Piętka, Dariusz

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Dariusz Piętka (Warsaw, Poland)

THE PHILOSOPHY OF STANISŁAW LEŚNIEWSKI

Introduction

Stanisław Leśniewski was one of main representatives of the Lvov–Warsaw philosophical and logical school. His work falls on the years 1911–1939 and can be divided into two periods: the early one, which can be referred to as grammatical, and the late one, referred to as formal. The former period involved, above all, epistemological and metaphysical analyses focusing on such issues as those of existential propositions, the truth, principles of non–contradiction and excluded centre as well as analyses concerning the status of general objects. The late theories include protothetics (generalized propositional calculus), ontology (generalized calculus of names) and mereology (theory of collective set).

The research of Leśniewski was focused on the foundations of mathematics, which resulted from historical context. Indeed, the beginning of the 20th century was a period of turbulent development and of theoretical transformations in mathematics and its foundations. It was in that period that the foundations of mathematics were upset by various antinomies which were discovered at that time. Most famous antinomies, which as a matter of fact were examined by Leśniewski in his search for the possibility to avoid them, are the antinomies of the set of sets which are not one another's elements, and the antinomy of the set of all sets. The work of Leśniewski focused on seeking such method of consolidating mathematics as to avoid the problems encountered by researchers of the classical set theory. This was to be supported, first of all, by mereology which was being created from 1916 onwards, as well as by generalized systems of propositional calculus and calculus of names.

The article presents the views of Leśniewski both from the first and the second period of his work. Although his articles from the period of 1911–1915 do not usually get much attention, they should not be completely forgotten as the opinions expressed therein are reflected in his later output. Above all, the method of practicing philosophy changed radically. The purpose of deliberations presented below is a synthetic discussion of the views of Leśniewski, but also presentation of certain connections between his opinions from the grammatical and formal periods.

Leśniewski was opposed to pure formalizm. He called his calculus of name as *ontology* with a reference to Aristotle's first philosophy which was understood as a knowledge of principles of being. The first philosophy is interpreted as a general theory of objects[1]. According to Sobociński[2] and Woleński[3], Leśniewski was metaphysicist in logic. He belived the logic describes the world. In this meaning we can say that Leśniewski's logic is a way of practice of philosophy. There is similar with mereology. The collective set is an aggregate consisting of parts. The theory of collective sets is the theory of objects' complexity[4]. Leśniewski's systems belong to philosophy. There are general theories of objects. We have to remember that in theoretical meaning Leśniewski's systems are uninterpreted systems as pure logical systems. They can be treated as logical system too. A choise of approach depends on problems which are put. We are interested in all results of Leśniewski's research and we do not limite to a pure logical research. In this meaning this paper treats Leśniewski's output as philosophy.

1. Early views

1. 1. Epistemology from the semantic point of view

While analysing epistemological issues, Leśniewski did it in a way which was later on referred to by Ajdukiewicz as semantic theory of cognition. The object of analysis in the theory of cognition defined in this manner are not mental processes of thinking, but logical propositions. Basically, all the works of Leśniewski classified as the early period are semantic analyses of propositions, because of – for example – their conditions of truthfulness.

Among Leśniewski's early works, his Ph. D. thesis deserves particular attention. The intuitions and deliberations contained therein were continued and expressed in ontology. Beginning from the definition of existential sentence as a sentence whose predicate has a positive or negative form of the verb *to exist* or of synonymous verbs, Leśniewski analyses situations in which such sentences are true as well as situation in which they are false[5]. In that purpose he performs a fundamental division of existential sentences into positive and negative ones, along with emphasising problems connected with negative existential sentences. In fact, if an existential sentence has a negative predicative in the predicate using the word *is*, this leads to a contradiction. In this case, the predicative co–means the feature of non–existence. *Non–existence* is the synonym of the word *non–being*, and that is contradictory with the word *being* referring to the object denoted by subject of the sentence. The analyses performed by Leśniewski constitute an attempt to resolve the classical Platonic problem described by the Athenian in *The Sophist.*

According to Leśniewski, a common and intuitive view is that both positive and negative sentences can assume the value of truthfulness or falsity. An opposite statement has its origins in that the forms of existential sentences

[1] Cf. S. Leśniewski, *O podstawach matematyki*, p. 162.

[2] Cf. B. Sobociński, *In Memoriam Jan Łukasiewicz* in: *Philosophical Studies* 4, 1/1957, pp. 40–43.

