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THE INFLUENCE OF CULTURAL BACKGROUND ON THE DEVELOPMENT OF MATHEMATICS

The goal of this article [1] is to present certain trends occurring in mathematics since the beginning of its existence. These trends will be regarded in a broader cultural context of the development of mathematics. Such an approach to this 2000-years-old discipline, developing mainly in a cumulative way, will allow to explain some peculiar phenomena occurring in its history, which would have been impossible without considering a broad background. The problem of origins and peculiarity of the Greek mathematics can be an example of that. The approach mentioned above will help to understand also some contemporary phenomena with very old origins, like for example the recent revaluation of interrelations between mathematics and informatics, [2] or the predominating role of the Platonic realism in philosophy and ideology of mathematics. One of the causes of such an approach is a considerable resistance, inertia, of the phenomena in question. To prove that, it is enough to remind oneself that Euclid's *Elements* or their revised editions have been obligatory manuals of geometry till recent times—2000 years after their creation.

The irregularity of the development of mathematics throughout its whole history is the most important problem needing to be explained, and requiring an analysis of long-lasting trends. The history of mathe-

[1] The preliminary version of this paper was published in Polish (with English summary) in "Zagadnienia Naukoznawstwa" 4/52 (1977), pp. 549–570.

[2] The term "informatics" used in continental Europe covers both "computer science" and "data processing." For information about the subject and methods of informatics see e.g. F. L. Bauer and G. Goos, *Informatik, eine einführenden Übersicht*, Berlin-Heidelberg, 1971. The outline of the history of informatics since the 16th century (vol. 2, Appendix C) is given in the same source.

matics, such as it is, stresses the continuity of its development and similarities between various periods. It also demonstrates the functioning of intra-mathematical factors of development, and the role of distinguished personalities. Due to such an approach, some important growth characteristics escape unnoticed. It is easy, for example, to overlook the fact that in the course of its 2000-year-old history, mathematics had only two periods of flourishing development within the circle of the European thought. The first period embraced the very beginning of mathematics from the 6th century B.C. up to the beginning of our era. Mathematics was at that time very distinctly limited to Hellenic and Hellenistic cultures. The second period started in the 16th century and lasts till now. Besides, mathematical thought developed in some periods outside this sphere (e.g. in mediaeval Islamic countries influenced by it). It is worth noting that outside those periods, mathematics lost its continuity and had to be revived painstakingly wherever the conditions were favourable.

As it follows from our investigations, those favourable conditions (despite the accepted opinions) did not always mean the possibility of practical mathematical applications. More important was the situation in culture and social life, giving high prestige to scientific cognition (or more broadly—to learning the truth about the objective reality). In scientific cognition, the "Platonic" viewpoint, attributing to mathematics the central role in the description and explanation of the reality, had to be strong and even predominating.

This work is only a starting point of a large research program on the socio-cultural conditions of development of mathematics, the concepts and results of which will be published elsewhere. We hope that they will not only allow for a deeper insight into mathematics and its place in the civilization, but will also throw a new light on more general problems of the history, philosophy, and the theory of science.

ORIGINS OF MATHEMATICS

Usually, when speaking about the origins of mathematics, one used to refer to its practical sources, recalling the results of anthropological and archaeological investigations (some knowledge of arithmetic and geometrical ornamentation in almost all primitive cultures), the ethymology of the word 'geometry,' or to the utilitarian nature of mathematics in the ancient civilizations of Egypt, Mesopotamia and China. Such way of argumentation conforms to the long-lasting tradition. It was Proclus already who wrote in his summary of the history of Greek geometry: "Since it behoves us to examine the beginnings both of the arts and of the science with reference to the present cycle [of Universe], we say that according to most accounts, geometry was first discovered among Egyptians, taking its origin from the measurement of areas. For they found it necessary by reason of the rising of the Nile, which wiped out everybody's proper boundaries. Nor is there anything surprising in that the discovery both of this and the other sciences should have its origin in practical need, since everything which is in process of becoming, progresses from imperfect to the perfect. Thus the transition from perception to reasoning, and from reasoning to understanding, is natural. Just as exact knowledge of numbers received its origin among the Phoenicians by reason of trade and contracts, even so geometry was discovered among the Egyptians for the aforesaid reason." [3]

The quoted excerpt makes a good summary of various trends of history, deriving the Greek geometry from mathematical achievements of earlier Mediterranean civilizations.

At present, two such sources are usually mentioned: the Egyptian geometry and Babylonian arithmetic. But the sole statement of obvious borrowings does not explain the problem of why the Greek mathematics differed to such extent from its original sources? Why doesn't it show major influence nor the existence of most characteristic features of Middle Eastern sources (such as Egyptian fractions or Babylonian positional system)? Questions of this kind make one doubt in the correctness of Proclus' hypothesis, or at least ask about the factors of such a deep transformation, and tempt one to find out if they are not more important than the borrowings or foreign inspirations.

Let us examine briefly these two sources. We know very little of the Egyptian geometry. From later Greek sources (Democritus, Plato, Herodot, Proclus, Diogenes Laertios) we learn that the Egyptian priests had much practice in making complex measurements. But this practical knowledge should be compared rather to our geodesy than geometry. [4] We should assume, therefore, that the Egyptian and Greek geometry were quite different phenomena as far as the methodological, social and cultural viewpoints are concerned.

The same can be said about the Babylonian and Egyptian arithmetic. Some rudimentary arithmetic was known in almost all, even most primitive cultures. Historically, traces of such skills have been found in the oldest civilizations. [5] The development of this rudimentary knowl-

[3] Proclus' *Summary*, translated by I. Thomas is quoted after H. Midonick (ed.), *The Treasury of Mathematics*, Hormondsworth, 1968, vol. 1, p. 407.

