Grodziński, Eugeniusz

Philosophical Foundations of Many-Valued Logic

Organon 24, 21-39

1988

Artykuł umieszczony jest w kolekcji cyfrowej Bazhum, gromadzącej zawartość polskich czasopism humanistycznych i społecznych tworzonej przez Muzeum Historii Polski w ramach prac podejmowanych na rzecz zapewnienia otwartego, powszechnego i trwałego dostępu do polskiego dorobku naukowego i kulturalnego.

Artykuł został zdigitalizowany i opracowany do udostępnienia w internecie ze środków specjalnych MNiSW dzięki Wydziałowi Historycznemu Uniwersytetu Warszawskiego.

Tekst jest udostępniony do wykorzystania w ramach dozwolonego użytku.





Eugeniusz Grodziński (Poland)

PHILOSOPHICAL FOUNDATIONS OF MANY-VALUED LOGIC

By the truth value of propositions we mean the degree of their accordance with reality. A true proposition is one which says, first, that something is so and so (stating a positive fact) and indeed this something is so and so in reality; or, second, one which says that something is not so and so (stating a negative fact) and indeed this something is not so and so in reality. A false proposition is one which says, first, that something is so and so (stating a positive fact), while it is not so and so in reality; or, second, one which says that something is not so and so (stating a negative fact) while in reality it is so and so. Classical mathematical logic, as well as its predecessor, Artistotelian logic, are both two-valued logics because they employ just two truth values of propositions : truth and falsity. The Polish logician, Jan Łukasiewicz, laid foundations for a new logic, a three-valued logic (which he later developed into many-valued logic). Łukasiewicz assumed that apart from propositions which are unquestionably in accordance with reality (true propositions) or unquestionably at odds with reality (false propositions), there are propositions about which all we can say is that it is possible they are in accordance with reality but they may as well not be in accordance with reality. Łukasiewicz attributed to those propositions a third value—apart from truth and falsity—that of possibility, defining them as "neuter propositions" (i.e. propositions which are neither true nor false).¹

If we are to introduce the notion of "neuter" propositions into scientific discourse, and this is probably necessary, we must begin with defining certain things in a more accurate manner. When we say about a proposition that it may be true but it may as well be false, as a rule we give expression to our uncertainty. The proposition we have in mind is in fact true, or it is in fact false, but we are unable to establish its actual truth value because we lack the necessary

¹ Cf. J. Łukasiewicz, Z zagadnień logiki i filozofii [Some Problems in Logic and Philosophy], Warsaw, 1961, pp. 125, 153.

information. This specific ignorance is regarded as temporary in some cases and as definitive and irremovable in other cases. When we hear the proposition "Apart from thinking living beings in our planet Earth, thinking living beings also exist on some other distant planets," we are perfectly aware that this proposition is true or false already now, but, because we have virtually no communication with those remote planets, all we can state at this stage is the *possibility* of both the truth and falsity of this proposition. Still, we trust some communication with those planets may be set up in the future, and then we shall be able to see whether the proposition about the existence of rational life there was true or false.

This may not be the reaction of an agnostic philosopher asked to establish the truth value of the proposition "A physical world, distinct from human consciousness, does really exist." He knows that this proposition has one of two classical truth values, being either true or false, but he says the proposition *may* be true and it *may* be false, because he does not know which of these two truth values actually holds for this propositions. So, we have to do with an agnostic who has no hope whatever for people ever knowing for sure whether or not a physical world exists. He thinks the human mind is incapable of grasping that kind of knowledge. Accordingly, he believes the answer to the question about the existence of the physical world is bound to be inconclusive for ever. In his view, a physical world may exist, yet it may as well not exist.

Generally speaking, whenever we think that a proposition is true or false but are unable to establish its truth or falsity (whether we regard our uncertainty as temporary or as irremovable), three-valued logic should not be taken to work at all, because classical two-valued logic is perfectly enough to establish the truth value of such propositions. The notion of possibility is applied, as the foundation of a third truth value, not so much to the state of our knowledge of the proposition's truth value as to the proposition itself. Thus, a proposition has a third truth value, that of possibility, when there is no way of knowing if it is in accordance with reality or not, and when the reality itself is to "blame" for that, because it has not yet been definitely determined.

It follows that only those propositions can have a third truth value which concern the future, i.e. propositions formulated in the future tense. The proposition "In fifty years' time, the danger of nuclear war will have disappeared once and for all" is now neither true nor false (having, as it does, a third truth value), because the reality of the world fifty years from now has not yet been established and it may turn out to be this or that. Both the present and the past have already taken their final shapes, and facts which exist now cannot possibly not exist, whereas facts which existed in the past cannot possibly not exist in hindsight. Accordingly, propositions about facts of the present or the past may be in accordance with them, in which case they will be true; or else they may not be in accordance with those facts, and thus false. But such propositions cannot have a third truth value. The proposition "John attended a conference yesterday" (there is no doubt about John's identity) is either true (when John did attend the conference) or false (when he did not). This proposition cannot be either true or false, for it is impossible that John neither did nor did not attend the conference. But the proposition "John maybe attended a conference yesterday" does not express the fact itself of John's presence at or absence from that conference but only reflects our ignorance of whether or not that fact did occur.

Not each proposition about the future has a third truth value. When there is no uncertainty about the future, propositions about it are either true or false. The proposition "A year from now, it will once again be January" is true if said in January, whereas the proposition "A year from now, it will be July" is false if said in January. A third truth value, as pointed out before, pertains solely to those propositions which concern an uncertain, not-yet-determined, future. But "hard determinists," followers of Laplace, are of course convinced that whatever has happened, is happening, or will happen even in an infinitely remote future, was, is, and will be, determined to the tiniest detail. This view does not necessarily imply a determining intervention by a supernatural force, but it does presuppose immanent determination, i.e. the belief that the universe is entirely and absolutely governed by causality which decides the course of any event, and nothing like randomness can really exist.