[3] Cf. J. Woleński, *Filozoficzna szkoła lwowsko–warszawska*, Warszawa 1985, p. 139.

[4] J. A. Stuchliński, *Definicja zdania prawdziwego w języku logiki i językach opartych na logice*, p. 39.

[5] S. Leśniewski, *Przyczynek do analizy zdań egzystencjalnych*, p. 329–332.

are commonly used to symbolise some of such meanings whose adequate symbols would be non–existential sentences. For instance, the forms of existential sentences *people exist*, *a square circle does not exist* are used for verbal symbolisation of meanings whose adequate verbal symbols are the sentences *certain beings are people*, *no being is a square circle*.

While analysing the hypothesis claiming that all sentences can be reduced – without a change in meaning – to existential sentences, Leśniewski notices a threat that sentences with a negative linking verb could never be true. For example the sentence *Paris does not lie in China* would be false[1]. In fact, if the sentence has an existential nature, one ought to state that the lying of Paris in Chine does exist, whereas the non–lying of Paris in China does not exist. That sentence would symbolise non–existence of Paris in China. The question arises whether language can symbolise something which does not exist?

Therefore, the fundamental subject of inquiry becomes the issue of adequate symbolisation of object related meanings by linguistic expressions. In this context Leśniewski specifies the primary types of inadequate symbolisations and adequate non–existential symbolisations corresponding with them: (a) the inadequate sentence *Only objects* A *exist* is matched by the adequate non–existential symbolisation of *All beings are objects* A, (b) the sentence *Objects A exist* is matched by the adequate symbolisation of *Some beings are* A, (c) the sentence *Object* A *exists* is matched by *One being is* A, (d) negative sentences *Objects* A *do not exist* and *Object* A *does not exist* are matched by the non–existential symbolisation of *No being is object* A[2]. One can easily notice that very similar formulas can be found in ontology. (a) and (b) are affirmative sentences from the square of opposition, with the former constituting one of the factors of conjunction of the right side of the axiom. The objective is to avoid discussion of general objects in ontology – that is why ontology discusses all specific objects and points to a certain group in this manner. Point (c) on the grounds of ontology can be referred to as definition of the *ex* (i. e. exists) predicate[3].

According to Leśniewski, there exists a principle allowing creation and analysis of adequate symbolisations of object related meanings. In his opinion, conclusion of adequacy consists in analysing the attitude – in relation to the speaker's symbolisation intentions – to the above mentioned (a), (b), (c) and (d) schemes. All sentences ought to symbolise possession by the object (symbolised by the grammatical subject of the sentence) of characteristics co–denoted by the predicate. It results from the above that the analysis of adequacy or inadequacy of sentences in relation to meanings symbolised by them is finally based, as Leśniewski claims, on phenomenological analysis of the speaker's symbolisation intentions[4].

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[1] S. Leśniewski, *Przyczynek do analizy zdań egzystencjalnych*, p. 340.

[2] S. Leśniewski, *Przyczynek do analizy zdań egzystencjalnych*, p. 341.

[3] These definitions and the axiom are given in farthest article part.

[4] S. Leśniewski, *Przyczynek do analizy zdań egzystencjalnych*, p. 344.

Intuitions and certain ideas concerning the character of existential propositions were used by Leśniewski in his early polemics, in particular those with Tadeusz Kotarbiński and Jan Łukasiewicz. In the first case, sentences concerning the future, that is referring to events which do not exist yet, is raised. The polemics leads Leśniewski to two questions; first, whether a moment will come in the future when the proposition currently true will not be true, and second, whether there has ever been a moment in which the proposition currently true was not true?[1]

First of all, Leśniewski excludes the view that a proposition is only true if it continues physically. As truthfulness is a property of the proposition, it would only continue throughout existence of the proposition, for example, during its utterance. Assuming that a given truth is not eternal, in other words that there would once come a time t_i when the proposition "*A* is *B*" will be false even though it is true at present, i. e. t_0. If "*A* is *B*" is false in t_i, then in t_i it will be true that "~(*A* is *B*)". "*A* is *B*" will thus be contradictory in t_0 and in t_i. This conclusion cannot be accepted on the grounds of the principle of non–contradiction stating that if one of two contradictory propositions is true, then the other one must and thus always is false. One must, therefore, conclude that if "*A* is *B*" is true in t_0, then "~(*A* is *B*)" in t_i is a false proposition. Claiming that a propositions which is currently true will become false in the future leads to a contradiction. As a consequence, that assumption must be rejected and one must conclude that each truth is everlasting[2]. Correspondingly, one can justify the claim that each truth is immemorial[3].