[4] Compare Aristotelian differentiation between sciences (*episteme*) and arts (*techne*): *Metaphysics* (*Met.*) I, 981 a 25–981 b 30; *Ethica-Nicomachea*, 1139 b 14–1142 a 30.

[5] See e.g. B. A. Frolov, *Numbers in the Graphic Arts of Paleolitic Age* (in Russian), Novosibirsk, 1974.

edge to the level of an efficient tool of processing numerical data was as inevitable a condition of creation of large centralized social structures as the existence of some specific economic, technical, and cultural factors (e.g. the knowledge of alphabet, some ideological system, etc.). Those factors undoubtedly underwent a simultaneous evolution, and the development of the former influenced the growth of the latter (and vice versa). Thus they coexisted in such civilizations as Egypt, Babylonia, China, Creta, Mykenes or precolumbian cultures in America. All these civilizations represented a similar level of mathematical knowledge. In spite of some differences, in all these countries mathematics was an efficient tool allowing for processing of considerable amounts of numerical data indispensable in management of large centralized social structures. The range of problems, methods used, and the social functions, indicate that the above cases present not so much underdeveloped mathematics but rather well developed informatics. [6]

Obviously, the similarities between mathematics and informatics in cases discussed above could not be stated before the latter became an independent scientific discipline (which occurred in recent ten years). Nevertheless, at present we can ascertain that the Greek and the Babylonian mathematics were quite different disciplines, only incidentally called by the same name. Following this approach, the problem of the "Greek miracle," i.e. of the transformation from the starting point to the Hellenic mathematics, changes into the problem of genesis of mathematics (geometry) in the Hellenic background as a new, original phenomenon which does not have much in common with similar phenomena in other cultures.

When explaining the origin of the Greek mathematics, we shall refer to K. Popper's statement [7] that Euclidean geometry is in fact a cosmological theory derived from the Platonian philosophical doctrine. Cosmological concepts, i.e. attempts to describe the Universe as a whole, and investigate its laws, principles and causes, very early started to draw on mathematical concepts. [8] O. Neugebauer traced the exact links between mathematics and astronomy (constituting a part of widely understood cosmology) in Egypt, Babylonia, and Hellenistic Greece. [9] The Greek tradition ascribes the beginnings of both geometry and cosmology to the same person—Thales. It is significant that most of theorems traditionally

[6] For the analogous point of view on Babylonian mathematical techniques, but without drawing such radical conclusions see: D. E. Knuth, *Ancient Babylonian Algorithms*, "Communications of the Ass. of Comp. Math." 15 (1972), pp. 671–677.

[7] K. Popper, *The Cosmological Origins of Euclidean Geometry*, in: I. Lakatos (ed.), *Problems in the Philosophy of Mathematics*, Proc. Int. Colloq. in the Philosophy of Science, London, 1965; Amsterdam, 1967, pp. 18–20.

[8] Frolov, *op. cit.*, pp. 118–145.

[9] O. Neugebauer, *The Exact Sciences in Antiquity*, Providence, (R.I.), 1957.

assigned to Thales have no practical applications, and their pure nature becomes more intelligible when they are understood as statements concerning the cosmological system rather than mathematics. [10]

Thales' cosmology, or more generally—cosmology of philosophers from the Miletus School, represented the philosophy of nature, whose further evolution carried it far away from mathematics. That is why it is cosmologic hypotheses, made by Pythagoras and his school, that are of more interest to us. Pythagoras draws our attention as the person who (according to Proclus) "transformed this study [i.e. geometry] into the form of liberal education, examining its principles from the beginning and tracing down the theorems immaterially and intellectually; it was he who discovered the theory of proportionals and the construction of the cosmic figures." [11] Let us add that it was in the Pythagorean school where the term 'mathematics' was created, as well as the division of mathematics into four parts: geometry, arithmetic, astronomy and music (preserved up to the end of the Mediaeval Ages in the form of *quadrivium*—a higher level of student scientific initiation). Therefore, further growth of mathematics was to a large extent determined by the ideology and form imposed on it by Pythagoras and his followers.

The characteristic feature of Pythagorean cosmology was the statement that "everything is a number," which, according to Aristotle, was due to the fact that "they discovered that types and properties, and other kinds of things, seemed to have been created like numbers, and numbers are something primordial in the whole nature; therefore they found that the rules of building the numbers were the rules governing the Universe, and the heaven was a musical harmony and numbers." [12] Thus, in the Pythagorean school there appears the idea that the scientific cognition has to find in nature the harmony described in mathematical terms. This concept, giving mathematics the central role in science, was one of main stimulants of mathematical studies in natural sciences. [13] In the next chapter we will show how this concept was transformed, in Plato's approach, into the program of development of mathematics, which in a very short time led to the creation of mathematical paradigm [14]—to Euclid's *Elements*.

It is therefore in the conditions of development of the Greek cosmol-

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[10] For the critical analysis of mathematical results ascribed to Thales see: S. Kulczycki, *Z dziejów matematyki greckiej* [From the History of Greek Mathematics], Warsaw, 1973, p. 22; for the whole production see: S. Oświęcimski, *Thales—the Ancient Ideal of a Scientist*, in: *Charisteria Thaddaeo Sinko*, Warsaw, 1951.

[11] Cf. Midonick (ed.), *op. cit.*, p. 408.

[12] Aristotle, *Met.* 985 b 31 (see also *Met.* 985 b 27, 986 a 3).

[13] E. Franck, *Plato und sogenannten Pythagoreer*, Halle, 1923, p. 21.

