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FROM TIME TO PLACE: THE PARADIGM CASE

The world-order in philosophical cosmology can be founded upon time as well as space. Perhaps the most fundamental question pertaining to any articulated world-view concerns, accordingly, their ontological and epistemological priority. Is the basic layer of notions characterized by temporal or by spatial concepts? Does a world-view in its development show tendencies toward the predominance of one set of concepts rather than the other? At the stage of its relative maturity, when the qualitative and comparative phases have paved the way for the formation of quantitative concepts: Which are considered more fundamental, measurements of time or measurements of space? In the comparative phase: Is the geometry of the world a geometry of motion or a geometry of timeless order?

In the history of our own scientific world-view, there seems to be discernible an oscillation between time-oriented and space-oriented concept formation.[1] In the dawn, when the first mathematical systems of astronomy and geography appear, shortly before Euclid's synthesis of the axiomatic thought, there were attempts at a geometry of motion. They are due to Archytas of Tarentum and Eudoxus of Cnidus, foreshadowed by Hippias of Elis and the Pythagoreans, who tend to introduce temporal concepts into geometry. Their most eloquent adversary is Plato, and after him the two alternative streams are often called the Heraclitean and the Parmenidean world-views. But also such later and far more articulated distinctions as those between the statical and dynamic cosmologies, or between the formalist and intuitionist philosophies of mathematics, can be traced down to the original Greek dichotomy, although additional concepts entangle the picture. And doctrines such as the theory of the *klîmata*, the kinematics of the 14th-century Schoolmen, Thomism, Evolutionism, the dialectics of nature, and Vitalism, could hardly be adequately discussed without a reference to the pivotal status of temporal concepts.

[1] A concise summary appears in G. J. Whitrow's *Natural Philosophy of Time*, London and Edinburgh 1961 (esp. Chapter III, *Mathematical Time*).

It looms also behind the emphasis on geometrical construction in the history of analysis and synthesis, and behind the heuristical ideas in the development of calculus[2], although Cauchy, Dedekind, Cantor and Weierstrass adhered to the formalist view. The necessary precision in the concept formation was gained by abandoning the predominance of temporal concepts.

This paper discusses the paradigm case of such a predominance which I shall associate with the rise of Greek mathematical astronomy and geography, *i.e.* with the first mathematical analyses of the motion of the Sun and other celestial bodies.[3] The point of special emphasis is Eudoxus' thought which assumes a synthetic manifestation in his main instrument, the *arachne*. This view-point is not adopted merely in order to gain a better understanding of the beginnings of our world-view, of Zeno's paradoxes and Plato's criticism of the contemporary geometry. It is adopted because it sheds light also on the end of a long tradition. For paradoxical though it may sound, Eudoxus' contribution to the development of modern analysis was found fresh after its dormancy of two millennia by Dedekind and Weierstrass[4], who witness the elimination of temporal concepts from the terminology of calculus.

THE CIRCLE AND THE LINE

Approaching the interrelation of time and place from the direction of temporal notations, we meet with the dilemma: Should we begin with the concept of linear time, or with cyclic time?

There are two contexts where cyclic and linear time both seem to occur, and they suggest that these two concepts of time are interrelated in the same way as the circumference and the diameter of a circle.

The first context is the determination of the length of day and night, at the summer

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[2] One must only recall Archimedes' work on spirals and his methodological fragment, Napier's invention of logarithms, Galileo's *Protophysik*, Hamilton's discovery of quaternions, Kronecker's criticism of Cantor, or Newton's invention of the calculus of fluxions.

[3] For our purposes Greek mathematical astronomy and geography need not be discussed separately. However, since we are going to discuss the very early stages of their development, it might be more appropriate to speak about time and place than about time and space. For their connection in the Greek paradigm see E. Maula, *Ancient Shadows and Hours*, *Annales Universitatis Turkuensis*, Ser. B, Tom. 126, Turku 1973; E. Maula, E. Kasanen, P. Kasanen, J. Mattila, *Meridian Measurements from the Nile to the River Tornio 1736—1737*, The River Valley as a Focus for Interdisciplinary Research (eds. E. Maula and E. Erkinaro) Oulu 1980; E. Maula, E. and P. Kasanen, J. Mattila, A. Szabó, *Man's Orientation in Time and Place*: *The Discovery of the Relation between Temporal and Geographical Measurement*, Proceedings of the XVth International Congress of the History of Science, Edinburgh 1977; E. Maula, *Man's Self-orientation in Time and Place*, Proceedings of the First International Week on Philosophy of Greek Culture, September 19–25, 1977, Chios, Greece = Diotima 1979.

[4] See Sir Thomas L. Heath, *A History of Greek Mathematics* I–II, Oxford 1921 (esp. I, pp. 325-327, 385). It is noteworthy that Eudoxus' general theory of proportion is applicable to incommensurable as well as to commensurable quantities, and "equally applicable to geometry, arithmetic, music and all mathematical science". There is a marked tendency in Eudoxus to develop general theorems—in contradistinction to theories pertaining to the departmental division of sciences, as we would call them.

and winter solstices and at the equinoxes, by means of the ratios of the day-arc and night-arc (of the tropical circles of the celestial sphere) on the one hand, and by means of the corresponding ratios of parts of the diameters of these tropical circles on the other hand. It is not known when the measurements of the day-length by means of the *klepsydra* and the *gnomon*, and by means of observations of the night sky, first became interpreted in terms of the celestial sphere. Probably it was not later than about 400 B.C. when the hypothesis of the sphericity of the Earth was already widespread. At any event, from Hipparchus we gather that Eudoxus (c. 390–337 B.C.) used such expressions (*e. g.* Hipp. in *Arat. et Eud.* I, 2 22; I, 3, 9).

They are customarily explained as ratios of arcs, divided by the horizontal, plane at the solstices at noon. However, in this interpretation it is not easy to connect the ratios (5:3 and 12:7) to latitudes known from Eudoxus' biographical tradition. I have suggested (in *Ancient Shadows and Hours*) that they could be interpreted as ratios of the corresponding parts of the diameter of the tropical circle instead. This interpretation will connect them, using the value for the obliquity of the ecliptic obtained in my reconstruction of Eudoxus' cosmology[5], with the latitudes of Rakotis (Alexandria) and the ruins of Babylon with a surprising accuracy, the error being in the region of 16'. After the reconstruction of Eudoxus' main instrument, the *arachne*[6], I now think that the ratio 5:3 alone pertains to the diameter, while the ratio 12:7 pertains to the tropical circle. This is a conjecture based on the length of the radius of the tropical circle. It is 12 units, the radius of the circular plate, *enoptron*, of the instrument being 13 such units. For the Pythagorean triangle with sides 13, 12, 5 determines Eudoxus' value for the obliquity of the ecliptic.[7]

On Fig. 1 the *arachne* is erected at the latitude of Rakotis. The ratio 5 : 3 appears as the ratio of the parts of the diameter of the tropical circle (SQ : QS') and also as the ratio of the *gnomon* to its equinoctial shadow (AO : AC = OP' : QP'), which is equivalent to the cotangent of the latitude (φ).

The interrelation of the circumference and the diameter of a circle are attested already in the Papyrus Rhind, where the value $\pi = 256 : 81 = 3.160$ is given (in 1700 B.C. probably referring to much earlier times). In the diameter of the tropical circle of the *arachne* $r = 12$, and the two nearest whole number lengths for

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5 E. Maula, *Studies in Eudoxus' Homocentric Spheres*, Commentationes Humanarum Litterarum, Vol. 50, Helsinki 1974, and *The Constants of Nature. A Study in the Early History of Natural Law*, "Philosophia" IV, Athens 1974.

6 E. Maula, *Points of Contact between Physics and Philosophy in their Early History. The geometrical Spider*, On the Foundations of Physics (eds. P. Lahti and A. Siitonen), Report Series in Theoretical Physics, Turku University, Turku 1975, and *The Spider in the Sphere. Eudoxus' Arachne*, "Philosophia" V/VI, Athens 1975/6.

7 Note that the acute angle obtained (*a* in the attached figure) is about 22°37' and thus somewhat smaller than the true value in Eudoxus' day, $\varepsilon = 23°44'$, but this is in fact a strong argument on behalf of the reconstruction, since it is known that Eudoxus postulated a (fictitious) deviation for the third sphere of the Sun, a deviation from his chosen obliquity of the ecliptic. Moreover, the same method which explains the periods and axial inclinations of all other planets in his system, explains even this solar deviation (I = 1°16').

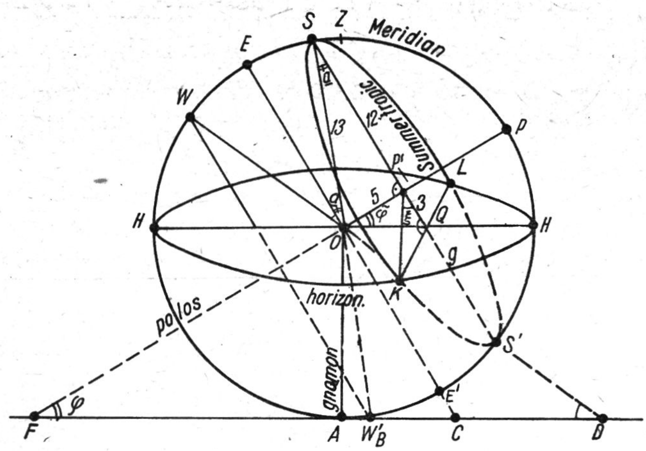

Fig. 1. The main circles and the Arachne in Rakotis: AO—gnomon, FP—polos, HH—plane of horizon, SS′—diameter of the summer tropic, WW′—diameter of the winter tropic, KP′Q = ξ—half the night arc, COA = QOP′ = φ—latitude

the rectification of the quadrant are $c/4 = 18$ and $c/4 = 19$. We thus obtain two approximations to π as $c : d = 18 : 6 = 3 : 1$ and $c : d = 19 : 6 = 3.166...$

It must be remembered that in the first place these are ratios expressing the interrelation of cyclic and linear time. While cyclic time is associated with stellar observations of the tropical circle, the *arachne* measures the day and night by means of the ratios on its diameter, and this can be associated with the concept of linear time. But simultaneously we have obtained two approximations to π. The full importance of this discovery will be seen when we remember that the Earth, too, was assumed to be an example of a spherical body. This will connect the meridian measurements to the same set of problems.[8]

In order to obtain further approximations to π we start from the ratios 3 : 1 and 19 : 6 which suggest an algorithm. This algorithm, which closely resembles to the Pythagorean approximations to surds, is

$$d_i = 2d_{i-2}+d_{i-1} \qquad (1)$$
$$c_i = 2c_{i-2}+c_{i-1}$$

where $c_i{:}d_i \approx \pi$, when $i = 1, 2, \ldots, 6$).