The early output of Leśniewski is also determined by one of classical philosophical issues, namely by the issue of so–called first principles of thinking and existence, the principles of non–contradiction and excluded centre. Similarly to Łukaszewicz, Leśniewski distinguishes between ontological and logical principles. That is why this matter is partly covered by epistemology and partly by metaphysics[4]. Leśniewski believes that the logical principle of excluded centre ought to be rejected. By means of numerous counterexamples, he attempts to demonstrate falsity of that principle. In order to do that, Leśniewski uses, first of all, the contradictory propositions from the square of opposition with the following forms: *each ... is ...* and *some ... are not ...* , as well as *none ... is ...* and *some ... are ...* . Basing on those schemes, he provides examples of such existential propositions which are neither true nor false. Indeed, neither the proposition *each person exists (is existing)* nor the sentence *some people do not exist (are non–existent)* can be true[5].

Beginnings of mereology can be found in the year 1914. In order to

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[1] S. Leśniewski, *Czy prawda jest tylko wieczna, czy też wieczna i odwieczna?*, p. 506, p .513.

[2] S. Leśniewski, *Czy prawda jest tylko wieczna, czy też wieczna i odwieczna?*, pp. 506–507

[3] S. Leśniewski, *Czy prawda jest tylko wieczna, czy też wieczna i odwieczna?*, pp. 513–514.

[4] Even if Leśniewski gives the proof of ontological principles of non–contradiction (it puts his proof in borders of metaphisics), in fact he begins proving from some definition of notion of true. This argument has metalogical sense. Cf. J. Woleński, *Szkoła lwowsko–warszawska w polemikach*, p. 52.

[5] S. Leśniewski, *Krytyka logicznej zasady wyłączonego środka*, pp. 325–328.

resolve the antinomy of the set of sets not being one another's elements, Leśniewski introduces a definition of set which is different from the classical and distributive one, by a differently defined relation of subordinating an element to a given class. In an intuitive and non–constructive manner Leśniewski presents his idea for collective understanding of a set. In his opinion, Russell's paradox disappears if the expression *an object subordinated to class* K was used, for example, in the following meaning: *either half P of sphere Q is subordinated to the class of quarter of sphere Q*. In this case, the relation of subordination is *de facto* the relation of adjunction of a part to a certain whole[1]. In this context, an element (part) could not be identified with a set (whole). Two years after the deliberations on classes not subordinated to one another, Leśniewski specifies his theory of sets in the collective sense. Basically, it already has the same form as mereology contained in the series of articles entitled *O podstawach matematyki* [*On Foundations of Mathematics*] published in *Przegląd Filozoficzny* in the years 1927–1931. He bases it on the primary term *is a part* and introduces the notion of ingrediens and mereological class[2].

1. 2. Metaphysics

The term *metaphysics* is used here in opposition to the later *ontology* as in the early period Leśniewski dealt with, for example, the issue of perfect objects' existence. Therefore, his analyses covered more than formal relations between objects only. By undertaking this issue, he got involved in the classical and medieval metaphysical dispute concerning the existence of real objects. Among others, the dispute concerned the question if individuals remain in certain relations towards universals, which would guarantee their existential identity. On the other hand, the later ontology examines purely formal properties of objects and is a non–interpreted theory, i. e. does not deal with any specific, really existing beings.

In the pre–formal period of his work, one can notice a certain programme of metaphysics which will guide Leśniewski in subsequent years. He understood metaphysics as a system of propositions concerning all objects in general[3]. However, while discussing all objects in general, in his opinion one cannot mean allegedly existent general objects[4]. It is, in a way, a reference to Aristotle's concept of metaphysics according to which metaphysics is a domain of science concerning everything, in opposition to detailed sciences whose scopes are limited to objects of a given kind. Leśniewski emphasises that *in general* does not mean *general*. Metaphysics does not deal with ideas, forms or any kind of so–called *commons*.

Referring to that period, particular attention ought to be paid to his attitude concerning universals, which was influenced on a form Leśniewski's systems, particulary, on a form ontology and mereology.