[14] We use this term according to T. S. Kuhn, *The Structure of Scientific Revolution*, Chicago–London, 1962.

ogy, or, more generally—the Greek philosophy, that one should look for the causes of the creation of the geometric vision of the Universe, determining further growth of geometry, of the whole mathematics, and of other sciences based on it. It would thus be especially important to determine the causes of such an orientation in cosmological studies, and to explain why they have been developed in such a way. Unfortunately, current analyses are not fully satisfactory in this respect, and a presentation of our suggestions concerning this point would lead us too far away from our main subject. Anyhow, as it can be seen, the question about the originality of the Greek mathematics has been thus changed to embrace the problem of the originality of the whole Greek culture; the problem that had been analysed (without drawing final conclusions) ever since Aristotle.

The Pythagorean tradition has for long established the opinion that mathematics is a science about numbers and geometrical figures.[15] In current attempts of classification of the sciences it is also stressed that mathematics "explores quantitative and spacial relationships in reality as a whole.[16] But the rapid development of mathematics nowadays and the process of mathematization of other sciences made the number of problems investigated by mathematics increase to such an extent that it is impossible to describe them briefly, or even to enumerate the most important ones. Any attempt to define the subject of mathematics must inevitably lead to philosophical problems concerning the nature and the way of existence of objects examined by it. Since these problems belong to most difficult and most controversial ones in philosophy, we must doubt whether this route will provide us with a satisfactory explanation of the nature of mathematics. The shift of the balance from mathematical objects to the aspects to reality investigated by mathematics (as in the above quotation) does not eliminate this difficulty. Therefore, the contemporary attempts at definition what mathematics is, are focused on its methodology. The description of the method, although it does not give a full insight into the problem, seems to be simpler and more satisfactory both for "pure" mathematics and for its applications.

It is sufficient to assume, for our purpose, that the method used in mathematics relies on a wide application of deduction in order to motive the theorems formulated. The first fully deductive course of mathematics were Euclid's *Elements*. For 200 years they had been not only a compendium of elementary mathematical knowledge (hence the name

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[15] Compare the title of a very popular (even now) book on mathematics: H. Rademacher and O. Toeplitz, *Von Zahlen und Figuren*, Berlin, 1933.

[16] E. Olszewski, New Principles of Classification of Sciences: A Proposal (forthcoming).

of elementary mathematics), but also a manual of geometry and the example of mathematical accuracy. Therefore, the explanation of the origin of the form of *Elements* will also provide the explanation of the origin of the deductive method, and, with acceptance of the methodical definition of mathematics—also of mathematics as a whole.

In the opinion of Proclus, Euclid "deserved admiration pre-eminently in the compilation of his *Elements of Geometry* on account of the order and of the selection both of the theorems and the problems made with a view to the elements. For he included not everything which could have been said, but only such things as he could get down as elements. And he used all the various forms of syllogisms, some getting their plausibility from the first principles, some setting out from demonstrative proofs, all being irrefutable and accurate and in harmony with science. In addition to these he used all the dialectical methods, the divisional in the discovery of figures, the definitive in the existential arguments, the demonstrative in the passages from the first principles to the things sought, and the analytic in the converse process from the things sought to the first principles." [17]

From this quotation one can see that the deductive method of *Elements* can be derived from the Eleatic dialectics, i.e. from a method of philosophical analysis of the problems by means of confrontation of opposite points of view. Originally, those viewpoints were expressed by interlocutors in dialogues (as in Plato's works) and represented true opinions of authentic opponents. Sometimes, the aim of the discussion was to gain a consensus bringing the opponents nearer to the objective truth. Sometimes, as for example in sophistic discussions, the aim was to destroy the adversary and to win the discussion then treated as a game. In this latter case the hypothesis presented by the winner did not have to be true; it could even contradict the personal experience of the debaters, could be paradoxical.

A. Szabo analysed [18] in a detailed way this origin of the deductive method—Euclid's pattern as well as the usual procedures of mathematicians. According to this analysis the postulates of *Elements* supply the common grounds for opponents in discussion (i.e. the judgments obvious for both sides, from which discussion could start). The axioms are supplementary demands on the part of the person starting the discussion addressed to the potential adversary. The proof a contrario is an eristic procedure, relying on a temporary acknowledgment that the opponent

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[17] Cf. Midonick (ed.), *op. cit.*, p. 413.

[18] A. Szabo, *Greek Dialectic and Euclid's Axiomatics*, in: Lakatos (ed.), *op. cit.*, pp. 1–8; *Wie ist Mathematik zu einer deduktiven Wissenschaft gevorden*, "Acta Antiqua Ac. Sc. Hung." 4 (1966), 1–4, pp. 109–152.

is right, in order to demonstrate the absurdity of his approach. The principle of the excluded middle can also be derived from the rules of the game in the discussion ...

Therefore, also the problem of the origin of the deductive method (as it was previously the case with the cosmological subject of mathematics) leads us to more general issues—the origin of dialectics, the role of the dispute, of discussion and argument in the Greek culture (especially in the social and political life); to the causes of the fact that in the system of *polis* a discourse or a speech became a political instrument, a basis of all authority, a tool of management and control of other people. Briefly speaking, we approach the problem of the "erosion of power" characteristic for ancient Greece, and lasting from the Doric invasion in the 12th century B.C. up to the reign of Alexander the Great, [19] and the sources of *polis* with its democratic system.

Thus the Greek mathematics was organically bound with the Greek (and Hellenistic) culture as a whole and to such an extent that it is impossible to understand its genesis and nature irrespectively of all the cultural and social phenomena underlying it.