"Hard determinists" are staunch adherents to two-valued logic. They regard any proposition—not only those concerning the present or the past, but also those about even the remotest future—as true or false already now. Accordingly, they do not recognize the existence of propositions having the truth value of possibility. A "hard determinist" will insist that when we assert something that is, is possible, then we do so solely out of ignorance about how things are (were, or will be). If we could know everything, we would have no use at all for the concept of possibility itself.

"Hard determinism," however, is the credo of only a tiny minority of philosophers today. Apart from indeterminists, who are another tiny minority, most philosophers now subscribe to what can be called "soft determinism," a theory which does recognize causality, but, along with this principle, also recognizes randomness and its great role in nature and human life. Thereby "soft determinists" acknowledge the indeterminate character of many future events and accept the third truth value for all those propositions which confirm or deny the truth of all indeterminate future events already now.

Naturally, the third truth value of propositions concerning an indeterminate future is by itself a temporal event. When the future which is described in such propositions becomes the present, they cease to be "neuter" and become true or false ones depending on whether or not the facts which are stated in them come about in reality.

The possibility of occurrence of a fact often amounts to the measurable likelihood of its occurrence. When you cast a die with six sides, each marked with spots from one to six, then it is possible that the die falls with the side showing three spots facing upwards, and the likelihood of this particular outcome is one-sixth. When we consider a logic as three-valued but not as many-valued, then each possibility of truth (or falsity) of a proposition about an indeterminate future should be considered equal to other ones, for it constitutes the third truth value of the proposition whether its likelihood is high, low or infinitesimally small. If in a lottery involving one thousand tickets 999 are winning numbers, then, in three-valued logic, the proposition "My number will be drawn in this lottery" is "neuter" in the same way as the analogous proposition about a lottery in which only one number in one thousand tickets wins.

But we can refuse to confine ourselves to the three values of truth, falsity, possibility, and to pledge ourselves in favour of a many-valued logic in which, next to truth and falsity, there are many more values, each carrying a different degree of likelihood. Thus, if there are 999 lotteries, each involving one thousand tickets, and chances of winning grow in the order of natural numbers from one per thousand up to 999 per thousand, then in each next lottery we will be facing a growing truth value of the proposition. "My number will be drawn in this lottery," beginning with 0.001 and finishing at 0.999.

If we add to these 999 lotteries a zero-lottery, i.e. one in which no ticket wins, as well as a one-thousand-lottery, i.e. one in which all tickets win, then the proposition "My number will be drawn in this lottery" will be false (and only false) in the zero-lottery but true (and only true) in the one-thousand-lottery. So, we will have altogether 1,001 lotteries, and the proposition, "My number will be drawn in this lottery," will take 1,001 truth values ranging from falsity through successive degrees of growing likelihood up to truth.

In the logical field between falsity and truth you can also place an infinitely great number of growing (from falsity to truth) or decreasing (from truth to falsity) likelihoods. An infinitely-many-valued logic will be the result of such an operation.

TRUTH VALUE AND "ZERO TRUTH VALUE"

If I ask my friend to determine the truth value of the proposition "I, Eugeniusz Grodziński, resident at that and that address, am going to die on an even day,"² he will probably react this way: "The day of your death has not been determined in nature. It depends not only on any objective cause, say on the irreversible process of ageing, but also on many contingent factors which cannot possibly be foreseen at this stage ; you may, for instance, be involved in a fatal accident. Therefore the proposition you have just uttered has the third truth value, as it is neither true nor false. It will become true or false the moment you die."

² As far as I know, an analogous example was cited for the first time, although in a different connection, by Professor Kazimierz Ajdukiewicz.

Agreeing with my friend's opinion,³ I will ask him to do another thing, saying, "Tell me the truth value of the proposition 'I am going to die on an even day, please. Notice that now I am referring this proposition neither to myself nor to any other person, but I am mentioning it merely by way of example."

My friend will probably have this to say : "This proposition, again, is neither true nor false, but it has a third truth value. Whichever person you may refer to, it is true that he or she may die on an even as well as an odd day. As long as that person is alive, the day of her or his death is unknown."

But at this point I would disagree saying, "The proposition 'I am going to die on an even day' has no truth value at all when applied to no specific person, indeed it does not lend itself at all to definition in the aspect of truth value. Consider, for example, the thing known as 'book,' or the word 'book,' for that matter ; you would not say about them that they are true or false, for, quite simply, you do not put a question about their truth value at all. This holds, *mutatis mutandis*, for the proposition 'I am going to die on an even day.' As long as we do not nor can know who that I is, we cannot possibly check this proposition as true or false. If it does not lend itself to this kind of verification, then it does not lend itself to the attribution of a truth value to it, either."

The conclusion to draw is clear : Only a proposition that can be attributed a third truth value—i.e. one about which you can justifiably say that it will become true or false at some point in the future—can be said to be neither true nor false. If the truth or falsity of a proposition can never be established because of its specific structure as an elliptic proposition—then we should not say about such a proposition that it is neither true nor false but only that it does not lend itself to the attribution of a truth value, or that it is unsuitable for logical valuation.