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[1] S. Leśniewski, *Czy klasa klas nie podporządkowanych sobie, jest podporządkowana sobie?*, p. 65.

[2] S. Leśniewski, *Podstawy ogólnej teorii mnogości*.

[3] S. Leśniewski, *Krytyka logicznej zasady wyłączonego środka*, p. 312.

[4] S. Leśniewski, *Krytyka logicznej zasady wyłączonego środka*, pp. 312–320.

Throughout the history of philosophy, the dispute concerning existence of universals assumed different forms. Four basic attitudes can be distinguished here[1]: (a) extreme notional realism in which spontaneous and real existence is attributed to general objects, (b) moderate notional realism, that is the view which accepts real existence of general objects, but in a non–spontaneous manner, (c) conceptualism which claims that no general objects exists in reality. Only notions can be general, (d) nominalism, which does not accept existence of any general objects but only general names. The nominalist attitude of Leśniewski seems to have played a key role in all his later works, especially in relation to the form of ontology and mereology. In fact, he also takes up this issue in the later period[2]. The argument of Leśniewski is supposed to justify non–existence of universals irrespective of the multiplicity of concepts concerning general objects[3].

A common feature of all general object is the fact that each of them is a general object in relation to a certain specific group of individual objects. It then has the characteristics common to all individuals of a given group. Leśniewski believes that if a feature is one which is not possessed by all individuals but by some of them only, then a general object cannot possess this feature. If a certain object P is a general object corresponding with individual objects $x_1, x_2, x_3, \dots, x_n$, that is

(i) $Px_1 \wedge Px_2 \wedge Px_3 \wedge \dots \wedge Px_n$

then for each individual object one can find the feature S, which is not common to all individual objects $x_1, x_2, x_3, \dots, x_n$, that is

(ii) $[Px_1 \wedge Px_2 \wedge Px_3 \wedge \dots \wedge Px_n] \wedge [Sx_1 \wedge \sim Sx_2 \wedge Sx_3 \wedge \dots \wedge Sx_n]$,

and therefore

(iii) $Px_1 \wedge Sx_1$ and $Px_2 \wedge \sim Sx_2$.

If the property of attribution in x together with S (P is a general object attributed to x together with S) is marked by α, which is the property of a higher order in relation to property S. Thus, the general object P has the feature of attribution in x together with S and at the same time it does not have this feature:

(iv) $\alpha(P) \wedge \sim\alpha(P)$,

and that is impossible. Therefore, P does not have all features common to individuals of a certain group. P would have to possess the feature of existence in x together with S and of non–existence in x together with S. As this is impossible for any general object, thus, general objects do not exist.

2. Mature Period

2. 1. Semantics and Semantic Categories

Leśniewski was a precursor of scientific development of semantics. Actually, these problems were present already in the first period of his work,

[1] K. Ajdukiewicz, *Zagadnienia i kierunki filozofii*, pp. 110–113.

[2] S. Leśniewski, *O podstawach matematyki*, pp. 183–184.

[3] From the point of view of better readability of argument on unexistence of universals I present it in somewhat manner changed with reference to original version. I have taken advantage of Bocheński's idea. J. M. Bocheński, *Zagadnienie powszechników* in: J. M. Bocheński, *Logika i filozofia*, Warszawa 1993, pp. 101–103.

when he focused on the issues of truth that ranks among metalinguistic concepts. The second period of his work brought about the application of formal apparatus to the analysis and construction of semantic problems. His thorough remarks and reflections in this respect, discussed and analysed during the lectures at Warsaw University, had a great impact on the views of his students and colleagues, particularly on one of the most outstanding Polish scientists of the 20th century Alfred Tarski whose contribution to the development of Polish semantics cannot be overestimated[1]. Tarski gained fame with his work *The Concept of Truth in Formalized Languages* in which he constructed a semantic definition of classical truth. He stated that it was Leśniewski who had first provided precise conditions for an adequate definition of truth.

Leśniewski assumed that semantic concepts which denote certain linguistic objects form part of metalanguage. He also enhanced differences between those interrelated elements. Due to this distinction he acknowledged that semantic antinomies are of extra–logical and metalinguistic character. He distinguished them from logical paradoxes present within formal systems. He assumed that in the language which is not constructionally structurised, namely when from the constructional point of view it does not allow an unlimited number of semantic categories, but is ideally complete and semantically closed to the inclusion into the system of expressions belonging to any given possible semantic categories, laws of classical logic cannot be sustained[2].