The results may be given in Table I.

It will be seen that correct lower and upper approximations are obtained alternatingly.

The oldest recorded meridian length, $c = 400\,000$ stades (reported by Aristotle,

[8] An interrelation of temporal and geographical measurements, especially Eratosthenes', is argued for also by Prof. Harry Bowsher in his paper *Cultural Influences on the Evolution of Ancient Weights and Measures, The River Valley as a Focus of Interdisciplinary Research.* It is a computer study in ancient metrology based on extensive data concerning ancient artifacts and monuments recently measured.

TABLE I

c	d	$c : d$	Meridian circle : diameter of the Earth	Measurement
3	1	$3 : 1 = 3.0$	$= 300\,000 : 100\,000 = 240\,000 : 80\,000 = 180\,000 : 60\,000$	Dicaearchus? Eudoxus (1)?
19	6	$19 : 6 = 3.166\ldots$	$= 253\,270 : 80\,000 = 190\,000 : 60\,000$	Posidonius, Ptolemy and followers (1) Eudoxus (2)?
25	8	$25 : 8 = 3.125$	$= 250\,000 : 80\,000$	Eratosthenes (1)
63	20	$63 : 20 = 3.15$	$= 252\,000 : 80\,000$	Eratosthenes (2), Hipparchus
113	36	$113 : 36 = 3.1388\ldots$	$= 214\,700 : 68\,400$	
239	76	$239 : 76 = 3.1447\ldots$	$= 215\,100 : 68\,400 \approx 240\,000 : 76\,000$	Eudoxus? Posidonius?

De caelo 298 a 9 ff), associated by Tannery (*Recherches sur l'histoire de l'astronomie ancienne*, 110 f.) with Eudoxus. It does not ensue from our algorithm and may represent an isolated measurement not connected with the other problems indicated. In our interpretation, Eudoxus determined the latitudes of Rakotis ($\tan \varphi = 3 : 5$, in several ancient sources) and Cnidus ($\tan \varphi = 3 : 4$, reported by Hipparchus erroneously as the latitude of Athens and adjacent regions; $\varphi = 36°52'$, from a Pythagorean triangle with sides 3, 4, 5, however, pertains to Cnidus and the nearby Cos, while the true latitude of Athens is very nearly 38°). What is known about Eudoxus' observation of Canopus (α Carinae) and about the distance between Cnidus and Egypt (with which Cnidus had traditional contacts of commerce, not to mention Eudoxus' research voyage), rather suggests that he obtained, by means of his *arachne*, an arc of 1/60 of the meridian and a circumference of either 300 000 or 240 000 stades.[9] Furthermore, Eratosthenes' two reported measurements, as well as Posidonius' reported change from 240 000 to 180 000 stades become intelligible. They

[9] Eudoxus' paradigm for meridian measurement, which became canonical through the subsequent periods up to Maupertuis' expedition to the River Tornio, Northern Finland, in 1736–1737, has been discussed in our paper *Meridian Measurements from the Nile to the River Tornio 1736–1737* mentioned above. In the *arachne* the magnitude of an angle is measured in terms of ratios of whole numbers, $q : p = \tan(\alpha/2)$. If $q : p = 1 : 19$, then the whole angle $\alpha = 6°02'$ or very nearly 1/60 of a circle.

represent different approximations to π (and different lengths postulated for the radius of the Earth).

It might be noted that Archimedes' estimates $3\frac{1}{7} > \pi > 3\frac{10}{71}$ do not ensue from our algorithm. However, it is known that in his basically geometrical procedure Archimedes first obtains an approximation $1351 : 780 > \sqrt{3} > 265 : 153$. This result has never been explained satisfactorily (cf. Heath, *A History of Greek Mathematics*). The intermediate step naturally affects the approximations to π. But we have been able to show[10] that the approximations to $\sqrt{3}$ can be obtained by another algorithm, viz.

$$\text{(2)} \qquad \begin{aligned} a_i &= a_{i-1}+d_{i-1} \\ d_i &= 3a_{i-1}+d_{i-1} \end{aligned}$$

(where $d_i : a_i \approx \sqrt{3}$ when $a_1 = d_1 = 1$),
which itself is a generalization of the best known Pythagorean algorithm yielding approximations to $\sqrt{2}$,

$$\text{(3)} \qquad \begin{aligned} a_i &= a_{i-1}+d_{i-1} \\ d_i &= 2a_{i-1}+d_{i-1} \end{aligned}$$

(where $d_i : a_i \approx \sqrt{2}$ when $a_1 = d_1 = 1$).

Hence not only an algorithmic method for the approximations has been established, but even the known exception can be explained by means of such an algorithm. Moreover, also Eutocius' note on Apollonius obtaining a better approximation to π than Archimedes becomes understandable. For the arithmetical mean from the two last approximations by (1) indeed is better than that from Archimedes' two estimates. Finally, if we make the coefficient in (1) one instead of two, the "classical" approximation (after Archimedes), $\pi = 22 : 7$ will be obtained. It may be noted, though, that still in the Hellenistic period often $\pi = 3$, a value which we meet also in the Hebrew Mishnat ha-Middot, being no doubt quite satisfactory for an architect.[11] There is a great number of interesting philosophical implications from the apparently extensive use of similar algorithms. They are discussed in my paper "The First Beats of a Dynamic World-View", and their general import tends to support our main thesis about the imparity of time and place. But there are also some more immediate mathematical implications that are of the greatest interest just here.

Before we leave this subject, it may be noted that there is also another context where the concepts of linear and cyclic time seem to occur side by side. That is the calendaric edifice of the luni-solar cycles. Our best sources about these cycles are Geminus' *Isagoge* and Censorinus' *De die natali* which profess to give a historical

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[10] In a paper called "The First Beats of a Dynamic World-View" read at a symposium on the reconstruction of the *arachne* arranged at Leningrad State University in October, 1976, mimeographed.

[11] See my review on Prof. Lehti's paper on *Mishnat ha-Middot*, in *Zentralblatt für Mathematik*, 291.

account of the development. This may be wrong, but at any event we learn that a whole number of lunar and solar rotations were supposed to occur either within an 8-year period (*octaeteris*, which Censorinus associates with Eudoxus): 99 lunar months are approximatively 8 years of $365\frac{1}{4}$ days or 2922 days (*Isagoge* 8.34–5); or within a 16-year period (*Isagoge* 8.36–41); or within the Metonic 19-year period = = 235 months of which 7 intercalary months = 6940 days = $19\times365\frac{5}{19}$ days (Censorinus, *De die natali* 18.8); or within a 59-year period (which Censorinus associates with Harpalus, before Meton; *De die natali* 19.2) or 730 lunations = 21 557 days; or within a 76-year period = 940 months, of which 28 intercalary = 27 759 days = $= 4\times19\times365\frac{5}{19}-1$ days $= 76\times365\frac{1}{4}$ days or the Callippic cycle (*Isagoge* 8.57 –60) which Ptolemy used in the *Syntaxis* as an alternative; or within a 160-year period (presupposed in Eudoxus' *octaetris*; cf. Heath, *Aristarchus*, 293); or yet within Hipparchus' 304-year period (Ptolemy, *Syntaxis* III. 3). We do not know how the ancients conceived of these relations between the lunar and solar cycles (and the computations of days and intercalary, "full" and "hollow" months may be simply arithmetical), but it is hardly a coincidence that the whole number ratios of the diameters and the circumferences indicated in the table based on (1) are met even here. For all these luni-solar cycles seem to have based either on the approximation $\pi = 3.0$ or on the approximation $\pi = 3.166\ldots$ This will be seen when we draw the corresponding circles so that whole number lengths are obtained for the radius and circumference. It may be noted that even the 59-year period (of Harpalus?) could be illustrated basically in the same way, if the value $\pi = 239 : 76$ is accepted. Hence all luni-solar periods can be accounted for.

THE BEGINNING OF TRIGONOMETRICAL TABLES

If the algorithm (1) is permissible, we have in the first table an illustration of a "theory-informed observation" and even of a "theory-informed experiment", long before the Islamic optics (where Sabra sees the beginning), before Galileo Galilei's *Protophysik* (where Mittelstrass might put the first occurrence), and before Newton's codification of modern physical thinking. Yet the immediate bearing of the algorithmic representation lies elsewhere. For it will give rise to an interrelation of parts of the diameter and parts of the circle, which is the problem solved also by the later Tables of Chords. Yet since arithmetical considerations give the impetus to the algorithms, it is clear that not geometrical but arithmetical methods must be applied.

We consider first the rectifications based on the two first approximations to π. A simple juxtaposition of ratios of line segments and ratios of the corresponding arcs will carry us surprisingly far. We juxtapose ratios of the day- and night-arc (or rather their halves, $\eta : \xi$) with ratios of the corresponding parts of the diameter of the tropical circle ($a : b$), giving also the lenghts of day and night in equinoctial hours, the corresponding tangents of the latitudes (as in the *arachne*) equivalent to the ratios of the equinoctial shadows to the *gnomon*, and for better assurance the latitudes in degrees. Table II will gather these things together.