THE PLATONIC PROGRAM AND PARADIGM OF MATHEMATICS

In this chapter we shall examine the philosophical assumptions and program underlying Euclid's *Elements*. Following the tradition and considering Euclid as the follower of the Platonic doctrine (Proclus: "In his aim he was Platonist, being in sympathy with this philosophy, whence it comes that he made the end of the whole *Elements* the construction of the so-called Platonic figures" [20]), we shall stress the role of Plato whose influence it would anyhow be difficult to overestimate. Proclus wrote about him that "he made the other branches of mathematics as well as geometry take a very great step forward by his zeal for them; and it is obvious how he filled his writings with mathematical arguments and everywhere stirred up admiration for mathematics in those who took up philosophy." [21] The Platonic school of thought provided the first attempts, prior to *Elements*, towards the systematization of geometry, linked directly with teaching of mathematics in Plato's Academy. Also in later periods mathematics developed under direct Platonic influence (in Neo-

[19] See Aristotle, *Athenaion Politeia*, III. 2–4. For political roots of peculiarities of Greek culture see J. P. Vernant, *Les origines de la pensée grecque*, Paris, 1962 (especially chap. 4 and 5); for connections with the origins of mathematics see O. Gigon, *Der Ursprung der griechischen Philosophie*, Basel, 1945.

[20] Cf. Midonick (ed.), *op. cit.*, p. 411.

[21] Ibid.

platonic circles) and till Modern Times the "Platonic realism is a working hypothesis of eac creative mathematician." [22]

The analysis of Plato's concepts which influenced further development of mathematics will be based on the text of the 7th book of the *Respublica*, providing the program of research and teaching mathematics (which, with slight modifications, is still currently used).

The characteristic feature of Plato's attitude to mathematics is a strict separation of pure and applied mathematics. It is also a feature of the whole Greek culture, in which the speculative wisdom was estimated much higher than practical skills. It can be traced e.g. in Aristotle's distinction and mutual relationship between *episteme* and *techne*. Thus Plato sees in mathematics "a something which all arts and sciences and intelligence use in common," [23] but when determining the predominating (and strongly established) role of mathematics in the education process of the elite of his utopian state, he recommends "to go and learn arithmetic, not as amateurs, but they must carry on the study until they see the nature of numbers in the mind only; nor again, in the spirit of merchants and traders, with a view to buying or selling, but for the sake of their military use, and of the soul herself; and because this will be the easiest way for her to pass from becoming to truth and being." [24]

According to these suggestions, the direct applications of mathematics were assigned the place outside science—among skills, to which both practical arithmetic (logistic) and practical geometry belonged. But one should distinguish those practical applications from the ones in other branches of pure science. Making a hierarchical classification of sciences starting from the purest ones and going to more applied ones, Plato enumerated them in the following order: science about numbers (arithmetic), planimetry, stereometry, then astronomy (after enriching the concepts of pure mathematics with the notion of motion). The first three sciences are concerned with eternal, unchangeable beings, therefore they are most important for the knowledge "of the eternal, and not of the perishing and transient." [25] The order of these sciences depends only on their level of logical complexity (and corresponds to the dimension of the Universe considered). One should add that stereometry in Plato's time only started to develop, therefore his inclusion of research

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[22] J. R. Shoenfield on IVth Int. Congress for Logic, Methodology and Philosophy of Science, Bucharest, 29 VIII–4 IX 1971 (unpublished).

[23] Plato, *Respublica*, 522 C (transl. by B. Jowett, *The Dialogues of Plato*, Oxford, 1875).

[24] Ibid., 525 C.

[25] Ibid., 527 B.

and education in the program proved considerable amount of courage in forecasting of the development of science (in order to accelerate investigations in the field of stereometry Plato postulated participation of the State in the research program [26]). Astronomy in form that Plato advocated was also a purely theoretical science, i.e. represented more of theoretical mechanics than a descriptive astronomy (astronomic observations had to be only a starting point for further mathematical speculations).

The hierarchical division of sciences made by Plato had manifold consequences. The most visible one was a strong link of mathematics and mechanics—the domain which in the future had to have the greatest impact on mathematics and to become the main field of mathematical applications. The second is due to the fact that the Platonic hierarchy was of not only a classifying nature but also of the evaluating one. Long before Kant a principle was stated that the value of a given science depends on the level of its mathematization. We showed elsewhere [27] how this orientation (inherent to science from the very beginning) determined later reductionistic trends.

The hierarchical classification of sciences was a natural consequence of the hierarchy of beings underlying Plato's cosmology. The latter had a gnoseological effect transgressing the frame of the philosophical system in which it originated. For if we assume that the external world we see is an imperfect reflection of some ideal state, then the primary objective of scientific cognition is to approach this ideal model (or rather original) of the given phenomena. Naturally, in such cognition mathematics must have a crucial role—as a science dealing with pure ideas. Thus a logical consequence of such an approach is the statement that the goal of scientific cognition is the mathematical description of a given section of the reality. It is one of the strongest trends in science (sometimes identified with the very essence of science [28]). Let us add that this opinion was also shared by the Pythagoreans (compare the previous chapter). But Plato made another step forward. Contrary to the Pythagoreans he assumed that numbers (and other mathematical ideas) exist independently of objects. [29] Such an approach gave mathematical research a more stable foundation than the Pythagorean point of view (and much more fruitful than could be provided by any practical application or empirical data about real processes).

Let us briefly mention the influence of Plato on the popularization

[26] Ibid., 528 B–C.

[27] J. Waszkiewicz, *Réponse à Anders Kock,* in: R. Jaulin (ed.), *Pourquoi la mathématique?*, Paris, 1974, pp. 27–30.

[28] According to neopositivistic philosophy.