A distinction between a deductive system of a closed number of semantic categories and a deductive system admitting an infinite number of grammatical forms sets the criterion for distinguishing a narrow or enlarged system of logic[3].

Leśniewski assumed two basic semantic categories: semantic categories – names and sentences – and the categories of functors[4]. Expressions belonging to those two basic categories can form much richer grammatical forms. Functors are functional expressions which form other expressions from expressions belonging to a definite semantic category (belonging to the same or a different category). For instance, the word *is* is a functor that allows to make out of two expressions belonging to the category of names an expression that belongs to the category of sentences, e. g. using two names *Peter* and *man* the functor *is* allows to construct an expression that belongs to the category of sentences: *Peter is a man.* And so in a similar manner developed can be the infinite wealth of functor expression forms which depend on the number of arguments or categories to which the arguments belong. A sensible expression in its whole can belong to one and only semantic category. If such expression has parts, some parts may belong to a different category than the whole. Each of the parts can, however, as a whole belong to one and only category.

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[1] E. C. Luschei, *The Logical System of Leśniewski*, p. 35.

[2] E. C. Luschei, *The Logical System sof Leśniewski*, p. 35.

[3] J. A. Stuchliński, *Definicja zdania prawdziwego w języku logiki i językach opartych na logice*, p. 11.

[4] The notion *functor* originates from Tadeusz Kotarbiński, and the notion *foundamental kategory* from K. Ajdukiewicz, *O spójności syntaktycznej* in: K. Ajdukiewicz, *Język i poznanie*, t. 1, Warszawa 1985, p. 223.

Leśniewski's theory of semantic categories has its source in Russell's theory of types, Aristotle's theory of categories, and Husserl's theory of meaning categories[1]. Leśniewski's conception was mainly presented by Kazimierz Ajdukiewicz who provided it with an elegant form, according to Woleński[2]. He provided each expression belonging to a given category with following indicators: *z* – a sentence, *n* – a name, *z/nn* – a functor (in this case, sentence–formative from two name arguments). Moreover, Ajdukiewicz also provided a simple way to check a syntactic sensibleness of expressions[3].

When analyzing the concept of assertion or negation[4] in *Principia Mathematica* he points to the ambiguity of explanations proposed by the authors regarding senses of specific terms and phrases. For, according to Leśniewski, it is unclear, for example, whether the sentence "$p \vee q$" should be interpreted with the use of the sentences "*p* or *q*", or perhaps with the use of sentences "'*p*' is true or '*q*' is true". For the first belongs to the theory language, and the latter to metalanguage. On the grounds of the analysis of particular phrases from *Principia Mathematica*, and in particular problems related to the meanings of assertion and negation, Leśniewski finally reaches a precise distinction between language and metalanguage[5].

2. 2. Metalogic

Leśniewski's system of logical languages is composed of phrases belonging to any conceivable semantic category. Variable expressions exist only as bound variables. In Leśniewski's systems quantifiers combine variables belonging to any semantic category, namely both name, sentence, or functor variables. Quantifiers, as distinct from *Principia Mathematica*, do not have any semantic functions. For this reason Leśniewski introduces only one universal quantifier. The implication being that they do not become entangled in existential assumptions. This theory is a pragmatic theory of quantification[6]. An existential quantifier is inscribed by means of universal quantification and two negations. Using a classical quantifier the universal quantifier can be shown as: ∀...(...), with a variable by its side that binds this quantifier in a given expression, e. g. $\forall x\ Px$ Existential quantifier can be inscribed as: ~∀...~(...). When we put variables into dotted places this can be inscribed as $\sim\forall x\sim(Px)$. Obviously, the quantifier does not have a presented form in a Leśniewski System (LS). A quantifier has a form of quoins and is defined by means of structural–descriptive names. The avoiding of any object and existential assumptions demonstrates a purely constructional character of LS.

Language expressions are defined by Leśniewski in a structural–descriptive way. Creation of a specific notation, different from the traditional

[1] S. Leśniewski, *Gründzuge eines neuen Systems der Grundlagen der Mathematik*, p. 14.

[2] J. Woleński, *Filozoficzna szkoła lwowsko warszawska*, p. 141.

[3] K. Ajdukiewicz, *O spójności syntaktycznej*, p. 229.