Let us look at other examples. The proposition "Yesterday at noon, air temperature in Warsaw was 10° C" has no truth value. That such a proposition has no truth value holds as much for this proposition (as long as that "yesterday" remains a pseudo-date) as for the proposition. "I am going to die on an even day" (as long as the person referring to himself or herself keeps his or her identity secret).

Still, there is a difference, albeit an indirect one, between the propositions "Yesterday at noon air temperature in Warsaw was 10°C" and "I am going to die on an even day," as far as their truth values are concerned. As elliptic ones, these two propositions cannot have their truth values determined. But as soon as their ellipticity has been eliminated, their truth values will be different from each other. The proposition "I am going to die on an even day" after the speaker has

³ To save readers from uncessary complications, I leave aside the fact that the proposition "I, Eugeniusz Grodziński, etc., am going to die on an even day" will become meaningless after my death. The dead do not speak, and if they did, I would have to rephrase this proposition to put it into the past tense and not in the future.

divulged his or her identity, will have gained a third truth value, that of possibility, and will preserve that value until the speaker's death, upon which it will become a true proposition or a false proposition.

But the proposition "Yesterday in Warsaw air temperature was 10°C" will become true or false the moment the date has been specified, without having to go through a "purgatory" period of temporarily holding a third truth value.

It is seen, then, that elliptic propositions, which include pseudo-definite terms such as I, you, he, they, today, yesterday, tomorrow, here, there, that, and so on, do not lend themselves to valuation whether or not they have such or other truth value ; in other words, they are "zero-value propositions." These propositions become free of this limitation only when by such or other operations they have been purged of their ellipticity.

Not only propositions with pseudo-definite terms are elliptic. All propositions which include vague terms are so. Is the proposition "33-year-old John is a young man" true or false? There is no way to answer that with one word. Peter, who is 70, will probably think John is a young man, while 15-year-old Paul will not think John is young at all. The vague word "young," of course, causes all the trouble, for different people tend to attribute different meanings to it and, within certain limits, there is no reason to accuse any of them of saying untrue things.

Can we justifiably say, then, that the above proposition is neither true nor false but has a third truth value, that of possibility ? Absolutely not. This proposition not only does not exist beyond the range of truth and falsity but, just the opposite, it is simultaneously true (for some) and false (for others). Clearly, without referring it to the person who is to use his or her best knowledge to establish the truth value of this proposition, it cannot be shown either to be true or to be false. So, it has to be recognized that the proposition "33-year-old John is a young man" does not lend itself to logical evaluation, while the propositions "33-year-old John is a young man in Peter's opinion" or "33-year-old John is not a young man in Paul's opinion" are perfectly suitable for logical evaluation.

There is another way of attributing a truth value to the proposition "33-year-old John is a young man." To this end, let us describe the vague word "young" using what Professor Kazimierz Ajdukiewicz called a "regulating definition." We can say, for example, "Each and every man who is not yet 35 years old is a young man, and 33-year-old John is a young man." With this definition of the word "young" the above proposition is obviously true. But we can also set different limits to young age, saying, for instance, "Each and everybody is young who is not yet 25 years of age, and 33-year-old John is a young man." If the age limit for youth is set at 25, then the proposition which states John's young age is false.

All this concerns propositions involving ill-defined words. They all have "zero value." To give a truth value to any of these propositions, it is necessary to provide it with a *more precise definition* in one of the above-indicated ways. Incidentally, let me point out that the course of making a proposition more precisely defined differs widely depending on which category of elliptic

proposition is involved, i.e. whether we are dealing with a proposition including pseudo-definite words or one with ill-defined words.

When in the proposition "It rained heavily today," we substitute different specific dates for the pseudo-date "today," we will obtain many different states of affairs. Some of them will be in accordance with the proposition "It rained heavily today (plus a date mentioned)" and in such cases it will be a true proposition. In other cases, this proposition will be at odds with reality, and hence false.

But if we want to render the elliptic proposition "33-year-old John is a young man" more precise and, for this purpose, we either supplement it with various regulating definitions of the ill-defined word "young" or mention individuals who hold different views about what it means to be young, then we are not affecting the original state of affairs at all. We are, now as before, talking about 33-year-old John, and the precision-imparting operation amounts to citing opinions on whether 33 years of age still makes one young (for regulating definitions also express the speaker's opinion).

Apart from truly elliptic propositions, there are also numerous *apparently* elliptic propositions. Such propositions are not "zero valued" ones. In fact, they do have some truth value.

Let us consider a proposition such as the following ones : "Rainfall is a useful thing," "Cancer ends in death," "Orders must be obeyed," and so on. In these and similar propositions, a certain feature is attributed to an object the name of which is in the plural. You can argue that the proposition, "Cancer ends in death," is false, if it is taken to mean that "Each case of cancer is necessarily terminal." Yet the same proposition is true if it is construed to mean, "Some cases of cancer are terminal." This proposition, then, is an elliptic proposition, and as long as we do not clearly say which of the two interpretations is right we cannot attribute any truth value to it at all, not even that of possibility.

But I think this argument is erroneous. Logic tells us that propositions formulated in this manner should be preceded—in the mind or on paper—by the major quantifier, i.e. they should be construed as, "All rainfall is a useful thing." "All cases of cancer are terminal," "Orders must always be obeyed." Indeed, this is the proper interpretation of such propositions in colloquial speech. This type of propositions, then, are not elliptic at all. They are always false when the addition of a major quantifier before them makes them so, but they are true when the major quantifier in front of them confirms their truth (e.g., "People are mortal" = "All people are mortal").