[29] Aristotle, *Met.*, 987 b 30.

of mathematical education. Before him, teaching of mathematics at the primary level practically did not exist, and only several philosophers (the Pythagoreans, Hippias) wanted to include it into higher levels of education.[30] Plato, on the contrary, included mathematics into curricula of all levels of his educational program. At the elementary level mathematics embraced simple numerical and geometrical relations of practical importance. At higher levels, pure mathematics had to teach philosophy of life, to shape the desired features of character. This ideology of teaching mathematics, as well as the program itself, the proposed methods, and even the selective role of teaching mathematics, very closely resembles the teaching mathematics as we conceive of in our times.

As far as the formal side of mathematics is concerned, the role of Plato is indirect. He developed a philosophical dialectic,[31] and Diogenes Laertios ascribed to him the introduction of the very term "dialectic" into philosophy. Plato developed also, and popularized, some ways of argumentation very typical in mathematics. But probably the most important contribution was the separation of the dialectical method from its genetic origin (democracy). Giving the philosophers the right to rule in his ideal republic, and giving mathematics the crucial role in philosophy, he opposed philosophy (and also mathematics) to the ideas of social democracy.

Let us emphasize this latter aspect, since it seems to be very important for the future development of mathematics. After having broken the genetic links with practical activity, after endowing mathematical concepts the rank of objective beings and after isolating dialectic (i.e. also deduction) from its social grounds, mathematics could become a fully authonomous, independent scientific discipline and a form of social activity able to function beyond the civilization, the epoch and the social formation which had given birth to it. Thus, according to our opinion, in respect to mathematics Plato played the role of midwife (to use his favourite term). He also indicated some of the trends of its further development, creating a program which was realized in other epochs and civilizations.

THE HELLENISTIC PERIOD

In previous chapters we stressed the role played by cosmology in the creation of mathematics. Now we shall analyse the links between mathematics and cosmology in later periods. Let us consider in this

[30] H. I. Marrou, *Histoire. de l'éducation dans l'antiquité*, Paris, 1948, Part I, Chap. 6.
[31] Aristotle, ibid.

examination the real and the postulated role of mathematics in the cognition of the Universe and the problem of mathematization of the tools of scientific cognition.

Plato and his followers gave mathematics the central role in the description of the Universe. Such an approach had orientated the astronomic research in the Hellenistic epoch. It was already Plato's disciple Eudoxus that had built the first mathematical model of the planetary movement (a system of concentric spheres) included into the Aristotelian cosmological system. His model was later enlarged (Apolonius, Hipparchus) by complex systems of epicycles and deferents. Its further development led to the creation, in the A.D. 2nd century, of the greatest astronomic achievement of the ancient civilization—Ptolomeo's *Almagest*. It supplied a mathematical description of the planetary system, using not only a very advanced geometry, but also Babylonian arithmetic and astronomical observations.

But in the same period a different view of the Universe started to develop. It was the system created by Aristotle and based on his physics. At the end of the Antiquity this view began to prevail in the Mediterranean culture. The basic difference between the two systems consisted in that Aristotle made the movement and change the main object of his physics, and based cosmology on it—contrary to the Platonic point of view. Since, according to Aristotle, "everything that moves must be moved by something else," hence each motion, each change had to have its cause (physics was a science about such causes). The role of mathematics in physics and cosmology was very restricted. It could be useful in the determination of relations between certain phenomena, could explain some features of processes observed, etc. But being the science about objects unchanging in time, it could not contribute much to the knowledge of the causes of changes observed, except of the so-called formal causes. This drawback of mathematics, as well as of logical reasoning, in explanation of the phenomena of change and movement was quite obvious for the ancients at least from the time of Eleats. Therefore, mathematics could not show the essence of physical phenomena. In the matter under consideration, the difference between the advocates of Platonic and of Aristotelian doctrines can be summed up in the question: is mathematical description the very essence of scientific cognition, or is it only one of the elements of analysis (and indeed a secondary one) and does it have to be necessarily completed by explanation of the causes made in physical terms? [32]

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[32] A. C. Crombie, *Augustine to Galileo: the History of Science A.D. 400–1650,* Melbourne–London–Toronto, 1957, Chap. 3.

This controversy was to play a crucial role in the late Middle Ages and in the Renaissance. The whole scientific development in this period can be regarded from this point of view. [33] But in the Antiquity it did not play such an important role. It was due to several factors. First, let us emphasize the fact that the perspective of the history of science, or more generally of the human thought, deforms in a peculiar way the image of the culture of this period, i.e. the Hellenistic culture. The weight assigned to specific phenomena depends on our contemporary value systems. We are inclined to expose those events which later gave birth to important discoveries, started the directions of research continued till our days, or otherwise had an effect on the shape of contemporary science. The same phenomena which in a diachronic analysis came to the fore, in the synchronic approach could pass almost unnoticed (and vice versa). When analysing the Hellenistic epoch from the viewpoint of the future achievement in the field of mathematics and the natural sciences, we deform the reality, and do not see how weak was the influence it exerted on the public life. Only in the classical epoch (4–5th centuries B.C.) of philosophical flourishment it played an important role in the Greek culture (or the social life). And even then we do not know how far this influence was felt beyond Athens, its geographical centre. If we take into account such events as the condemnation of Socrate, unsuccessful attempts of Plato and his disciples to play an important role in the social (or political) life, or embitterment of Aristotle due to discrepancy between his advice and Alexander's policy, we can get a rather bitter image reflecting how insignificant was the direct influence of philosophy even in the period of this highest development.