[4] S. Leśniewski, *O podstawach matematyki* (1927), chapter 1.

[5] J. Woleński, *Filozoficzna szkoła lwowsko warszawska*, p. 136.

[6] J. A. Stuchliński, *Definicja zdania prawdziwego w języku logiki i językach opartych na logice*, p. 61.

one used in the inter–war period, was meant to guarantee maximum precision to Logical Systems. Leśniewski regarded signs as physical objects. A sign, just like every physical object, owns its specific physical qualities, and therefore shapes; structure; it is located in a definite place in space and is an event placed on a definite time axis[1]. Physical attributes of expressions are defined in a descriptive way and they are given precisely defined meanings.

Directives for concluding or defining are formulated in metalogic. They have an essential impact on the shape of deductive theory, but they themselves do not belong to it. Their object is system expressions. Since the basic theory is protothetic, Leśniewski provides primitive metalogic expressions for protothetic. The vocabulary gradually enlarged would allow him to formulate directives and terminology explanations in a symbolic way. Thanks to this vocabulary Leśniewski was able to define specific sings and rules for protothetic in a structural–descriptive way. Four explanations below define the graphic shape of a quantifier as four quoins. In order to read these explanations one needs to know basic elements of metalogic vocabulary. Here are some abbreviations:[2]

A1 means the same as *axiom 1*
vrb means the same as *a word*
cnf(A) means the same *equivalent–shape expression A*
Uingr(A) means the same as *the final word belonging to A*
1ingr(A) means the same as *the first word belonging to A*
2ingr(A) means the same as *the second word belonging to A*, etc.

When using them one can formulate the first terminology explanation related to the shape of the quantifier inscriptions:

Terminology explanation 1: A ε vrb1 ↔ A ε cnf(1ingr(A1)).
Terminology explanation 2: A ε vrb2 ↔ A ε cnf(5ingr(A1)).
Terminology explanation 3: A ε vrb3 ↔ A ε cnf(6ingr(A1)).
Terminology explanation 4: A ε vrb4 ↔ A ε cnf(Uingr(A1)).

All the explanations have their reference to the first axiom of protothetic:

$\llcorner pqr \lrcorner \ulcorner \mathrm{E}(\mathrm{E}(\mathrm{E}(p\ r)\mathrm{E}(q\ p))\mathrm{E}(r\ q)) \urcorner$

Sign *E* is sign of equivalence. Having defined quantifier shapes, Leśniewski goes on to explain the position of quoins in the expression of a quantifier function. It is described in further explanations[3].

2. 3. Protothetic

Protothetic is a Generalized Propositional Calculus which contains propositional variables, functors of various categories, as well as functors binding both propositional and functor variables. As protothetic contains all the conceivable proposition–derivative categories, it can be said that it is a maximally rich system. Woleński claims that it is a system which can be in a way called absolute, for it is hard to imagine a stronger and richer Propositional

[1] J. A. Stuchliński, *Definicja zdania prawdziwego w języku logiki i językach opartych na logice*, p. 14.

[2] S. Leśniewski, *Gründzuge eines neuen Systems der Grundlagen der Mathematik*, pp. 60–63.

[3] S. Leśniewski, *Gründzuge eines neuen Systems der Grundlagen der Mathematik*, pp. 63–75..

Calculus[1]. Protothetic stems from the research into Equivalence Propositional Calculus. It is a system that can be based on various primitive terms, and therefore also on various axioms. Initially, Leśniewski's Propositional Calculus was based on the three following axioms[2]:

A1. $\forall p \forall q \forall r \{[(p \leftrightarrow r) \leftrightarrow (q \leftrightarrow p)] \leftrightarrow (r \leftrightarrow q)\}$

This axiom expresses the quality of equivalence which has been called by us transitiveness.

A2. $\forall p \forall q \forall r \{[p \leftrightarrow (q \leftrightarrow r)] \leftrightarrow [(p \leftrightarrow q) \leftrightarrow r]\}$

A2 characterises associativity of equivalence.

The following axiom is a specific proposition of protothetic which goes beyond the Classical Propositional Calculus (CPC), as it contains semantic categories of expressions with regard to CPC. Leśniewski accomplished this by analysing the following problem: by modes of which axioms and directives can one enhance the classical system of Propositional Calculus in order to receive the calculation that can be added the thesis:

(a) $\forall p \forall q \forall f [(p \leftrightarrow q) \rightarrow f(p) \leftrightarrow f(q)]$

with all its consequences[3]. The thesis contains functor variables of propositional arguments. As one–argument functors from a propositional argument are *it is false* or *it is true*, the thesis can be read that if *p* if and only if *q*, then *p* is true if and only if *q* is true, or if *p* if and only if *q*, then *q* is false. Functor "*f*" thus represents truth function.