From what has been said so far, does it follow that only those propositions are elliptic which include pseudo-definite or ill-defined words? The proposition "The book is on the table," includes no pseudo-definite or ill-defined words. It is nevertheless an elliptic (and hence "zero valued") proposition; it will remain so as long as it has not been supplemented with details saying which particular book is meant, which table, or exactly when does it happen that that book is lying on that table. Still, should somebody try to prove that each elliptic proposition does include pseudo-definite or ill-defined words, such a person would have a point. He or she could point out that as long as we do not know which particular book or which particular table are meant, the proposition "The book is on the table" is actually equivalent to "A book is on a table," and the indefinite article is typically a pseudo-definite word.

Propositions such as "The book is on the table," "New goods are for sale at the shop," or "The battle has been won," abound in colloquial speech (as well as in scientific literature). The overall number of such propositions is probably much greater than that of those which give accurate definitions of facts they refer to. It seems to follow that most propositions (sentences) we use in ordinary speech have no truth value at all and so do not lend themselves to logical appraisal. Fortunately, that is not so. People who communicate with one another using elliptic propositions usually specify in their minds—on the basis of their common body of knowledge—everything those propositions are about. This way, they cease to be elliptic and take a truth value, becoming either true or false. Thus, when John says to Peter, "The battle has been won," and Peter understands what John means (even though he may think the news untrue), then they both know which battle John has in mind, who fought it, when it happened, and which side was described by John as the winning one.

Were it not for the continually repeated procedure of supplementing elliptic propositions with extra information by people in their minds, people could not communicate at all if they depended on such propositions alone. People cannot communicate unless they take a stand on the truth value of utterances they exchange in the process of communication.

ON APPARENT "ZERO-VALUEDNESS"

In this section, I wish to challenge some authors who deny any truth value to certain propositions which, in my view, do have value; those authors regard such propositions to have what I propose to call "zero value." When referring to this specific quality, I am going to speak of "apparent zero-valuedness."

The British philosopher P. F. Strawson went perhaps further on that road than anybody else. In his widely-discussed article "On Referring" (1950), Strawson vehemently attacked Bertrand Russell's classification of all propositions into true, false, and meaningless. Strawson argued that propositions, including clearly meaningful propositions, have no truth value, and that only "statements"—not even all of them—are true or false. By "statement" Strawson denoted a proposition uttered by somebody in good faith that the state of things is as asserted in the proposition. Strawson, accordingly, made a plea for universal and unconditional "zero-valuedness" of propositions as such.

In this way, Strawson not only attacked Russell but also the entire body of two-valued logic (as well as many-valued logic, even though Strawson does not seem to have taken any special interest in that kind of logic) which attributes truth value to propositions. Although Strawson doggedly defended his position, I do not feel persuaded to his view. Strawson's position precludes the existence of any truth value in certain types of propositions which never take the form of statements but which, as most people feel, do have such value, being either true or false.

Guesses are a case in point. Suppose John says to Peter, "Guess whom of our mutual friends I bumped into today." "You met Paul," replies Peter. "Yes, it was Paul," John states. Was Peter's proposition "You met Paul" uttered in belief that that was what actually happened? Not at all. John and Peter have many mutual friends and John might as well have bumped into Andy, Bob or Charlie. But nobody (except perhaps Strawson) will deny that proposition "You met Paul" uttered by Peter was a true sentence in that particular situation.

Here is a slightly different illustration of our point. Adam hates Alex and wants to hurt him. Knowing that Alex has an old father in the countryside, Adam phones Alex and, without revealing his identity, says, "Your father died yesterday." This proposition, of course, is no statement; indeed, Alex is certain that is all not true. But, by an odd coincidence, Alex's father indeed died the day before. Accordingly, Adam did tell the truth, yet without uttering a statement.

Strawson's doctrine turns out to be particularly vulnerable in propositions which are blatantly at odds with reality. Nobody will seriously utter things such as the following propositions : "Some people have three heads," "Two and two make five." "Elephants are smaller than ants" or the like, and so such propositions never really get a chance to become statements. Yet once again common sense tells us they are evidently false, and not devoid of any truth value as Strawson wants it.

Some logicians deny any truth value to propositions which attribute such or other features to nonexistent objects. Let us again begin with Strawson, who, as we saw, denies any truth value to all kinds of propositions without exception, but, as will be seen, denies such value also to certain statements. Says Strawson :

Now suppose someone were in fact to say to you with a perfectly serious air : "The King of France is wise." Would you say, "That's untrue"? I think it is quite certain that you would not. But suppose he went on to ask you whether you thought that what he had just said was true, or was false; whether you agreed or disagreed with what he had just said. I think you would be inclined, with some hesitation, to say that you did not do either; that the question of whether his statement was true or false simply did not arise, because there was no such person as the King of France."⁴

A closely similar view was put forward by another widely known British philosopher, P. T. Geach, who points out that if France has no king, then the affirmative answer to the question "Is the present King of France bald?". "Yes, he is bald," will be "not false but simply out of place."⁵

⁴ P. F. Strawson, "On Referring," in : *Philosophy and Ordinary Language*, ed. by C. E. Caton, Urbana, 1963, p. 174f.

⁵ P. T. Geach, "Russell's Description Theory," in : *Philosophy and Analysis*, ed. by M. Macdonald, New York, 1954, p. 33.

I think both Strawson and Geach stretch illegitimately the meaning of proposition (statement) without truth value. The truth value of any proposition which attributes such or other features to nonexistent objects (e.g., "the present king of France") can be easily established in a process of verification which can as a rule be done in the mind and no empirical verification seems necessary.