TABLE II

$a : b$	$\eta : \xi$ (lower)	$\eta : \xi$ (upper)	shortest night	day: night (appr.)	Shadow: gnomon	latitude	locality
12 : 12	18 : 18	19 : 19	12^{h}	12 : 12	0 : 5	0°	Equator
13 : 11	19 : 17	20 : 18	$11^{h}22^{mn}$	$12\frac{2}{3} : 11\frac{1}{3}$	1 : 5	11°19′	
14 : 10	20 : 16	21 : 17	$10^{h}43^{mn}$	$13\frac{1}{3} : 10\frac{2}{3}$	2 : 5	21°48′	
15 : 9	21 : 15	22 : 16	$10^{h}04^{mn}$	*14 : 10*	*3 : 5*	30°58′	Rakotis (Alexandria)
16 : 8	22 : 14	23 : 15	$9^{h}24^{mn}$	$14\frac{2}{3} : 9\frac{1}{3}$	4 : 5	38°40′	
17 : 7	23 : 13	*24 : 14 = = 12 : 7*	$8^{h}43^{mn}$	$15\frac{1}{3} : 8\frac{2}{3}$	5 : 5	45°	Mid-Pontus?
18 : 6	24 : 12	25 : 13	8^{h}	16 : 8	6 : 5	50°12′	N of Borysthenes
19 : 5	25 : 11	26 : 12	$7^{h}15^{mn}$		7 : 5	54°28′	

Here we have, then, a representation of the imparity of time and place. If we proceed equally with, say, two hours in the length of the longest day of the year, the first step will take us to Rakotis, and the second one to Olbia on the north of Borysthenes (Dnieper) whereas the distances along the meridian (shown by latitudes) have become unequal. The values in italics are met in the literature and probably come from Eudoxus.

Hence if we proceed from time parallel of 16 hours (that being the length of the longest day of the year at that parallel) southwards, towards the Equator, the surface distance along the meridian corresponding to each equal period will become greater, and vice versa. Conversely, if we proceed with steps of equal length along the meridian from the Equator northwards, the length of day-light will become longer in increasing periods and vice versa. It is not a valid point that no actual motion is involved here. For take a measure like "a day's journey" or "a day's sailing", from sunrise to sunset, and we have the actual motion. Alternately, we can say that we are observing the Sun's spiral motion on the celestial sphere, a motion well known to Plato in his *Timaeus*.

When this notion of the imparity of time and place is added to that of the ever increasing amount of real time and effort needed in calculating more and more accurate approximations using algorithms like (2) and (3) (discussed in "The First Beats of a Dynamic World-View"), we discern a pattern lending itself to several philosophical implications.[12] Yet we need not be concerned with them here. Instead, we comment on the table just drawn.

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[12] This pattern seems to underlie a dialectics of nature and suggests an interpretation of the Platonic Forms as irrationals and/or integers to be approached, but never reached, by the successive approximations obtained by the algorithmic method.

First, we have computed the night-arc from the shorter rectification of the semicircle. If the longer one is being used, the length of the shortest night will be longer. Even though this does not affect the approximative ratio (of day : night) much, it is likely that the shorter rectification (with a monotonous increase of 5° for each 1/18 th of the arc) has been used. Witness the association of the day of 14 hours with Rakotis (Alexandria) up to Hipparchus. Second, in both cases the ratio of the shadow to the *gnomon* at the equinoxes (*i.e.* tangent of the latitude) will increase by 1/5 ths in the *arachne*. This does not mean that no other types of such ratios were used. There are different types even in the tradition of the tables of parallels, and in the later tradition of the *climata*, of which we here see a modest beginning. For instance, when Hipparchus gives the latitude of Greece (c. 37°, by which he means Athens and the adjacent regions) it is the ratio of the *gnomon* to its equinoctial midday shadow 4 : 3 (which truly implies $\varphi = 36°52'$, *i.e.* Cnidus or the nearby Cos rather than Athens), in his criticism against Eudoxus and Aratus (I, 3, 6). Judging from the context this is originally a Eudoxan ratio, and it hardly escaped Eudoxus' eye that the latitude of his home-town can be given by means of a Pythagorean triangle which is generated from an octave (For with $p = 2$ and $q = 1$, $p^2+q^2 = 5$, $p^2-q^2 = 3$, and $2pq = 4$). Third, in the *arachne* angles are measured by means of ratios of two relatively prime integers generating a Pythagorean triple, their ratio being equivalent to the tangent of the half angle. If $q : p = \tan(\alpha/2)$, then $2\,pq : (p^2-q^2) = \tan\alpha$. This makes one ask about the form of the ratios of shadow : gnomon in our table. Yet, for any purpose of reference, these ratios can be taken as tangents of half angles (*i.e.* as $q : p$). It is not the angle of the latitude, or its complement, which is now being observed by the *arachne*, for we started from the observation resulting in the reading $a : b$ (*i.e.* ratio of the parts of the diameter of the tropical circle).

All these arithmetical juxtapositions were superseded by the Tables of Chords. Therefore, if we find any evidence for their use, it will probably be from a period before Hipparchus. The results (the ratios) indeed come from an earlier period but we have not, so far, seen any vestiges of the method that has produced them. Hipparchus' method is different from our reconstruction. Yet it admits, although with more difficulty than ours, of an interpretation of the preserved results. Unfortunately there are very few historical sources suggesting how the pre-Hipparchian method looked like.

One such source is Ptolemy's *Syntaxis* I, 67, 22, where he gives two limits to the angular distance between the tropics saying that it is nearly the same estimate as that of Eratosthenes, which Hipparchus also used. But Eratosthenes had given the arc between the tropics as 11/83rds of the meridian circle. Theon of Alexandria (in Ptol. *Synt.*, ed. Rome, vol. 2, p. 528, 20) in his comment adds that the estimate was based on accurate measurement, and that the ratio 360° : 47°42′40″ is the same as 83 : 11. Here we have, then, a parameter value $\varepsilon = 23°51'20''$ that was used for a very long time indeed (the true value in Eratosthenes' day was $\varepsilon = 23°43'40''$, and of course ε is not a constant, although the ancients so believed). This ratio 11 : 83 is a notorious puzzle, and none of the explanations offered (by Delambre, Diller, Berger, or by Dicks himself, who discusses them in his book *The Geographical Fragments of Hipparchus*, University of London, 1960, 167–169) is satisfactory.

However, if $2\varepsilon = 11/83$rds of a circle, $\varepsilon = 22/83$rds of a quadrant $= 13\,200 : : 49\,822$ of a quadrant, which is very nearly 16 700 : 63 000 of a quadrant. In other words, Eratosthenes' measure for a quadrant being 63 000 stades (with $\pi = 3{,}15$), the exact distance of the tropic from the Equator (down to the nearest hundred stades, as also elsewhere), is 16 700 stades. From other fragments of Eratosthenes (preserved in Strabo 72, 132, 133) we can compute that Syene, the nearest well-known place to the tropical circle, was 16 800 stades from the Equator, corresponding to the practical value $\varepsilon = 24°$. This supports our reconstruction. It also suggests the explanation of Theon's mistaken view. For 13 200 stades+49 800 stades = = 63 000 stades. Hence Theon has confused a method with its outcome.

The nature of the method is, making $a = 13\,200$, $b = 49\,800$, $c = 16\,700$, as follows:

$$a : b = c : (a+b) \Leftrightarrow a^2+ab = cb \tag{4}$$

which is one way of stating the problem of the application of the area, a problem that is solved instrumentally also by Eudoxus' *arachne*. Moreover, since $a+b =$ = 63 000 (stades), the expression itself is another example of the method of juxtaposition. The original measurement probably pertained to the ratio of the two parts of the diameter of the tropical circle divided by the plane of the horizon at the summer solstice.

Finally, we may recall that when at the long last division into 360 equal parts of the circle appears in Greek sources (in Hypsicles' *Anaphorikos*, c. 180 B.C.), it is applied both to place and to time (*moirai topikai* and *moirai khronikai*), once more connecting these two basic concepts (In one Babylonian horoscope text from 410 B.C. the zodiac and the Equator have been divided into 360 parts; see A. Sachs, *Babylonian Horoscopes*, J. C. S. VI, 2, 54–57). But then already the Tables of Chords were in use, and the original arithmetical methods were almost forgotten, continuing their shadowy life in the tradition of the time parallels and *klimata* alone.

THEORETICAL CONTACTS BETWEEN TIME AND PLACE

So far we have been discussing the numerical results, the instrumental usages, and the reconstructed computing techniques of a superseded tradition. These belong to the praxis of certain ancient astronomers, that is to say, to their craftsmanship. The praxis may have been replaced by more accurate observation methods and more accurate computing techniques. There should be, however, some evidence of the continuity of the theoretical framework.

The essence of our reconstruction lies in the assumption of the use of the ratios ($a : b$) of line segments (parts of the diameter of the tropical circle) as a starting point. And our reconstruction is set forth within the framework of the plane geometrical circle model of the *arachne*. On the other hand, the later development is associated with Ptolemy's spherical model, where the use of ratios of arcs and the use of chords of double arcs are being prescribed. If a continuity of the theoretical framework between concepts of time and place is to be proved, therefore, we must show that what can be stated within Ptolemy's model, can be stated also within the *arachne*.

