our case: $AaB \vdash BiA$.

"If R belongs to all S, P to some S, P must belong to some R. For since the affirmative is convertible, S will belong to some P; consequently since R belongs to all S, and S to some P, R must also belong to some P; therefore P must belong to some R" (*Anal. pr.* I c. 6 p. 28b 7–11)—can be in extenso characterized as follows:

$$RaS \wedge PiS \rightarrow PiR$$

Proof:	(1)	$RaS \wedge PiS$	supposition
(DPR)	(2)	RaS	(1) : EC
	(3)	PiS	
	(4)	SiP	(3) : CS
	(5)	$RaS \wedge SiP$	(2), (4) : IC
	(6)	RiP	(5) : *darii*
		PiR	(6) : CS

Both direct proofs and similarly all other ones are expressed in the text of the *Organon* in great detail. With the exception of the obvious rules for the elimination and introduction of conjunction, Aristotle mentions in his verbal formulations nearly all steps.

The indirect proofs of the moods *baroco* (2nd figure) and *bocardo* (3rd figure) are, however, expressed in a much more condensed manner. In the first case Aristotle says:

"If M belongs to all N, but not to some X, it is necessary that N should not belong to some X; for if N belongs to all X, and M is predicated also of all N, M must belong to all X; but it was assumed that M does not belong to some X" (*Anal. pr.* I c. 5 p. 27a 37–27b 1).

This proof can be in extenso formulated as follows:

$$MaN \wedge MoX \rightarrow NoX$$

Proof:	(1)	$MaN \wedge MoX$	supposition
(IPR)	(2)	$\neg(NoX)$	supposition of IPR
	(3)	MaN	(1) : EC
	(4)	MoX	
	(5)	NaX	(2) : CONTRAD
	(6)	$MaN \wedge NaX$	(3), (5) : IC
	(7)	MaX	(6) : *barbara*
		inconsistency	(4), (7)

In the second one we find this formulation:

"If R belongs to all S, but P does not belong to some S, it is necessary that P does not belong to some R. For if it would belong to all R, R belongs to all S and P to all S; but it was assumed that it did not" (*Anal. pr.* I c. 6 p. 28b 17–20).

The corresponding proof of the mood *bocardo*, which is in the text expressed with a reversed order of premises, is then as follows:

	$RaS \wedge PoS \rightarrow PoR$		(respectively $PoS \wedge RaS \rightarrow PoR$)
Proof:	(1)	$RaS \wedge PoS$	supposition
(IPR)	(2)	$\neg (PoR)$	supposition of IPR
	(3)	RaS	(1) : EC
	(4)	PoS	
	(5)	PaR	(2) : CONTRAD
	(6)	$PaR \wedge RaS$	(5), (3) : IC
	(7)	PaS	(6) : *barbara*
		inconsistency	(4), (7)

Both proofs, expressed by Aristotle in a very condensed manner, contain explicitly only the supposition of the indirect proof in the form *NaX*, respectively *PaR*, the conclusion *MaX*, respectively *PaS*, deduced by means of the rule *barbara*, and the statement of the inconsistency between the lines (4) and (7).

These procedures are similarly applied in the "reductions" of imperfect moods in the second deductive system (*Anal. pr.* I c. 7) containing a reduced number of perfect moods as inference rules, namely, *barbara* and *celarent* only. The growth of formalization in the development of Aristotle's conception, representing presumably the latest stage of his investigation, is best exemplified by his view that the deductive systematization of his syllogistic can be achieved by various equivalent deductive systems with different moods, even moods of the second and third figure as primitive rules of inference (*Anal.* pr. I c. 45; II c. 10).

Under this reconstruction, first of all, Łukasiewicz's criticism of Aristotle's use of the indirect proof in the case of the moods *bocardo* and *baroco* is, doubtlessly unjustified. This fact is an immediate consequence of the view that the undemonstrated moods in every deductive system are conceived as primary rules of inference. Łukasiewicz (1951, 55f.) is, of course, well aware of the fact that if he considered these indirectly proved moods as inference rules, Aristotle's proofs by *reductio ad impossibile* would be correctly applied. He refuses, however, to admit it, since he is convinced that "Aristotelian syllogisms are [...] propositions".

Secondly, it helps us to settle a textual, and in its consequence an interpretational dispute as well, concerned with the formulation of the mood *camestres* (*Anal. pr.* I c. p. 5 p. 27a 9f.), where we have to decide between two variants: either "If *M* belongs to all *N*, but to no *X*, then *X* will belong to no *N*" [17] or "If *M* belongs to all *N*, but to no *X*, then *N*

[17] Comp. Patzig, *Aristotle's Theory of the Syllogism*, p. 186, note 17.

will belong to no *X*".[18] Taking into account the text of Aristotle's proof—

"For if *M* belongs to no *X*, *X* belongs to no *M*; but *M* (as was said) belongs to all *N*. *X* then will belong to *N*. For the first figure has again been formed. But since the negative is convertible, *N* will belong to no *X*. Thus there will be formed the same conclusion" [19] (*Anal. pr.* I c. 5, 27a 10–14)—we have no problems in reconstructing it in a system of natural deduction, only if we accept the reading of the second variant:

$$MaN \wedge MeX \rightarrow NeX$$

Proof:	(1)	$MaN \wedge MeX$	supposition
(DPR)	(2)	MaN	(1) : EC
	(3)	MeX	
	(4)	XeM	(3) : CS
	(5)	$XeM \wedge MaN$	(5), (2) : IC
	(6)	XeN	(5) : *celarent*
		NeX	(6) : CS