Thus, if we were to answer the question about the truth value of the proposition "The present king of France is wise," we would first have to rid this proposition of its elliptic character by substituting the words "reigning in 1990" for the pseudo-date "present." To establish the truth value of the proposition "The king of France reigning in 1990 is wise," we should recall, first, that any proposition which asserts a fact as existing in reality is true when that fact does exist in reality; and, next, that any proposition which asserts a fact as true whereas that fact does not exist in reality is false. The proposition "The king of France reigning in 1990 is wise" asserts a fact which does not exist in reality, because in 1990 France had no king, and a nonexistent king can be neither wise nor unwise, neither bald nor not bald; indeed he has no features at all that can be attributed to a real human being.

Accordingly, the proposition "The king of France reigning in 1990 is wise" is false, the way all propositions attributing any features to nonexistent objects are false. As far as the truth value of this proposition is concerned, it is of no importance at all whether or not the speaker knows that France was a republic in 1990. Whether or not he knows that, this proposition is false, for it is at odds with facts. This, to my understanding, shows that Strawson and Geach are wrong in their arguments.

But perhaps our position can be questioned by an adversary saying, for example, this: "King Lear presented by Shakespeare in his tragedy never existed, and yet the proposition 'King Lear loved his daughters' is true." But our adversary would be wrong. The proposition "King Lear loved his daughters" never had a referent in reality (because King Lear never existed), and therefore it was, and is, false. Anyone who has read Shakespeare's play will understand that the proposition "King Lear loved his daughters" is a superficial formulation. The in-depth formulation should perhaps be this : "One of Shakespeare's fictional (and hence nonexistent) characters is King Lear, who is presented as loving his daughters." Now this is unquestionably a true proposition.

From the question of truth value of propositions with subjects represented by names of nonexistent objects (empty names), let us now proceed to the more involved question of truth value of what are called performative utterances. To explain what these are, we should take a quick look back at the history of studies of such utterances. The term itself, "performative utterances," was introduced by J. L. Austin, the British philosopher of language, several decades ago. Austin classed all utterances (propositions and series of propositions) with one of two categories : reporting (or "constative"), and performative ones. Constative utterances report facts which are independent of the utterances referring to them, i.e. facts which exist even if no utterance about them is ever made. Performative utterances, for their part, do not report any facts independent of them but themselves create certain facts.

Propositions such as "Rome is the capital of Italy," "Mary is a teacher," or "I was in the theater last night" are constative utterances, whereas propositions such as these, for example, "Take this ring as a present from me," "I swear to speak the truth," or "I promise to quit smoking," are all performative utterances. Each of them creates a new fact, and none of these three facts—the fact of a donation, the fact of an oath, and the fact of a promise—would have arisen had those propositions not been uttered.

According to Austin, only constative utterances can be true or false, depending on whether or not they fit the facts to which they refer. As far as performative utterances are concerned, said Austin, they are unsuitable as carriers of any truth value because they say nothing about facts independent of them.

But Austin's view is debatable. It can be argued that performative utterances not only create certain fact but, at the same time, also supply information about the facts. If the fact reported in the utterance has come about, the utterance is true; if it has not, the utterance is false. When John tells Mary "Take this ring as a present from me" he not only creates the fact of a donation, in result of which the ring becomes Mary's property, but he also informs Mary and all those hearing his utterance about that fact. If a performative utterance did not fulfill a reporting function (secondarily) as well, John would have to say, first, "Take this ring as a present from me" to Mary, and next, "I am telling this you and everybody else hearing me." But this latter supplementary utterance is entirely superfluous. John's utterance "Take this ring as a present from me" is true when the ring legally becomes Mary's property. It is false when Mary does not become the owner of the ring, which is the case when John gives her a ring he does not legally own.

The truth value of "I swear to speak the truth" presents a slightly different problem. In this case, thruthfulness must not be confused with sincerity of the utterance. The person who is taking the oath may not have the slightest intention to keep his or her pledge. Nevertheless the proposition "I swear to speak the truth" he or she utters is true. Why? Because the proposition both constitutes the oath and informs about its existence. The oath itself, honest or not, was indeed made. Precisely because the oath was made, the perjurer should be called to account for having breached it.

Austin's view that performative utterances have no logical value is opposed by other analysts. For instance, G. J. Warnock, one of Austin's successors, says, "A saying which, by convention, counts as doing something could perfectly well be the saying of something true or false (though indeed, in such a case, truth or falsehood might not be the point mainly at issue)—so that the happy-unhappy distinction does not in any way exclude the true-false distinction."⁶ Warnock's

⁶ G. J. Warnock, "Some Types of Performative Utterances," in : *Essays on J. L. Austin*, Oxford, 1973, p. 73.

position is endorsed by I. Hedenius, a Swedish semiotician, who said, "A locutionary act is true or false. It is true if the performatory function of the utterance works smoothly, and false otherwise."⁷

But why should we draw attention to arguments in favour of the view that performative utterances have their truth value ? I think logicians who deny any truth value to the many different categories of assertory propositions are guilty of a certain superficiality in the study of such propositions, as though they stopped halfway through. As I see it, each meaningful proposition (I leave aside what are called hypothetical propositions, which I will return to later) has a truth value, with the only exception of elliptic propositions, i.e. propositions including pseudo-definite or ill-defined words (or both). But we know that elliptic propositions promptly get a truth value once they have been supplemented with expressions which eliminate the indeterminacy caused by the use of pseudo-definite or ill-defined words.

But, somebody could ask, why should we devote so much attention to propositions which are truly or allegedly "zero value" in an article on foundations of many-valued logic? Because we regard "zero-valuedness"—the unsuitability of certain propositions to have any truth value—as no less important than the truth value of other propositions. I am sure that if we ignored the question of "zero-valuedness," we would rip a large hole in our considerations of many-valued logic.