A representative case where Ptolemy computes one parameter of his model (taking as an example the latitude $\varphi = 36°$N, the longest day of the year $= 14\frac{1}{2}$ hours, the shortest night $= 9\frac{1}{2}$ hours, and $\varepsilon = 23°51'20''$) from the other parameters, is *Syntaxis* II, 2 (p. 60, Manit.). The proposition is as follows:

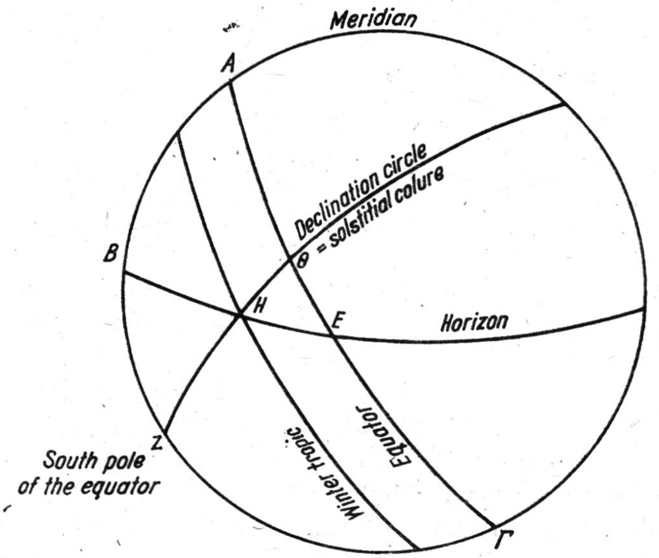

Fig. 2. Ptolemy's spherical model $A\Theta = \eta =$ half the day arc at winter solstice; $\Theta\Gamma = \xi =$ half the night arc at winter solstice; $AE = 90°$; $\Theta H = \varepsilon$; $HZ = \varepsilon'(\varepsilon + \varepsilon' = 90°)$; $HE = \delta$; $BH = \delta'(\delta + \delta' = 90°$, the point H indicating the sunrise in the eastern horizon in the morning of the winter solstice); $BZ = \varphi$

$$(5) \qquad \frac{\Theta A}{AE} = \frac{Z}{ZH} \; \frac{HB}{BE}$$

By means of the Tables of Chords (*Synt.* I, 11), the computations proceed as follows (see Fig. 2)

$$\frac{113^{p}37'54''}{120^{p}} = \frac{120^{p}}{109^{p}44'53''} \; \frac{HB(s2b)}{BE(s2b)} \text{; and}$$

$$\frac{\text{s2b HB}}{\text{s2b BE}} = \frac{113^p37'54''}{120^p} \cdot \frac{109^p44'53''}{120^p} = \frac{103^p53'23''}{120^p}$$

from which 2bHB = 120°, bHB = 60°, and bHE = 30° = δ, which was sought for.

It may be noted that both the solstitial and the equinoctial colures (which are declination circles "partly curtained by the horizon"), are discussed by Eudoxus (Hipp. in *Arat. et Eud.* I, 9, 3–4).

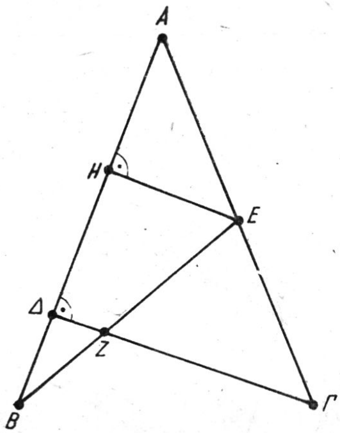

Fig. 3. Ptolemy's plane geometrical lemma

Similar computations occur at *Syntaxis* II, 3 (p. 62, Manit.) where the latitude (of Rhodes, $\varphi = 36°$) is being computed from the given values of other parameters, and elsewhere. In these cases, the parameters characterizing Ptolemy's system are the arcs corresponding to the angles ε, φ, ξ, η, δ and their complements (the arcs ξ, η being deduced from the lengths of the longest and shortest day and night of the year which the ancients conceived of in a symmetric fashion, paying apparently no attention to the effect of the atmosphere). It is typical for Ptolemy that he computes the latitude in two steps: first δ is obtained and the result is then used in computing φ (although φ could be computed even without knowing δ, as in the earlier *gnomon* observations).

As for the proofs of these propositions, Ptolemy refers to a lemma (I, 13).

(6) $$\Gamma A : AE = (\Gamma\Delta : \Delta Z)\ (ZB : BE)$$

The lemma is accompanied by a plane geometrical figure. It is worthwhile to compare the proof of the lemma and the reconstructed proof of the proposition (5), which we set abreast. A complete analogy is obtained through a simple transformation of the proposition.

1° Auxiliary drawing: HE parallel to ΔZ	Auxiliary drawing: ΠΘ parallel to BE
2° ΓA : AE = ΓΔ : EH (*Elem.* VI. 4)	AE : ΘA = BE : ΘΠ
3° Auxiliary parameter ΔZ introduced	Auxiliary parameter BH (BH = δ)

4° $\Gamma\Delta : EH = (\Gamma\Delta : \Delta Z)(\Delta Z : EH) = \Gamma A : AE$	$BE : \Theta\Pi = (BE : HB)(HB : \Theta\Pi) = AE : \Theta A$
5° $\Delta Z : EH = ZB : BE$ (*Elem.* VI.4)	$HB : \Theta\Pi = ZH : Z\Theta$
6° Substitution $ZB : BE/\Delta Z : EH$ to 4°	Substitution $ZH : Z\Theta/HB : \Theta\Pi$
7° $\Gamma A : AE = (\Gamma\Delta : \Delta Z)(ZB : BE)$ q.e.d.	$AE : \Theta A = (ZH : \Theta Z)(BE : HB)$ q.e.d.

In modern trigonometrical terms, (II, 2) gives rise to the formula $\sin\delta = \sin\eta \sin\varepsilon'$ and (II, 3) to $\sin\varphi = \tan\eta' \tan\delta$. In the proof of the lemma a reference is made to a plane geometrical application of Eudoxus' general theory of proportion presented in the fifth book of the *Elementa*, and also the technique of introducing an auxiliary parameter is characteristic of him (in the theory of the homocentric spheres). The plane geometrical figure of the lemma is there seemingly for its own sake. It has astronomical significance, however, as we shall see after having discussed the corresponding problems connected with Eudoxus' circular model, the *arachne*. It may also be noted that the same figure is helpful in the construction of harmonious point systems.

Turning now to Eudoxus' *arachne*, such as it once was set up for meridian measurement and for the determination of the latitude and other relevant parameters, close to the intersection of the main parallel of latitude and the main meridian of ancient geography, we see that exactly the same parameters will be obtained as in Ptolemy's spherical model. But in addition, a number of other interesting propositions, linking the *arachne* to the preceding *gnomon* measurements, will be obtained. The connections between them are based on simple geometrical considerations. Fig. 4 will be of some help. The triangles S′OQ and OBC on the one hand, and the

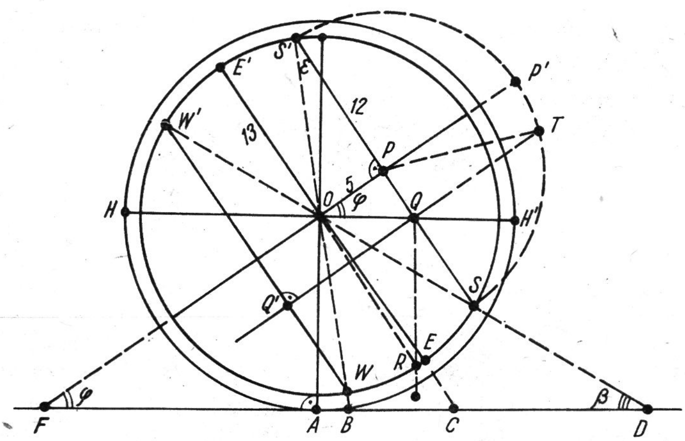

Fig. 4. The main parameters: the *arachne* indicates the summer solstitial, equinoctial, and winter solstitial shadows at noon (B, C, D), cast by the *gnomon* (AO), the *logotomos* (QQ′) referring ratios $(a : b) = QS' : QS$ of the diameter of the tropic circle to the circular plate (*enoptron*) of the instrument, the main arm of its measuring unit indicating the pole of the heavens (*polos*, FP′), and a plumbline (QR) indicating the angle (ROH) which, when the outer meridian ring is turned into ortogonal position, gives the point of the sunrise in the summer solstice. Denote the angles ABO, ACO, ADO, as α, φ', β. The angle $TPQ = \xi$, $POQ = AOC = \varphi$, $HOR = \delta$, and $DOC = COB = \varepsilon$

triangles QOS and OCD on the other hand, are similar. Hence $a : b =$ S′Q : QS $=$ $=$ CD : BC $=$ FD : FB (whence a harmonious points system is formed). Further, QR is the geometrical mean of a, b (*i.e.* QT $=$ QR $= \sqrt{ab}$). Noting that PS′$= (a+b)$: 2, PQ $= (a-b) : 2$, and QS $= b$, we can spell a number of interrelations between the parameters indicated, either in terms of a, b or in terms of other line segments of the figure.

For the sake of a convenient survey, all parameters may be given in the table III (in terms of modern trigonometrical functions).

TABLE III

	sin	cos	tan
φ	PQ : QO $=$ AC : OC $= \frac{a-b}{2}$: : QO	OP : QO $=$ 5 : QO	PQ : OP $=$ AC : AO $= \frac{a-b}{2}$: : 5
ε	OP : OS $=$ 5 : 13	PS : OS $=$ 12 : 13	OP : PS $= 5 : \frac{a+b}{2} = 5 : 12$
ξ	QT : PT $=$ QR : PS $= \sqrt{ab} : \frac{a+b}{2}$	OP : PS $= \frac{a-b}{a+b}$	TQ : PQ $= \sqrt{ab} : \frac{a-b}{2}$
δ	QR : OR $=$ QT : AO $= \sqrt{ab}$: : 13	QO : OR $=$ QO : 13	QR : QO $= \sqrt{ab}$: QO
α	AO : OB	AB : OB	AO : AB
β	AO : OD	AD : OD	AO : AD

These parameters may also be given in the form of a pyramid limited by four right-angled triangles (except the derived parameters $\alpha = \varphi'+\varepsilon$, $\beta = \varphi'-\varepsilon$). By means of a comparison of areas of these triangles (*Elementa* VI, 1), a number of connections between the parameters can be established; *e.g.*

(7) $\sin\varphi = \tan\xi' \tan\delta$ (cf. *Synt.* II, 3)

(8) $\sin\delta = \sin\varepsilon' \sin\xi$ (cf. *Synt.* II, 2)

(9) $\sin\varepsilon = \sin\varphi' \sin\delta'$

(10) $\tan\delta = \tan\xi \sin\varphi$

(11) $\cos\xi = \tan\varepsilon \tan\varphi$[13]

(12) $\sin\alpha : \sin\beta = a : b$

Modern notations can be replaced by references to parts of the *arachne*.