In the Hellenistic epoch philosophy played a very marginal role in social life. [34] At the time of Alexander the Great's conquests in the Eastern part of the Mediterranean, there occured a clash, unique in recorded history so far, of different cultures developing up to that time side by side (borrowing certain elements from each other, but preserving their own identity and independence). All the largest Mediterranean civilizations were contained within Alexander's empire, and Greek civilization faced the direct influence of Indian and Chinese cultures. It resulted in considerable cultural destruction since clashing with other civilizations the old cultural systems were undermined. It can be seen best on the example of the Greek religion. The ancient cult of the Olympic gods, linked inseparably with the system of the state-city broke

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[33] Ibid.; see also T. S. Kuhn, *The Copernican Revolution*, Cambridge (Mass.), 1957.

[34] In the analysis of the culture of this period we follow W. Tarn, *Hellenistic Civilization*, 3rd ed. (revised), London, 1952.

down violently, and became substituted by various syncretic religious systems connecting the elements of different cultures and mythologies (e.g. cults of Izis and Sarapis).

Let us enumerate major cultural processes characteristic for that period. We have already mentioned the search for new religious systems which often were of mystical and ecstatic nature. This, as a matter of fact, is a landmark of certain historical regularity which is difficult to explain: it has been observed that trends of this kind spread when the traditional cultural links become broken. Inadequacy of the traditional cultural links makes the situation of an individual a non-determined one and difficult to define in rational terms. This, in turn, favours the tendency to irrational attitudes. The uncertainty concerns also the future. There is an increasing demand for the knowledge about it, i.e. the need for all kinds of fortune-telling and divination. In the Hellenistic period this tendency was reflected in the development of fortune-telling activity and creation of astrology drawing on the Eastern (Babylonian) sources. The above activity brought about all kinds of attempts to exert an influence on the future, either by means of magical practics, or of sophisticated intellectual doctrines.

These factors must have had some effect on the development of philosophical thought. On the one hand, it became preoccupied by existential problems, on the other hand—wide and general religious quests must have been reflected in philosophy. The crisis occurred already in the first generation of the Athenian philosophers of the discussed period. In spite of a large variety of doctrines, all of them emphasized the individual existential problems of the human being (stoicism of Zeno from Krytion, Epicureism, Pyrron's, Tymon's and Arcesylaos' scepticism). [35] The former cosmological syntheses created by Plato and Aristotle were not continued. In the later period the mystical came to the fore, and prevailed in Neopythagorean and Neoplatonic doctrines (the latter was a philosophical synthesis of all the Greek philosophical trends).

Let us mention the last factor, i.e. the growing demand for politically active individuals in the enlarged area of the social life. In case of the political life of the Hellenistic world, but also in case of the Roman Empire, the qualifications of such political persons embraced mainly rhetorical skills (later also the knowledge of law) and some specific features of character. General philosophical background was no longer considered as necessary. Marrou showed [36] in what way the competition between

[35] In the Hebrew thought the Book of Ecclesiastes is going in the same direction.

[36] Marrou, *op. cit.*, Part II, Chap. 8.

rhetoric and philosophy was settled and left the former as a winner on the battleground. Teaching began to have a rhetorical and literary character. Mathematics was degraded and found its place in the system of elementary education, and in special, narrow studies in some philosophical schools. Astronomy was the exception, but it was also adjusted to the literary-rhetoric pattern. A literary work, *The Phenomena*, by Aratos (a poetic and superficial version of earlier astronomical treatises) became a foundation of astronomic knowledge. The lack of elementary mathematical background became so common, that together with the growth of Neoplatonic thought in the first centuries of our era, it became necessary to give supplementary courses in mathematics to young men who had received only literary education. That is why in the A.D. 2nd century Teon from Smyrna devoted his manual to mathematical knowledge useful in the studies of Plato.

In the spiritual life of the Hellenistic world religious quests prevailed over intellectual studies. In the latter years, rhetorics and literature gradually replaced philosophy. Philosophy, in turn, focused on the existential problems and became more directly linked with religious studies. Cosmologic studies were left a relatively narrow margin, and even there, the literary and astrological trends prevailed.

The marginal place of mathematics and cosmology predetermined the path of its further growth. After the rapid development in the 3rd century B.C. (Euclid, Archimedes, Apolonios), both mathematics and astronomy came to a standstill. Their revival started at the beginning of our era in Alexandria (we shall ommit the causes of this process).

Let us come back to the controversy between the Platonic and Aristotelian cosmological theory. Although it was not strong in the discussed period, it was present all the time and is visible in the dual methodology observed even in Ptolomeo's work. Beside the most famous *Almagest* which was the best mathematical description of Plato's concepts, he also wrote *Hypotheses Concerning Planets*, in which he tried to explain his mathematical description also on the basis of Aristotelian physics. Purely physical views are also present in *Almagest*. As we can see, already in that period there occurred a distinct separation between mathematical astronomy, which provided a relatively accurate mathematical description of astronomical phenomena (and thus became a good tool for astrological future-telling), and less developed cosmology, supplying a satisfactory physical explanation of the phenomena observed.

The nature of the Ptolemaic model was the reason of the long life of this division and of acknowledgment of the predominating role of physical cosmology. On the one hand, it was a considerable step forward on the way towards mathematization of the natural sciences, but on the other, the obtained model lost its explanatory power because of its complexity (and all the attempts to make it conform better with the results of observations introduced still more complex elements).

Thus, such a temporary settlement of the controversy between the two divergent trends in cosmology was worked out at the end of the Antiquity. It agreed with the eclectic nature of culture at the time. One thousand years later it became a starting point for a new stage of development of mathematics and cosmology in Mediaeval Europe.