TRUTH VALUE OF MODAL PROPOSITIONS

Modal propositions comply with one of the following patterns :

(1) Propositions including an operator of possibility (*problematic* propositions) : (a) "Is it possible that p." (b) "Is it possible that not p."

(2) Propositions including an operator of necessity (*apodictic* propositions) : (a) "It is necessary that p." (b) "It is necessary that not p." It should be pointed out that "It is necessary that p" means the same as "It is impossible that not p" while "It is necessary that not p" means the same as "It is impossible that p."

The variable p in these propositions stands not for propositions but for facts, for states of affairs. Thus, the model proposition "It is possible that p" does not stand for the utterance "It is possible that proposition p is true" but for the utterance "It is possible that fact p exists (has existed, will exist)." The model proposition "It is necessary that p" does not stand for the utterance "Proposition p is necessarily true," but for the proposition "It is necessary that fact p exists (in the present, the past or the future)."

The operators of possibility and necessity can refer either to a fact as a whole or only to an action or state of things which are parts of that fact. Thus we can

⁷ I. Hedenius, *Performativer*, quoted after M. Furberg, *Locutionary and Illocutionary Acts*, Göteborg, 1963, p. 198.

formulate pairs of equivalent propositions: (1) "It is possible that John has arrived" = "Perhaps John has arrived." (2) "It is necessary that winter is followed by spring" = "Winter is necessarily followed by spring."

Let me also point out that logic and philosophy of language are interested—apart from modal, i.e. problematic or apodictic, propositions, indeed even before these—in the essentially more important category of *assertory* propositions which do not say that this or that is possible or necessary but simply state that something is so and so or that something is not so and so.

The general formula of assertory propositions is, "Thing A has (not) the property B" or, simply, "A is (not) B."

Last but not least, we should mention *hypothetical* propositions (conditional sentences) of the formula. "If a fact (state of affairs) p holds, fact (state of affairs) q holds too" (or, simply, "If p, then q"). Hypothetical propositions have certain important properties as far as their truth value is concerned.

After these preliminary remarks let us now look at the truth value of modal propositions, beginning with problematical ones. This brings us to what appears to be a paradox. As pointed out before, the proposition "I am going to die on an even day," which is an assertory proposition, has the third truth value, that of possibility, because nature has not decided whether I should die on an odd or an even day now that I am still alive.

But the proposition which I have also just uttered, "I may die on an even day" has not the truth value of possibility but a more reliable one—it is true. This proposition will remain true (considering the time when it was said) even if death does get me on an odd day.

Let us look at yet another example. The assertory proposition "Mankind in the third millenium of our era will be happier than in the second millenium" has the truth value of possibility. This proposition can be verified as true or false only one-thousand-odd years from now, in the year 3001, provided of course that all necessary statistical data can be collected. But the problematic proposition "It is possible that mankind will be happier in the third millenium of our era than in the second millenium" is true already now and will remain so even in the remotest future. Even if the third millenium should turn out to have been much worse for mankind than the second one, a proposition uttered in 1990 (at a time when nothing yet was decided in nature about the following millenium) asserting the possibility of the third millenium being better cannot but must have been true, for such a possibility indeed did exist.

This brings us to the following conclusion : problematic propositions, i.e. those which indicate possibility in their formulation, *cannot* have the truth value of possibility. Can problematic propositions be false ? Certainly, they can. They are always false when they state the possibility of a certain state of affairs which is in fact impossible. Thus, the problematical proposition "It is possible that man will develop backwards, from age to infancy" is false. Many less obviously preposterous propositions than this one can be cited to show that problematical propositions may be false. It is false to say "John may call today to see us" if it is

known that John has just left for a very long trip to a far away country. The next conclusion to draw, then, is that problematic propositions can have one of two and only two truth values : they are either true or false.

Let us now look at the truth value of apodictic propositions. First observe that there are genuine apodictic propositions (or, simply, apodictic ones) apart from quasi-apodictic (i.e. only apparently apodictic) propositions. Both the one and the other categories look like apodictic propositions, because they use the operator "It is necessary (that)," although this operator may take entirely different meanings in each of the above-distinguished categories of propositions. In (genuine) apodictic propositions, the operator "It is necessary that p" is equivalent in meaning to "It is impossible (in the literal sense) that not p." Alternately, the operator "It is impossible that p" is equivalent in meaning to "It is necessary that not p."

Apodictic propositions are true when they state the necessity of a fact (state of affairs) or the impossibility of a fact (or state of affairs), and when that necessity or that impossibility do indeed occur in reality. Apodictic propositions are false when they state the necessity of a fact (state of affairs) or the impossibility of a fact (state of affairs), but when that necessity or that impossibility do not occur in reality.

Here are examples of true apodictic propositions: (1) "If all men are mortal, and Socrates is a man, then it is necessary for Socrates to be mortal" (logical necessity); (2) "If 10 is multiplied by 10, then it is necessary for the product to be 100" (mathematical necessity); (3) "If we let a body freely fall close to the Earth, then it is necessary that that object falls on it" (natural necessity); (4) "If John is taller than Peter, then it is necessary that Peter is shorter than John" (semantic necessity, following from the meanings of the terms "taller" and "shorter," respectively).

Examples of false apodictic propositions are easy to find. Here are a few : (1) "It is necessary that Socrates is immortal"; (2) "It is necessary that 10 times 10 is 200"; (3)"It is necessary that a body let fall freely close to the Earth moves towards the Moon"; (4) "It is necessary that if John is taller than Peter, then Peter is taller than John."