It remains to show that there is a close connection to the plane geometrical figure of Ptolemy's lemma, the astronomical and geographical significance of which will then be understood.

Since Ptolemy's computations pertain to the winter solstice and Eudoxus' two

[13] D. R. Dicks, *Early Greek Astronomy to Aristotle*, Ithaca, New York, 1970, pp. 19–23, leads this particular formula, which Ptolemy does not use, to characterize the early stages of Greek astronomy and mathematical geography up to Aristotle, contending (p. 23) that Eudoxus had reached this stage of understanding (without modern notation, of course).

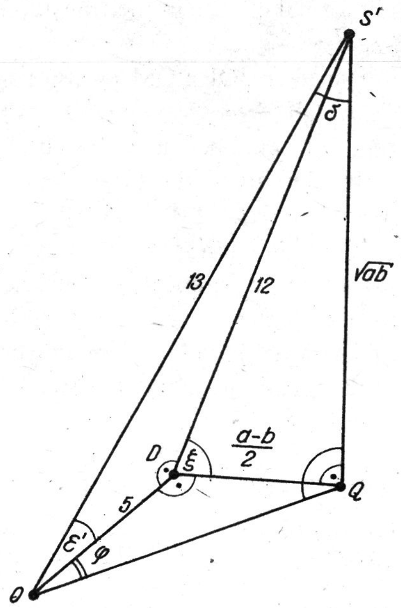

Fig. 5. The main parameters

ratios (a : b = 5 : 3 and $\eta : \xi = 12 : 7$) to the summer solstice, their connection may be shown within one figure. As the ancients always considered that the longest day and night on the one hand and the shortest day and night on the other hand

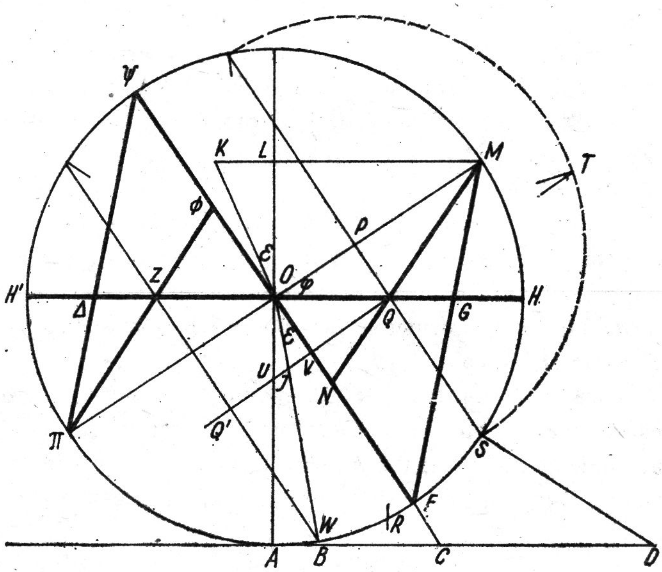

Fig. 6. The figure of Ptolemy's lemma at winter solstice (left) and at summer solstice (right) in Eudoxus' *arachne*

are of equal length, the only formal change in the transition from Ptolemy to Eudoxus is the substitution ξ/η.

It can be seen that for instance OP : OM = NQ : NM = ZΦ : ΠΦ = $\sin\varepsilon$, OQ : QR = OQ : OE = ZO : OΨ = $\cos\delta$ and PQ : OP = MG : GE = ΠΔ : : ΔΨ = $\tan\varphi$, while the night arc may be characterized also by means of the *logotomos* (QQ′), for seen directly from above the sun-lit half of the Earth is limited by the line KO extended so that KL : IM = JV : VQ = (a—b) : (a+b) = $\cos\xi$ (and by the same means JQ : UJ = $\tan\alpha$).

Therefore the continuity of the theoretical framework from Eudoxus' *arachne* and circular model to Ptolemy's spherical model seems likely, the mediating link being the plane geometrical figure of Ptolemy's lemma (the figure itself functioning as a forerunner of a quadrant[14]). It remains to show that Eudoxus' model is predominated by temporal concepts.

THE PRIMOGENITOR OF ALL SUN DIALS

Since the reconstruction of the *arachne* (in 1976), we have been studying its capacities. It is a model of the geocentric world-view (in cross-section), solves the problem of the two mean proportials and of the application of the area instrumentally, provides the *dynamis* (the square value of rectangle), generates conic sections as plane curves (*e.g.* a hyperbola can be drawn by means of it, for the measuring unit, which has given the instrument its name, the Spider, preserves a rectangle in all positions, and by means of rectangles with the same area a hyperbola can be drawn), and serves as an aid in mathematical invention (for instance, the heuristic ideas underlying the so-called Platonic solution to the problem of the two mean proportionals have been discovered, starting from Eutocius' hints). Moreover, it solves instrumentally the problem of combined spherical motions in Eudoxus' planetary theory. Here we have discussed, therefore, only a small number of its capacities, viz. those that pertain to the interrelation of time and place in Greek mathematical geography.

However, even in this restricted use the instrument will offer a surprise or two more. For in his book *De Architectura* Vitruvius, who has handed down the name of the Eudoxan instrument (without any description of its structure), gives a figure that, according to him, is the principle of the construction of the *analemma* curves for all sun-dials (VII, 3).

Looking at it we see that it is in fact the picture representing the *arachne* (which Vitruvius does not mention in this context) with the semicircles of the winter and summer tropics indicated. Hence the *arachne* can be used also in the construction of the *analemma* for all (other) sun dials, and it is, so to speak, the primogenitor of all sun-dials.

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[14] The divisions OQ : QG and MG : GE by way of the diagonal and ON : NE by way of the side (of a square) deserve close study in view of Timaeus 36C. The proof that MG : GE = = $\tan\varphi$ (by *Elementa* VI. 2) is easy when the square is complemented.

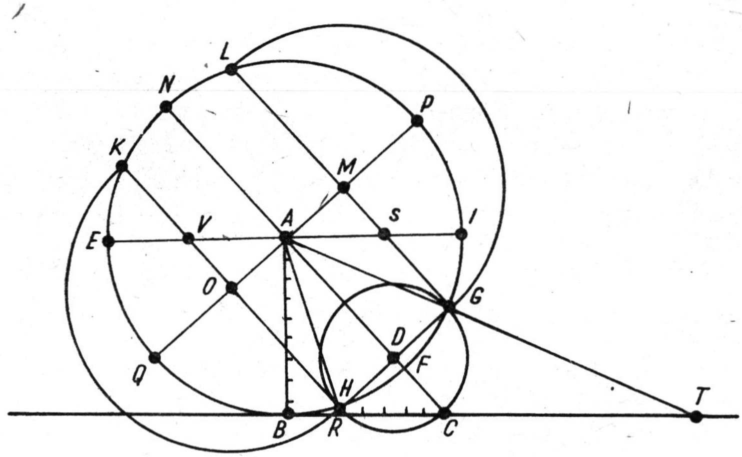

Fig. 7. Vitruvius' figure: HG = logotomos, ε = 1/15 of a circle = 24°

We need not dwell on the tropical circles any more; since the day and night arcs are given in their terms, temporal concepts predominate explicitly. It is unnecessary to discuss here the monthly curves, either. They can be constructed by means of the small circle, but may represent a later stage. The crucial question is: How were the *analemmata* of the summer and winter solstices and the equinoxes constructed? A customer wishing to have his sun-dial inscribed would know either the latitude (*i.e.* the ratio of equinoctial shadow to the *gnomon*), the ratio of the solstitial shadows to the *gnomon*, or the length of the longest day (by means of a *klepsydra*). The Sun's

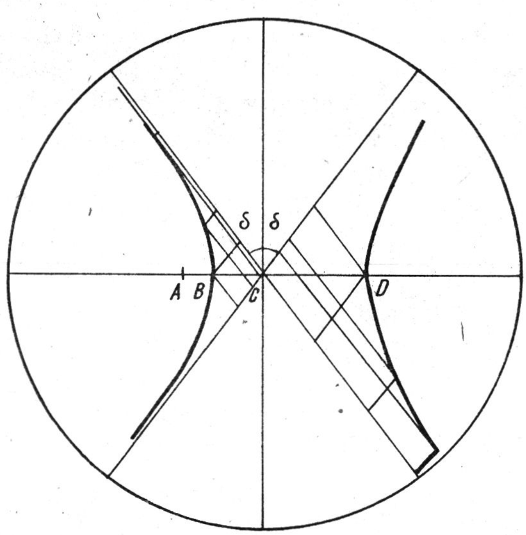

Fig. 8. The *analemma* curves for a plane sun-dial by means of parallelograms with the same area (two hyperbolas and a line)

daily path being very nearly a circle, its rays through the tip of the *gnomon* describe a slightly oblique cone. It is not hard to make a device for the determination of the curves of intersection with a spherical sun-dial. With the horizontal plane they are hyperbolas. Since the points A, B, C, D can be determined and the angles δ computed, they can easily be drawn.—The second surprise was the actual discovery of the *arachne* in Cnidus.