THE PLACE OF COSMOLOGY IN MODERN HISTORY OF MATHEMATICS

A more detailed analysis of the development of natural interrelations between mathematics and cosmology will be presented elsewhere. In this article let us turn to the next important period in this development. Let us draw the attention to a deep change Europe underwent starting by the A.D. 12th century. The reviving scientific thought was under the overwhelming influence of ancient philosophers, whose works were translated into Latin (from Arabian, and then from Greek originals). Aristotle's works played the major role among them. The coherence of his system appealed particularly strongly to the dogmatic and authority-orientated minds of Mediaeval thinkers. This is one of the main reasons of why mathematical speculations did not occur in the first stages of this scientific renaissance. The turning point in this process was the condemnation of the most extreme theses of Christian Aristotelianism (mainly Averroism, but also of some theses of Thomas Aquinas) by the bishops of Paris and Canterbury in March 1277.[37] As a result, other trends were intensified in the most famous universities (Paris and Oxford) that represented a more or less Platonic orientation. The nature of philosophy in the later period was quite different from the one of before 1277. The trends arose, mainly in Oxford, that were predecessors of the 16th-century mathematics (analysis of the notion of infinity, including investigations of the structure of continuum, examination of infinite series).

The second important phenomenon was the development of arithmetic under the influence of the Greek and Arabian mathematics (algebra). It favoured the gradual enlargement of the concept of numbers leading to the concept of real numbers in the 16th century and the appearance of complex numbers in solutions of cubic equations. The third of those processes was the Copernican revolution.

[37] E. Gilson, *History of Christian Philosophy in the Middle Ages*, Toronto, 1955, Part IX.

The most spectacular sign of changes was the interconnection of physics and mathematics (breaking the previous opposition of these two sciences). Tartaglia, Cardano and Stevin opened a long list of scientists working on the border of these two disciplines, and developing both theoretical mechanics and differentical and integral calculi. Thus, beginning from the 16th century, the cosmological theme in mathematics was linked with theoretical (or mathematical) physics. Starting from the discovery of Archimedes' exhaustion method in works of the ancients, mathematics in the 16th century underwent a basic change. According to Kuhn, Copernicus could have been called "the last great Ptolemaic astronomer." "The cosmological frame in which his astronomy was embedded, his physics, terrestrial and celestial, and even the mathematical devices that he employed to make his system give adequate predictions are all in the tradition established by ancient and medieval scientists." [38] This cannot be said about Galileo or Kepler who founded their investigations (and philosophical assumptions) on the newest mathematical methods unknown 100 years earlier.

The common development of this new mathematics and physics brought the old controversy between the Platonic and Aristotelian approach to the explanation of the laws of nature onto a different plane. Mathematization of mechanics was started under a distinct Platonic influence. Galileo wrote: "Philosophy is written in that book which stands forever open before our eyes, I mean the Universe; but it cannot be read until we have learnt the language and become familiar with the character in which it is written. It is written in mathematical language, and the letters are triangles, circles and other geometric figures without which means it is humanly impossible to comprehend a single word." [39] But the winning modern Platonism differed greatly from its ancient model. For Plato the physical world had been only an imperfect image of the transcendent world of the ideal mathematical forms. Therefore, physical cognition was not absolutely true and could be only a mediate stage on the way to the absolute Truth. For Galileo, on the contrary, the world of nature consisted of mathematical beings and laws which could be learned exactly and accurately. Hierarchical Platonic worlds (ideal and material ones) were thus intermingled. Their second separation by Descartes placed them on more equal levels. A methodological expression of this dualistic conceptions of the Universe was the assignment of almost equal significance to mathematical description and to the causal explanation

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[38] Kuhn, *op. cit.*, p. 128.

[39] G. Galilei, *Il Saggiatore*, question 6 (quoted from A. C. Crombie, *Augustine to Galileo*, vol. II, Cambridge, Mass., 1961, p. 142).

of changes undergoing in the world. Thus "two major themes dominate the scientific revolution in the 17th century—the Platonic-Pythagorean tradition, which looked on nature in geometric terms convinced that the cosmos was constructed according to the principles of mathematical order, and the mechanical philosophy, which conceived of nature as a huge machine and sought to explain the hidden mechanisms behind phenomena." [40] In the mechanistic approach, mechanics is not fully reducible to mathematics. But all phenomena in nature can be reduced to mechanical ones after sufficiently deep analysis, reducing all change to the local motion.

At the end of the 17th century, Newton in *Philosophiae Naturalis Principia Mathematica* (1687) established a common paradigm for mechanics and calculus (the same, but for the calculus only, was worked out by Leibniz). Newton liberated mechanics from mechanisms. A purely mathematical character of mechanical laws made it a wholly rationalistic discipline that could be developed by means of deductive methods. The search for mechanisms provoking the observed changes became unnecessary. At the same time, the second part of the mechanistic credo, the conviction about the reducibility of physical phenomena to mechanics, was preserved in science for a long time—up to the end of the 19th century.

The common foundations of the analysis and mechanics allowed for mutual inspiration of these two joined disciplines, namely, the application of mathematics to the analysis of mechanical facts (Riemann still used the physical interpretation whereby to demonstrate the existence of Green function equal to zero on a given surface). So these two disciplines developed side by side thanks to the efforts of the same persons and to the dual nature of the problems considered. Beside Descartes, Pascal and Newton, one can enumerate here other scientists dealing with these two disciplines, such as Euler, d'Alembert, Lagrange, Laplace, and even Gauss or Cauchy. Despite this, the situation raised doubts from the point of view of mathematical foundations of the calculus.