Is there anything like necessary truth or necessary falsity of apodictic propositions? For example, can we say that the proposition "It is necessary that Socrates is mortal" is *necessarily* true, whereas the proposition "It is necessary that Socrates is immortal" is *necessarily* false? We will say nothing of the sort, for that would be too much of a good thing, or, to put it in a more learned fashion, that would be a completely useless pleonasm. Apodictic propositions, accordingly, are true or false, but not necessarily true or necessarily false.

We said above that, unlike assertory ones, problematic propositions have only two truth values but not the third truth value of possibility. How about apodictic propositions ? Can they have the truth value of possibility ? Yes, they can.

Suppose John says, "Even in a million years from now, it will be necessary that anyone living then must die sooner or later." We are inclined to acknowledge already now the truth of this apodictic proposition. But, on second thoughts, you may find this is not as easy as it looks. You cannot rule out the possibility that during that one million years, if man is still alive, a way of preventing ageing as well as all potentially fatal diseases will have been discovered, and that may make all or some people immortal.

In view of this possibility, which is at least theoretically plausible, we should recognize the proposition "Even in a million years from now, it will be necessary that anyone living then must die sooner or later" as neither true nor false today, that is, we should attribute the third truth value to it. Generally, it can be said that the third truth value pertains to those apodictic propositions which shift the necessity of some facts to a yet indeterminate future, and even that necessity itself has not been embraced by determinacy yet. However, this is rather an artificial word structure, one you will rarely find in colloquial or even literary or scientific language. The third truth value, then, is of lesser significance for apodictic than for assertory propositions, and among the latter-named ones there are many including such or other assertions about a yet indeterminate future.

This brings us, surprisingly enough, to the following general conclusion : many-valued logic has customarily been associated, one way or another, with modal (problematic or apodictic) propositions. But on closer inspection it turns out that many-valued logic may not exist at all for problematic propositions, for they take only two truth values (either truth or falsity), whereas for apodictic propositions the third truth value is less important, much less anyway than for assertory propositions.

Let us now look at propositions described above as quasi-apodictic. John says, "It is necessary that Peter and Sophie marry" (in other words, "Peter and Sophie must marry"). What is the meaning of the operator "It is necessary that" in the proposition uttered by John ? Does it mean that John wants to say that it is impossible (logically, naturally or otherwise) for Peter and Sophie not to marry ? No, John certainly does not have this kind of impossibility in mind.

Consequently, the proposition he utters is not an apodictic proposition (authentically apodictic) but only a quasi-apodictic one. More precisely, the operator of necessity exists solely in the superficial layer of the proposition but not in its deeper meaning. What did John have in mind, then? Perhaps, deep in his meaning he wanted to say, "Only if they marry each other can Peter and Sophie be really happy"?

But of course it is also possible than John, who knows Sophie is pregnant with Peter, means this, for example, "Only if Sophie marries Peter will their child have a legitimate father."

Or take another example. Paul, a great soccer fan, says, "It is necessary that I watch this great match." Again, does he mean to say it is impossible that he does

not watch the match ? No, that is not what Paul is saying. The in-depth meaning of his utterance is probably this : "Watching this match will be really a treat for me, and missing it will be an awful disappoinment."

In the in-depth structure of John's and Paul's utterances, the operator "It is necessary that" does not occur any more. Nor are there appearances of apodictic propositions, but there are assertory propositions in the in-depth structure. We know that the logic of assertory propositions is a three-(or more)-valued one. Thus, the proposition "Only if they marry each other can Peter and Sophie be really happy," has a third truth value, for at the moment Peter's and Sophie's marital satisfaction, however likely it may be, is just anybody's guess.

How about Paul's utterance ? When he thinks "Watching this match will be really a treat for me," then although he thinks about the future (a near future, for that matter), we can regard that future as determined in the sense that Paul is certainly right thinking that watching the match will make him happy. So, the proposition uttered by Paul should be regarded as true, in its deeper sense.

The general conclusion to draw is that the truth value of *quasi*-apodictic propositions is the same as that of assertory propositions (for this is what quasi-apodictic propositions essentially are), and not as the truth value of authentic apodictic propositions.

*

I pointed out before that there is no point in speaking about the necessary truth or necessary falsity of apodictic propositions, for that would just be too much of a good thing. But should the terms "necessary truth" and "necessary falsity" be pulled out from scientific language for good ? Of course not. Assertory propositions should be described as necessarily true or necessarily false whenever their content fits the reality. Thus, the proposition "I am mortal" is necessarily true, while "I am immortal" is necessarily false. But this instantly raises a further important question : Should not we introduce—in the category of assertory propositions—apart from the three truth values of truth, falsity and possibility, two more truth values—necessity (necessary truth) and impossibility (necessary falsity) ? In my view, a flat "No" is the right answer to this question.

The truth value of possibility has been distinguished as a third value only because possibility is neither truth nor falsity. But necessary truth, of course, is one form of truth, opposite to what is called contingent truth, the way analytical truth is customarily opposed to synthesizing truth and *a priori* truth to *a posteriori* truth. Analogous divisions, of course, can be pointed out in the realm of falsity. All special forms of truth together constitute the logical value of *truth*, while all forms of falsity in their entirety constitute the logical value of *falsity*. Accordingly, no form of propositional truth separately from other ones, nor any form of falsity on its own, are the given proposition's separate truth values.