THE RETURN TO CNIDUS

In August, 1976 I was invited by the Technical University of Istanbul to deliver a lecture on "the oldest computer in the world".[15] I took along the bronze reconstruction of Eudoxus' *arachne* and some members of my research group. The Turkish colleagues told me about an American archaeologist, Professor Iris Love, who had been excavating at Cnidus for several years, and, being pleased with my lecture, arranged a voyage from Halicarnassus (modern Bodrum) to Cnidus. We reached Bodrum after some two weeks' travel in Anatolia, and already this journey showed good omens.[16] Even today, more than a year and a half after the journey, it is still impossible to digest the whole content of the experience. Six millennia, a greater number of civilizations, innumerable museums and archaeological sites—different people and variations of the Turkish kitchen and wines, changes of temperature from the freezing point to desert heath, changes of altitude from one and a half kilometers to the sea level and below (I had my frogman's gear with me), the fauna and flora of land and sea—the peasant's fields, the bazaars, the caravanserais, tourism, the silvan shadows and the mosques, the cry from the minaret top and the giant foot-prints in stinking corners—the monumental and the bizarre, life in caves, cities, and lofty eagle's nests.

In the Vikings' Miklagard, in Hagia Sofia itself, the midday sun, through windows looking due south at the right height, reached the centre of the church, reminding us of the astronomical tradition of Plato's Academy, which had guided the architects, the last academicians. In the Chora Church, again, we saw a magnificient fresco illustrating the three modes of human time intervined, Christ and Mary representing their own past, present, and future in one and the same composition. In Kirsehir, at Cacabey Mosque, originally a Seljuk roofless observatory (1272) with a round pool of water at the centre and a peculiar balcony above it at the side wall, we witnessed the continuation of the "mirror astronomy" once begun by Eudoxus with his *arachne* and its bronze mirror, the *enoptron*. We also remembered the related

[15] See E. Maula, E. Kasanen, J. Mattila, A. Siitonen, "*Knidoslu Eudoksos'un 2350 Yil Önce Gelistirmis Oldugu Dünyamin En Eski Hesap Mahinasi Arahne*", *Eletroteknik* (Istanbul Teknik Universitesi), Istanbul 1976.

[16] My paper "The Return to Cnidus" mimeographed, University of Oulu, was our first report written immediately after our return to Finland. This chapter has been based on it, although the discussion is based also on some papers of Iris Love which I have obtained only presently.

observatory of Samarqand (1428), the heir of Nasiraddin at-Tusi's (1201–1274) observatory at Hülägü's capital Meragh (in modern Azerbaijan). In the lands of Islam the religious need of finding the *qibla*, the direction of Mecca, had nourished the astronomical geography making use of the principles discovered in Ptolemy's *Megale Syntaxis* since the Arabic translations in Caliph al-Ma'mun's days (813–833). Yet I think that even today the technique of finding the correct direction has not been really understood.

But we met also other messengers. At the station of Perge, near Antalya and Aspendos (where the famous architect Zeno in the second century A.D. built one of the best preserved ancient theatres), we got acquainted with a running tortoise, perhaps a descendant of Achilles' winner. It accompanied us to the birth-place of the Ionian philosophy of nature. When we left for Bodrum, it remained quite satisfied at the theatre, no doubt enjoying the company of the great spirits of Thales, Anaximandros and Anaximenes. On the voyage from Halicarnassus to Cnidus, a dolphin in the best ancient style led our way. And finally at the southern harbour of Cnidus, we met a lonely donkey, although we were unable to tell whether it was Eudoxus' incarnation or a reminder of our folly in trying to return through 2300 years to Eudoxus' home-town, which is situated on the uttermost tip of a long, mountainous promontory near Cos.

Once the capital of the Dorian Hexapolis, later famous for Praxiteles' statue of Aphrodite Euploia (the epithet has been met at the borders of the *oikoumene*, in Olbia), for Eudoxus' observatory and its medical school competing long with the Asklepieion of Cos, and for its olives and wines (to witness: even today the red Halikarnas remains the best wine we ever tasted), it fell into oblivion in the seventh century A.D. In the fourth century B.C., in Eudoxus' day, the city, surrounded by its two harbours connected with one another by a canal, was replanned in squares, and just this town has been excavated by Iris Cornelia Love, our divine guide. In long seven years, not ever knowing of each other, we had both been inspired by Eudoxus. The rendezvous was worth a life's long work.

We began our *anabasis*, led by Iris Love, from the Small Theatre, seating some four thousand spectators and looking to the sea due South. Somewhat higher (the stepped North South streets, seven in number, have a gradient of 1 : 3)[17], there is an appropriate site for the Bouleterion, the sea-lanes of the Eastern Aegean clearly in sight. Besides a vast ceramic industry (65 % or more of the 40 000 stamped amphora handles found in Athens are Cnidian)[18], her olives and wine, exported to the Black Sea, Italy and Egypt, the prosperity of Cnidus was based on her excellent position guarding, together with Rhodes and Cos, the main commercial routes. It is not without a good reason that the Cnidians ordered from Praxiteles a statue of Aphrodite of the Fair Voyage.

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[17] I. Love, *Knidos-Excavations in 1967*, "Türk Arkeoloji Dergisi" (TAD) XVI. 2, Ankara 1968.

[18] Ibid., p. 135.

Above the Bouleterion once was an equally scenic taverna, as proved (in the archaeological understatement) by "a considerable number of broken wine-jars". At this stage we told Iris Love that it is certainly here that Eudoxus, who also preferred red roses to pansies, must have been meditating the secrets of life. To her surprised inquiry into proof, we pointed towards the *arachne* glittering in sunshine on top of the cabin of our boat down at the harbour, saying that it was exactly by the same method, practised in a sauna in Hauho, that we had reconstructed the instrument and its main capacities entirely from literary sources, which give, even them, only its name and some numerical results of the measurements and computations. Moreover, we told her that the instrument, which she had not seen as yet, would give the solution to the enigmatic town-plan with the square *insulae*, the distances of 60 meters between the centres of the main streets[19] (which is not in accordance with the Hippodamian principles, nor like Sostrates' design in Alexandria)[20], and the exact NS and EW directions of the streets.

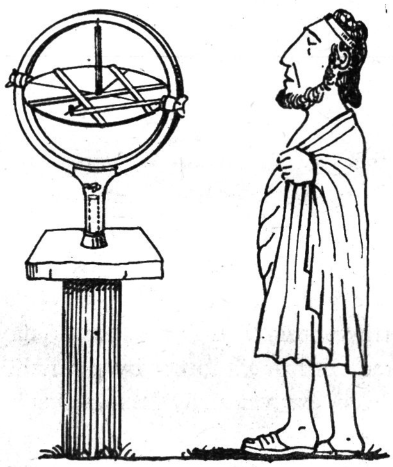


Fig. 9. The reconstructed *arachne*; general view; from Insinööriuutiset / The Engineer's News, 18.11.1977; Finnish Patent No 54205; At the Museum of Technology, Helsinki, Finland

This began one of several, more and more penetrating examinations of our work, so that by each step upwards, along a terraced NS street, we approached our common goal with ever more difficulty. But Iris Love became satisfied with our inner seriousness combined with an irresistible outer exhilaration.

At any event, when we reached the backbone of the town-plan, the main EW street (although we did not know it as yet), and saw an excavated part of a completely round cistern, we immediately told her that this must have been Eudoxus' obser-

[19] I. Love, *Knidos-Excavations in 1968*, TAD XVII. 2, Ankara 1969, p. 123.
[20] See TAD XVI. 2, p. 134.

vatory. And so it was, according to her considerations too. Her argument was based on the town-plan, ours on the idea that it was not an ordinary cistern at all, but rather another "mirror" resembling the bronze *enoptron* of the reconstructed *arachne*, and a similar arrangement at Cacabey. And we both knew Strabo's testimony that the observatory was somewhat above the roofs of the houses. Iris Love told us that such a round, extremely shallow "cistern" was unique in her experience, too. However, if it belonged to an observatory, the site was most appropriate, for the whole panorama of the city and the surrounding seascape evolved in front of us, albeit the ruins were hiding further details from the eye.

Now follows the final rise to a cliff a little North of the main street, Iris Love's fair step leading the way from a precarious stone to another. We knew that the climax was there, just as surely as in making love. And like Making Love it was, after all preludes, after the long ascent through mundane things towards the pure intellectual enjoyment, just as in Plato's *Symposium*. And next the Sudden, the *exaiphnes* of the *Parmenides* which changed everything in no time. We looked down and saw a round temple below us, a solitary stone pillar standing at its centre. We recognized it immediately for what it was: Our seven years' dream in stone. Eudoxus' *arachne* was still shooting the rays of the setting sun from far below—but its stone copy was sixty times magnified.

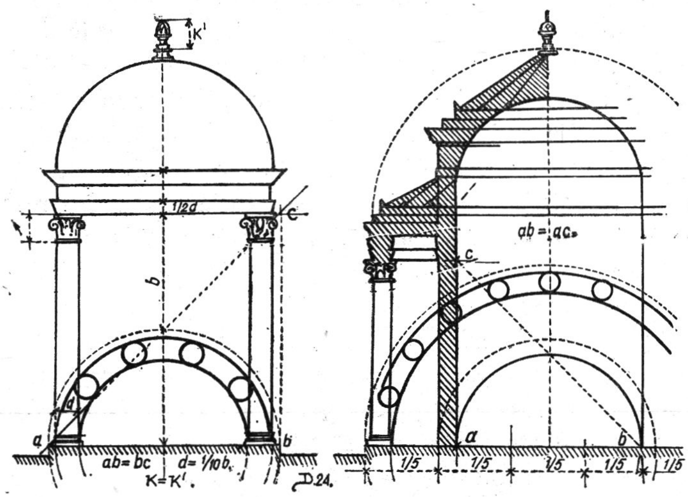


Fig. 10. The circular temple according to Vitruvius (from Durm)

Iris Love told us that on this site, under the *monopteros* of Aphrodite first excavated[21], there had been the present round edifice of the same dimensions, where the statue of Aphrodite (its original head may be Head No 1314 in the British Mu-

[21] See I. Love, *Excavations at Knidos in 1971*, TAD XX. 2, Ankara 1974, p. 106. Although the bronze coins bearing the head of Aphrodite Euploia are not accurately dated, in the vicinity it was discovered also a terracotta head of a woman from the first half of the 4th century B.C.

seum) probably was situated. However, the original *monopteros* was a unique temple, for it was roofless and housed the stone *gnomon* excavated in situ. The monument was surrounded by eighteen columns (reminding us of the rectification of a quadrant when $\pi = 3.0$), which is in accordance with the Vitruvian canon, but its rooflessness and the *gnomon* separate it from the temple at Tivoli and from Vitruvius' description (4, 8, 1–3) pertaining to round temples, some of which indeed are without a cella like the Cnidian edifice.