Already at the beginning of the development of analysis, there were two ways of establishing its foundations. The first was to enlarge the range of basic mathematical concepts with the notion of motion (continuous) and perhaps with other mechanical concepts. It was indeed the development of astronomy in the Platonic meaning of the word. This was what Newton had done. He began his *Tractatus de Quadratura Curvarum* by a characteristic definition: "I consider mathematical Quantities in this Place not as consisting of very small Parts; but as

[40] R. S. Westfall, *The Construction of Modern Science: Mechanisms and Mechanics*, New York, 1972, p. 1.

describ'd by continuous Motion. Lines are describ'd, and thereby generated not by the continued Motion of Points; Superficies by the Motion of Lines; Solids by the Motion of Superficies; Angles by the Rotation of the Sides; Portions of Time by a continual Flux; and so in other Quantities. These Geneses really take Place in the Nature of Things, and are daily seen in the Motion of Bodies."[41] In the above excerpt we can also see the opposition against another possibility, namely of basing the analysing on the notion of the infinitesimal. The most consistent attitude in this respect was represented by de L'Hospital under the influence of Leibniz.[42]

Both possibilities caused so much controversy that neither of them could serve as a generally accepted basis for a formal mathematical discipline providing a geometrical standard of accuracy. The adaptation of each of them required the abandonment of a strongly rooted conviction that the objects of mathematics were represented by geometrical objects and numbers. In the classical approach the geometric figures were those which could be defined on the basis of Euclid's geometry. Plane figures, for example, were: polygons and circles (mentioned already by Euclid), conic sections introduced by Apolonios and a certain amount of other curves (and parts of the planes bounded by them) such as spiral, conchoid, etc. Since the beginning of the 16th century the figures defined by means of algebraic relations between coordinates of the points were accepted. But the concepts of line or figure still lacked the level of generality indispensable in the analysis. The same concerned the gradually enlarging notion of the number and the knowledge of properties of the numerical axis, fundamental from the viewpoint of the analysis. Therefore the analytical concepts not only surpassed the accepted list of mathematical objects, but also no possibility was seen of reducing them to geometry or to the theory of numbers. Let us add that the extension of the list of objects was not satisfactory from the point of view of gaining logical perfectness of mathematics. Paradoxes and philosophical difficulties connected with such basic notions as motion, continuum, and infinity (including infinitesimale), were too well known from the time of Eleats.

The modern approach to the calculus, based on the concept of limit, is the result of a compromise suggested by d'Alembert. The concept of limit is similar to that of an infinitely small quantity, although it does not refer to the actual infinity. It preserves also something of the mechanistic intuition of motion. Cauchy was the first to base his

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[41] Cf. Midonick (ed.), *op. cit.*, vol. II, p. 189.

[42] A. Robinson, *The Metaphysics of the Calculus*, in: Lakatos (ed.), *op. cit.*, pp. 28–46.

course of analysis consequently on the notion of limit. A little later Weierstrass defined this notion on the grounds of real numbers theory. That is how in the middle of the 19th century the analysis wholly became a discipline of pure mathematics, and a borderline between mathematics and theoretical physics. Since that time it is totally independent from physical reality and lives its own life; it has its peculiar set of problems (only locally connected with physical applications), examines its own objects (often contradictory to the primary physical intuitions) and can be investigated without any preparation in physics (first courses in the calculus are usually prior to the instruction in these sections of physics to which it can be applied). It does not exclude, of course, the possibility of application of the analysis to physics, nor the inspirations of physics for analysis. But the distance between them, as well as the number of intermediate areas have been increasing.

CONCLUSION

Our brief survey will stop at the point which is very significant for the cosmological role of mathematics. Although the systematization of the analysis on the grounds of the theory of real numbers has brought this branch of mathematics to the point determined by the Pythagorean-Platonic vision of mathematics, it coincided in time with more important events that made a revolution in mathematics and established its position in the system of science.

The ontological status of mathematics was based from the very beginning of its existence on the distinguished role of geometry, i.e. the science concerned with the most fundamental, spatial relations in the Universe. Both philosophy and science accepted this special position of geometry considered as the very basis of the scientific cognition of the surrounding reality. The creation of the non-Euclidean geometry proved that a different description of the space is logically possible. But philosophy faced a serious problem of how to prove that it is Euclidean geometry that is the geometry of our Universe. The answer was presenting itself: the proof should be provided by physical experience. It was already Gauss, one of the creators of non-Euclidean geometry, that attempted to make such an experiment. The physics became responsible for the choice of proper geometry. For some time it seemed that physics was all for Euclidean geometry, on which the magnificent structure of Newtonian mechanics had been founded. The philosophy of mathematics was thus shifting towards conventionalism. This new attitude was formulated in 1905 by Poincaré. According to him, the Euclidean geometry itself was a sort of convention as to the way of expression: it was also possible to describe mechanical events using the non-Euclidean space, but it would be perhaps a less convenient tool.[43]

With this turning point was connected an increasing role of mathematics as a universal tool in science. Manifold successes of mathematized physics and the fact that mathematics plays within it only the role of a tool, created a precedent used later by other sciences. But besides of a rapid expansion of mathematical methods and the development of other mathematical branches, it caused that mathematics can explain the surrounding reality only intermediately through other scientific disciplines dealing with this reality. Let us add that as all tools, mathematics can one day be substituted by another, more perfect and effective one. Although such a situation is difficult to imagine nowadays, let us not forget that 100 years ago the elimination of the Euclidean geometry from cosmology seemed equally (or even more) improbable. And we can see even now, difficult and ineffective methods of mathematics are being replaced (on the local scale as yet) by more efficient numerical or simulation methods of informatics.

Thus, after losing its cosmological role and being transformed into a tool, mathematics is pushed to the margin of major philosophical and ideological controversies of our time and it will probably play a less important role in the future than it used to play in our history.

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[43] H. Poincaré, *La valeur de la science*, Paris, 1905; id., *L'espace et géometrie*, "Révue de métaphysique et de morale" 3 (1895).