One more point. Quasi-apodictic propositions are those which are apodictic

in form but assertory in substance. There is another such opposition of propositions, namely propositions which are assertory in form but apodictic in substance. Such propositions will be called crypto-apodictic propositions. The proposition "All people are mortal" is a textbook example of crypto-apodictic propositions. The in-depth structure of this proposition includes the operator of necessity, namely the tacit formulation "It is necessary that" everybody dies, and this in turn means "It is impossible that even one man lives on for ever in this world."

Not all propositions preceded by the major quantifier "all" are crypto-apodictic. The proposition "All students in our form have good marks in history" is not crypto-apodictic but an ordinary assertory proposition. It does not have any in-depth structure such as "It is impossible (logically or naturally) that any student in our form gets bad marks in history." The truth value of crypto-apodictic propositions is the same as that of (authentically) apodictic propositions.

TRUTH VALUE OF CONDITIONAL CLAUSES

The truth value of conditionals (hypothetical propositions) is a broad question which deserves to be treated separately. In this section. I wish to make just a few illustrations to draw the pertinent conclusions. Suppose that John, thinking Peter is an amateur of opera, says, "If Peter came to Warsaw yesterday, he went to the opera right away." What is the truth value of this conditional, and how can it be checked? The question appears very simple indeed. First, we have to check whether or not Peter arrived. If he did, then we must check if he went to the opera the previous day. If he indeed went to the opera on that day, then we can say the conditional uttered by John is true. Should we establish that Peter did arrive but did not go to the opera, then the same conditional would have to be recognized as false (the truth or falsity of the proposition, by the way, would have existed right from the moment John uttered it, even before the facts had been checked).

But should it turn out Peter did not come to Warsaw, the problem would be very complicated. In such a case, the conditional "If Peter came to Warsaw yesterday, he went to the opera" becomes unverifiable, for nobody can check whether or not Peter would have gone to the opera, had he indeed come to Warsaw. Admittedly, if Peter's intentions and habits are known to me, I can try to establish that fact which never actually happened with a certain degree of likelihood, but likelihood is not certainty. If we wish to establish the truth value of this proposition assuming Peter did not come to Warsaw after all, then we can follow one of two ways : either we recognize that under these circumstances the proposition "If Peter came to Warsaw yesterday, he went to the opera" has the third truth value, that of possibility, or we conclude that under the circumstances the conditional does not lend itself to any truth appraisal at all, i.e. that it is "zero-valued." At first glance, the former of the two ways will probably appear the more convincing of the two. However, on second thoughts the latter of the two will probably be chosen. We remember, of course, that the third truth value pertains to propositions only temporarily, until the future has become "determined" which deprives the proposition of the truth value of possibility leaving it with the first or second truth values, i.e. either as true or as false. The proposition "If Peter came to Warsaw, he went to the opera" does not concern a determined future but a past which has been determined once and for all, and—if Peter never really came to Warsaw—this proposition can never be verified as true or false. Under these circumstances and with this assumption, we can recognize the conditional considered here as a "zero valued" proposition. As pointed out before, this does not deprive us of the possibility to study the likelihood of Peter's visit to the opera house, should he come to Warsaw after all.

Or, John already knows Peter has not come to Warsaw and therefore formulates his utterance right away as a so-called "unreal past" sentence saying, "Had Peter come to Warsaw yesterday, he would have gone to the opera." What is the truth value of this proposition ? In the light of what we have just said, the answer is easy ; this proposition neither does nor can have any truth value; it is "zero-valued."

Are all unreal-past conditionals "zero-valued"? No, they are not. Suppose not a single drop of rain fell in Warsaw on July 10, 1985. The conditional "Had it rained in Warsaw on July 10, 1985, the streets in that city would have been wet" is of course an unreal-past conditional. Yet it is certainly not a "zero-valued" conditional. Indeed, it is a true proposition. How come there is such a difference in the truth value of two conditionals which have the same structures ? The conditional "Had it rained, the streets would have been wet" represents the relation of "strict implication" between the "if" and the "then" parts of the conditional (protasis and apodosis, respectively) such that whenever the "if" part is true the "then" part is *necessarily* true. Indeed, due to the laws of nature it is impossible that while it is raining the streets should remain dry. All conditionals of this type are true even when the "if" and the "then" parts taken separately are false, as the example now considered clearly shows.

The conditional "Had Peter come to Warsaw yesterday, he would have gone to the opera" does not represent the "strict implication" relation, for it is not physically impossible for Peter to have come to Warsaw and yet not gone to the opera, that is, it is not physically impossible for the "then" part to be false despite the "if" part being true. When in this type of conditional the protasis also turns out to be false (because Peter did not come to Warsaw in our case), then the conditional cannot possibly be checked, and so it has no truth value at all.

Let us now look at a conditional which says much the same thing but its predicate is in the future tense. Says John, "If Peter comes to Warsaw tomorrow, he will go to the opera right away." What is the truth value of this conditional ? As it was being uttered, it concerned a yet undetermined future. It was possible, after all, both that Peter would come to Warsaw and that he would not, and, in the event he did come he might as well have gone to the opera as not. So, at the moment it was uttered, the above conditional had the third truth value. The situation changed on the following day. If Peter did come to Warsaw and did go to the opera, the conditional has passed from the stage of the third truth value and became true ; if Peter did arrive but did not go to the opera, then the conditional "If he comes to Warsaw, then he will go to the opera" has also lost its third truth value, becoming false.

Lastly, if Peter did not come to Warsaw, our conditional, while losing the third truth value, which is perforce only a temporary value, has lost all value in the logical sense and has become "zero-valued," i.e. nonverifiable.

The logical appraisal of the above-given individual examples can of course be extended to embrace any number of conditionals of different forms.