It is no wonder that this monument ($d = 17.30$ metres) was later, at the second stage of building, turned into a temple. But we saw in it, as it was reconstructed by the archaeologists, a majestic world-model of the geocentric system, the system so ingeniously mathematized by Eudoxus, first time in the history of science.

The moment was so laden with vision, not to say clairvoyance, that it left us speechless. The sun was setting. At the autumnal equinox approaching it would set due West, in the direction of the main axis of the town-plan. The details of the long ascent now assumed their full meaning. We had reached a vantage point in time and place, where the ancient tradition of observatories, starting from the megalithic stone-rings moving westwards until they achieved the Atlantic, had created a fulminant shrine for a mathematical genius. From there in turn the radiation spread over a number of exact sciences, completely bright after two thousand years.

And we also saw in our eyes the meaning of the town-plan. Its main streets intersecting at right angles, seven in the NS direction and four in the EW direction, looked like a magnificent map lattice of the then known inhabited world, *oikoumene*, spanning by latitudes and longitudes based on equal intervals of time the lenght from the Pillars to India, and the breadth from the Equator to Borysthenes and Olbia.

But higher than that one could not go. Just as in Eudoxus' double method of analysis and synthesis (see "Philosophia" IV, Athens 1974), after reaching the culmination point, we had to descend. The *katabasis* could begin.

Slightly W from the round monument Iris Love had excavated six smaller altars and one greater one set apart from the rest. The seven planets, perhaps, and the star dedicated to Aphrodite, the most prominent of them? But in the vicinity there was also a perfectly rectangular water container of excelent masonry and provided with a system of double, at times triple pipe-lines on the outside. Obviously a great precision was aimed at, but for what purpose? No one has given a definite answer, but since our reconstruction of the *arachne* so fully corresponds to the round monument, I am perhaps allowed to submit an explanation of this find also.

I presume that it is a *klepsydra* and that the pipe-lines are there for the regulation of the outflow of water. I had predicted (in my paper "The First Beats of a Dynamic World-View", written in 1975) the use of a *klepsydra* for mathematical purposes. For in as much as the outflow can be regulated, a constant motion can be created and consequently, by means of a simple device, the central curve of the ancient mathematics, known as the *quadratrix*, can be produced.

But the vicinity of the round building warrants the presence of the *klepsydra*

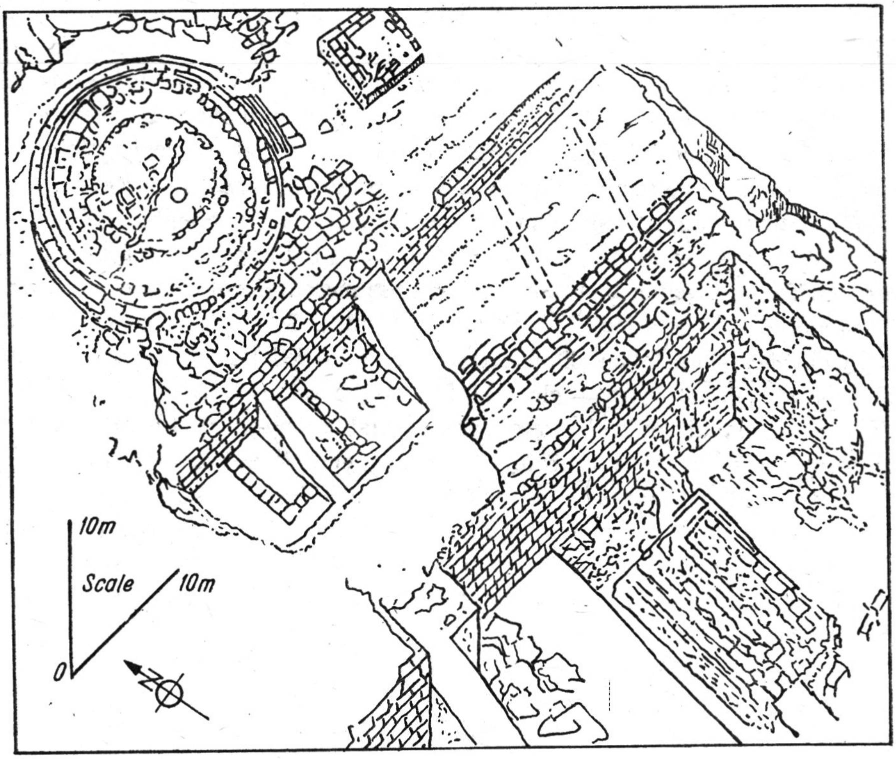


Fig. 11. The second stage of building, hence no *gnomon* at the centre (from I. Love, *Excavations at Knidos 1972,* TAD XXI. 2, 110)

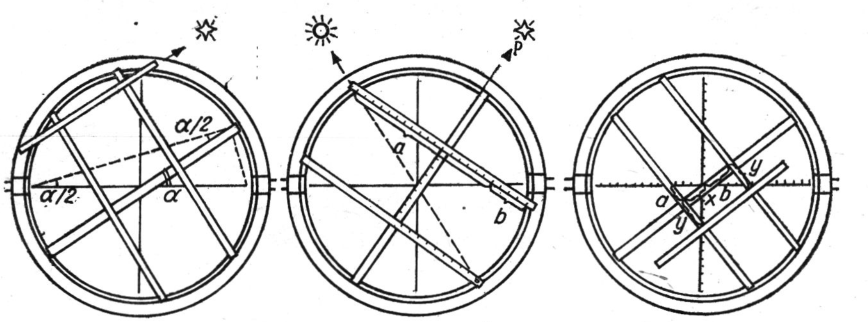
also for other reasons. For this building is an appropriate place for solar and lunar observations from the SE to the NW horizon (there is a cut in the rock of the island Triopion, permitting the observation of sunset at winter solstice; note also that in his criticism Hipparchus uses the term "ocean" for the horizon). Moreover, the rooflessness permits also the use of the round edifice as a *pelekinon* sun-dial. Eudoxus' observatory, again, is better suited for observations pertaining to zenith. From this it would follow that his Canopus observations were probably executed in the round building.[22] These observations are connected with his attempt at an estimate of the circumference of the Earth, for Canopus is "the star visible in Egypt" (Hipparchus), where Eudoxus once made an expedition. All this supports the idea that the whole quarter was dedicated to scientific activities during Eudoxus' life-time, and perhaps in other times also.

Descending a little more, Iris Love led us to a third, and in my opinion, decisive piece of evidence. It was a sun-dial scooped in the form of a semiparaboloid, out

[22] See Strabo (c 119) for these observations. According to Iris Love, TAD XX. 2, p. 108, the terrace below the round building may have been in large part an open area. This makes the horizon observations possible, because of the gradient of the streets. Cf. also Pseudo-Lucian's (end of the A.D. 3rd century) description of the temple where Aphrodite's statue was held.

of a block of stone. It was once provided with a horizontal *gnomon* (now lost, but clearly indicated by a groove). Vertical incisions show the hours, and horizontal ones the equinoxes and the solstices. According to Derek de Solla Price this design (some 40 specimens are known) "was constant from at least the fourth century B.C. until late Roman times".[23] It is very interesting to note that by means of the reconstructed *arachne* conic sections indeed can be produced, and that according to the tradition Eudoxus' pupil Menaechmus invented them (see "Philosophia" V/VI, Athens 1976). But in addition to this single sun-dial, there is the gigantic *arachne* and the *klepsydra*. Taken together they show that precise measurement of time was probably the foundation of the scientific activities in Cnidus. So our descent continued although darkness had fallen. Down, down we followed the invisible path toward the harbour again, toward a glow of fire at the waterfront.

There our captain, a real pirate and a veritable lady-killer, turned out to be a cook also, grilling a whole lamb for our late dinner. Wine was rich and Iris Love and her younger colleagues the merriest of companies. We were happy together, and I told them about our work leading step by step to the reconstruction of several kinds of measurements known to have been executed by Eudoxus, until just half a year before our meeting the *arachne* arose. They now saw the reconstructed instrument for the first time.



Figs 12—14. *Arachne* in the measurement of angles
Arachne in the determination of $a : b$
Arachne in the solution of $a : x = x : y = y : b$

And then occurred the last great surprise of our voyage. Iris Love recognised a part of the instrument, the *dioptra*, which also serves as a transversal cutting two intersecting lines in the same ratio (hence Vitruvius' term: the *logotomos*). She had excavated this part in the vicinity of the round edifice. It had been labelled, with a query, as "a bronze key to the gate of the temple of Aphrodite". This description could fit the notch at one end bent at right angles to the arm (although such an extre-

[23] See TAD XVI. 2, p. 137 (Iris Love speaks of "elliptical sun-dial", but it was in fact semi-paraboloidic); the *gnomon* of 17.9 cm would have given an accuracy of half a degree. The photoragphs do not show the geometrical drawings engraved on the backside of the sun-dial.

mely simple key could hardly fit the prosperity of the temple). Yet it does not fit at all the small hole at the other end similarly bent. It was the auxiliary arm of the *arachne*, provided with sights, which had been excavated in the vicinity of the monumental world-model of the geocentric system.