The first variant of the mood *camestres*

$$MaN \wedge MeX \rightarrow XeN$$

is inconsistent with the above mentioned proof which does not end with *XeN*, but with *NeX*. Further, it is incompatible with the function of the syllogistic terms from the Aristotelian and traditional point of view: *N* is subject of the conclusion, and at the same time the major term; *X* is predicate of the conclusion, but the minor term as well. Finally, if we attempted to overcome this discrepancy by interchanging the premises, we would obtain the mood *cesare*. This possibility, however, implies that Aristotle proved one imperfect mood twice, while omitting the proof of another second-figure mood.

What is still a more important consequence of this reconstruction, is the following very plausible explanation of the controversial issue "How can the syllogisms themselves be proved?", which has already been mentioned. Having in mind the difference between primary inference rules and unproved formulae which, of course, can be—after being proved—adopted as secondary inference rules, if it seems useful, we can per analogiam make a similar distinction between perfect syllogisms as inference rules, and imperfect syllogisms as implicative propositions. Assuming this interpretation, it seems to be clear that the imperfect moods are not proved from the perfect syllogisms but by means of them.

[18] *E.g.* W. D. Ross (ed.), *The Works of Aristotle*, Oxford 1928, p. 27a, note 2.
[19] I interpret the term "syllogismos" in line 27a 14 in the same meaning as "symperasma". Comp. similarly *e.g. Anal. pr.* I c. 9 p. 30a 16; *Anal. post.* I c. 17 p. 80a 19.

This interpretation leads, of course, to the conclusion that every valid Aristotelian syllogism is an inference rule, if accepted without any proof, and equally well an implicative proposition, if it is proved in a given deductive system. There can be objected that this explanation implies a confusion between derivability and implication in Aristotelian logic. I do not think that this objection is appropriate, since we can hardly presuppose such a modern distinction for the Aristotelian conception of logic.

In order to support my standpoint, I accept Łukasiewicz's explanation of Aristotle's procedure of the *conversio syllogismi*, the "antistrofé" of *Anal. pr.* II c. 8–10. This procedure, which is in Corcoran's reconstruction unconsciously or consciously ignored, is described as follows: "For it is necessary, if the conclusion has been changed into its opposite and one of the premises stands, that the other premise should be destroyed. For if it should stand, the conclusion must also stand" (*Anal. pr.* II c. 8 p. 59b 3–5). This description, as Łukasiewicz (1951, 57) rightly argues, expresses exactly what we know today as the compound law of transposition in propositional logic.

Aristotle uses this procedure to obtain from the mood *barbara* the moods *baroco* and *bocardo* in a very simple way as follows:

"Let the syllogism be affirmative, and let it be converted as stated. Then if A belongs not to all C, but to all B, B will belong not to all C. And if A belongs not to all C, but B belongs to all C, A will belong not to all B" (*Anal. pr.* II c. 8 p. 59b 28–31).

These conversions can be interpreted either as transformations of the mood *barbara* into the moods *bocardo* or *baroco*, *e.g.*

$$AaB \wedge BaC \rightarrow AaC \Rightarrow AaB \wedge AoC \rightarrow BoC \qquad (\textit{baroco})$$

or in the form of an inference rule, *e.g.*

$$AaB \wedge BaC \rightarrow AaC \vdash AoC \wedge BaC \rightarrow AoB \qquad (\textit{bocardo}).$$

In both interpretations, the syllogisms themselves have to be conceived as logical laws, not as inference rules.

Aristotle uses these conversions in order to construct triads of mutually transformable syllogisms containing one valid mood from every explicitly acknowledged figure. They differ, as he explicitly says, from proofs by *reductio ad impossibile* in this: "conversion takes place after a syllogism has been formed and both premises have been taken, but a reduction to the impossible takes place not because the contradictory has been agreed to already, but because it is clear that it is true" (*Anal. pr.* II c. 11 p. 61a 21–25). This explication confirms what has already been said: the indirect proof is correctly applied, if we interpret the perfect syllogisms as inference rules, whereas the conversion of syllogism is validly related to transformations of syllogisms interpreted as logical laws. At the same time, it reflects two different approaches to syllogistics: one of them (*Anal. pr.* I) has to be considered from our point of view as a deductive theory elaborated in the form of natural deduction, the other one (*Anal. pr.* II) as an attempt to examine the deductive relations holding between syllogisms, at least, intuitively under a stronger influence of conceptions later on systematically studied in ancient propositional logic. It substantiates also the bifold interpretation of the Aristotelian syllogism.

The evidence of the quoted passages together with the suggested interpretations justifies, as it seems to me, the adopted methodological assumptions as well as the critical evaluation of Łukasiewicz's and Corcoran's one-sided reconstructions of Aristotle's syllogistic from the standpoint of contemporary formal logic.