It was here that the research programme of "saving the phenomena" was executed, in the round edifice which certainly was an adequate site for Aphrodite Euploia also. There it was, high above us, only dimly lit by the seven planets and the eternal stars.

We remained sitting there on the lap of the night, discussing the astronomical odds against a mere coincidence as an explanation for our meeting there and then. It must be Eudoxus' logical soundness, and his intellectual penetration, witnessed by the fact that Dedekind and Weierstrass only one hundred years ago could continue exactly from the point where Eudoxus had left his message (*Elementa* V, Definition 5), that provides part of the explanation. In the similar fashion Eudoxus' paradigm for the meridian measurement held its own up to Maupertuis' expedition to the River Tornio in 1736–1737, and the ideas contained in his theory of the homocentric spheres (refined by Callippus, Aristotle, Ptolemy and others) laid the foundation for mathematical astronomy up to Galileo Galilei.

The farewells long after the midnight were bitter-sweet. Next morning we set sail for Halicarnassus, where I gave a talk at the Museum, reviewing what we had seen in Cnidus and promising to return again.

A VIEW ACROSS THE AEGEAN

Next autumn I returned to the Aegean, to Cos, seeing Cnidus only through my field-glass. But I had a clear idea where I should go: to the point nearest to Cnidus on the SE tip of the island. For that is the point where the latitude can be expressed as an acute angle of the fundamental Pythagorean triangle with sides 3, 4, 5 being generated from the octave (for if $p : q = 2 : 1$, then $a = p^2+q^2 = 5$, $b = p^2-q^2 = 3$, and $c = 2pq = 4$). But I did not know exactly what there was to be found.

Now the general picture seems clearer. Since 430 B.C. when Hippocrates won fame by healing the pest in Athens, the importance of the Asclepieion in Cos had all but eclipsed the medical school in Cnidus (where Hippocrates had studied). In 366 B.C. the Asclepieion was removed to its present site near the modern town of Cos, and foundations laid for the magnificent institute, which was to collect great treasures.

On the other hand after Konon's victory at the sea-battle of Cnidus, the ancient fame of Cnidus, as a shrine of Aphrodite, as the capital of the Dorian *hexapolis*, and as a site of a medical school, was in need of a new accentuation. Hence the Cnidians invited their famous countryman Eudoxus back to his home-town as a lawgiver. According to the archaeologists' dating the new monumental centre of Cnidus (based on mathematical principles, as we have seen) was built in Eudoxus' time, and the statue of Aphrodite Euploia was ordered from Praxiteles. The beauty of the new town-centre must have affected the choice of patients, when they had to make a decision between Cos and Cnidus.[24] Moreover, Eudoxus could point out that the very site of the town was unique, because its latitude was generated from the harmony of music.

The only possible answer of the priests in Cos was to show that health is even more important than astronomy and mathematics, and that the real latitude generated from the octave pertains to the NE shore of Cos. And miraculously, almost overnight, the divinity opened three healing sources on the very spot where water, land, air, and fire (in the form of the three volcanic sources), meet one another.

Being in good health myself, I crept deep into the interior of the leftmost source, now almost extinct, and discovered remnants of old archwork without mortar buried in sand. Perhaps the hand of a human demiurge had assisted the miracle? I also took samples of the volcanic salts, which are being analysed, and enjoyed the healing powers of the other two sources.

The old competition between the schools of Cnidus and Cos, it seems to me, explains the finds in Cnidus in Eudoxus' day. Yet in 336 B.C., only a year after his death, the present Asclepieion in Cos was built, and the palm of final victory was in Cos. A crack in the podium of the round building in Cnidus bespeaks an earthquake: there was no one to continue Eudoxus' work.

However, even though Eudoxus' home-town fell into ruins, his ideas did not. Some of them we have mentioned, but there are also others which deserve further study. For instance the sun-dials and the different projections used, eventually leading to the astrolabe[25] and the quadrant. In the field of mathematical heuristic, Fermat's greatest idea, scribbled on the margin of his Latin copy of Diophantus, calls for a comparison with Eudoxus' ideas pertaining to the Pythagorean triples. Also the transition from the homocentric spheres to the epicycles and eccentric motions remains to be studied.[26] And the genealogy of the sun-dials, deriving from the *analemma* is an obvious avenue for further research.

The most characteristic feature of the results so far obtained is the Eudoxan emphasis on time and temporal measurement. It is discernible in the use of the algorithms (1, 2, 3) and in the very choice of the specific days of the year for the observation. In his circular model, the same feature is seen both in the connection with earlier *gnomon* observations[27], *e.g.* in the formula (12), and in the reconstructed

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[24] Small marble copies of Aphrodite, other souvenirs, bronze coins with the head of Aphrodite, and a cave full of ceramics, from over 1400 years from the 7th century B.C. on, witness the commercial aspect of the activities connected with the round edifice; see TAD XXI. 2, p. 90 f.

[25] See E. Maula, *Points of Contact between Physics and Philosophy in their Early History, Report Series in Theoretical Physics,* University of Turku, Turku 1975.

[26] See E. Maula, *An Ancient Principle of Indeterminancy, Report Series in Theoretical Physics,* University of Turku, Turku 1977.

[27] See A. Szabó and E. Maula, *Anaximandros und der Gnomon, Acta Ant. Acad. Sci. Hung.*, 1977, and Der längste Tag und der Schattenzeiger: Untersuchungen zur Fruhgeschichte der griechischen Astronomie und Trigonometrie, ibid. (forthcoming in 1980).

beginning of the trigonometrical tables. Finally, it pervades the formulae (7–11), be that through the measurement of day and night or through the observation of the sunrise. Indeed, the *arachne* is an instrument to be used in the geometry of motion in general, albeit we have here discussed it only in the analysis of the Sun's spiral motion.

This analysis has bestowed us with a theory of mathematical geography, where the interrelation of time and place assumes a most interesting philosophical form. For denote the geometry of place outlined $G(x, y)$ and the geometry of time adumbrated $G(t)$, and the most universal generalization is

$$G(t)\ G(x, y) \neq G(x, y)\ G(t), \tag{13}$$

as we can gather from the theory of the *klimata* or hour-parallels, and perhaps also from the map-lattice in Cnidus. It underlies the modern system of time-zones and parallels of latitude also, although we seldom consider the latter from a temporal point of view. On the other hand, temporal concepts succur Eudoxus' unparalleled fertility and heuristic insight in the development of the exact sciences.[28]

Our way of reasoning is reduced—especially within the analytical school of philosophy—to merely analogical patterns for time and place, and there are marked tendencies toward an elimination of heuristics.[29] This may be warranted occasionally, as it was in the precision of modern calculus. But it is certainly unwarranted in the discussion of many earlier stages of development. Witness the futility of explanations given to Plato's criticism against the contemporary geometers. And witness also the standard analyses of Zeno's paradoxes.

For better assurance, consider how the analogical way of reasoning is imputed to Zeno without ever asking whether he conceived of time and place in the similar fashion. Trying to do justice to the logical acumen of the Elea, we may well group the paradoxes against motion according to whether time is discrete or a continuum. But once we begin with discrete time, we almost certainly also assume that place and motion are discrete. And if we begin with continuous time, by the same token place and motion will become continuous. Yet neither the extant fragments nor Zeno's silence warrant these analogous assumptions.

However, one might argue that this is not too dangerous because at least there is only one metric for both time and place. But we have seen that it is not the case. Within the truly scientific framework of ancient mathematical geography, it is not the case that a motion once commenced will cover the same distance in the same time, disregarding the direction and place of the motion. In fact, we often impute

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[28] See the papers by A. Siitonen, J. Mattila, E. Kasanen, T. Airaksinen, J. Mittelstrass and A. Szabó in *Analysis, Harmony and Synthesis in Ancient Thought*, Acta Universitatis Ouluensis 1977, eds. A. Siitonen and J. Mattila, Oulu 1977.

[29] A striking case is discussed in P. Marchi's, J. Agassi's and J. R. Wettersten's review *The Death of Heuristic*?,,Philosophia"1978 (USA),on J. Hintikka's and U. Remes' *The Method of Analysis*. The reviewers show that since the authors cannot distinguish between a proof and a proof procedure, their book becomes a series of *non sequiturs*, "not only a dud with no redeeming virtues; it is also misleading, confusing, and perhaps disingenious" (as the critics say in their MS).

to Zeno and his contemporaries our own notions of speedometers and fast vehicles, not to mention a logic of induction to enable them to generalize from a few cases of measured motion. The fixed stars will move with uniform velocity in simple circular motion, but the motion of the planets already is far more complex, since they seem to speed up and slow down their movements. This is the problem posed by Plato and solved by Eudoxus, and even though the component motions of his spheres were uniform (in which case they probably are not circular but spiral), their combinations aim at the "saving of the appearances", which are not uniform. In Zeno's case, again, the imparity of time and place leads to an even more interesting picture. For an arrow sent to fly by a mighty bow toward North will be slowing down at each consecutive moment of time, until its motion eventually stops altogether. And if the handicap between the myth-armoured Tortoise and the steel-tendon Achilles will be measured northwards, both will be running slower and slower, as an outside arbitrator at rest could say, finally standing still. Furthermore, if the arrow or the two competitors go southwards, different, but equally interesting results would be reached.

These interpretations of the Zenonian paradoxes will ruin the case of Parmenides' opponents even more effectively than the current ones reducing motion literarily to absurdities. And if we want to replace here the ancient analysis of the Sun's motion, which underlies our interpretation, by other paradigms for the interrelation of time and place, there are other cases, where the predominance of temporal concepts is to be understood. Even if one would dispense with the geometry of motion and much of Greek mathematical heuristic, there still are the all-pervading myths of Sisyphus and Tantalus and Chronus which show that time for the Greeks played a special algorithmic and ateleological role, hardly appreciated by us as yet, although Eudoxus certainly penetrated the crust of merely analogical reasoning as regards the interrelation of time and place more than two millennia ago.