The third axiom enhancing the prototheic by means of (a) has the following form:

A 3. $\forall g \forall p \forall f \{ g(p\ p) \leftrightarrow \langle \forall r[f(r\ r) \leftrightarrow g(p\ p)] \leftrightarrow \forall r\{f(r\ r) \leftrightarrow g[(p \leftrightarrow \forall q\ q)p]\}\rangle \leftrightarrow \forall q[g(q\ p)] \}$,

It comprises the principle of extensionality for propositions and the principle of bivalence[4].

A1, A2, and A3 imply all the theses for the Generalized Propositional Calculus. Research into protothetic has proved that different axiomatics for that system are possible. Leśniewski himself finally contained all the three axioms in the following single axiom of protothetic[5]:

$\forall f \forall p \forall q \forall r \forall s \forall t\ (p \leftrightarrow q) \leftrightarrow \forall g(\, f(p\, f(p\ \forall u\ u))) \leftrightarrow \forall u\ (f(g\ u) \leftrightarrow g(((r \leftrightarrow s) \leftrightarrow t)q) \leftrightarrow g(((s \leftrightarrow t) \leftrightarrow r)\ p)$.

In 1945, Sobociński shortened it to the following[6]:

$\forall p \forall q\ (p \leftrightarrow q) \leftrightarrow \forall f\ (f(p\, f(p\ \forall u\ u)) \leftrightarrow \forall r\, f(q\ r) \leftrightarrow (q \leftrightarrow p))$.

2. 4. Ontology

Elements of Leśniewski's ontology are know first of all thanks to Tadeusz Kotarbiński and Bogusław Sobociński. Leśniewski presented fragments of his

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[1] J. Woleński, *Filozoficzna szkoła lwowsko warszawska*, p. 145.

[2] S. Leśniewski, *Gründzuge eines neuen Systems der Grundlagen der Mathematik*, p. 33.

[3] S. Leśniewski, *Gründzuge eines neuen Systems der Grundlagen der Mathematik*, p. 30.

[4] J. A. Stuchliński, *Definicja zdania prawdziwego w języku logiki i językach opartych na logice*, p. 25.

[5] S. Leśniewski, *Gründzuge eines neuen Systems der Grundlagen der Mathematik*, p. 59.

[6] B. Sobociński, *An Investigation of Protothetic*, pp. 201–206.

theory in two papers only, the first of them being *Über die Grundlagen der Ontologie* and the second Chapter 11 of *The Foundations of Mathematics*[1]. Stanisław Leśniewski's Ontology (LO) is currently a calculus of names and constitutes an essential broadening of Aristotle's syllogism. This system is superposed over protothetic in which laws for quantifiers are also formulated. From the point of view of traditional logic it can be said the LO is a system superposed over the quantifier calculus[2].

LO's vocabulary contains only expressions belonging to one primitive category: category of names. Apart from this, it also contains functional expressions. LO's primitive term is functor ε whose meaning has been defined in the only axiom of ontology[3]:

AO. $A \varepsilon a \equiv \exists B\, B \varepsilon A \wedge \forall B \forall C(B \varepsilon A \wedge C \varepsilon A \rightarrow B \varepsilon C) \wedge \forall B(B \varepsilon A \rightarrow B \varepsilon a)$.

In order to build Propositional Calculus Leśniewski continued formulating subsequent definitions[4]:

D1. $\forall A \forall B \forall C\, (A \varepsilon B \vee C) \leftrightarrow (A \varepsilon B \vee A \varepsilon C)$

Definition of a name–formative functor from two name arguments *or*, namely the equivalent of *or* in the Propositional Calculus.

Definition of a name coniunction functor read as the word *and* and of a shape analogical do D1:

D2. $\forall A \forall B \forall C\, (A \varepsilon B \wedge C) \leftrightarrow (A \varepsilon B \wedge A \varepsilon C)$

Definition of a name negation:

D3. $\forall A \forall B[A \varepsilon \sim B \leftrightarrow \exists a\, (A \varepsilon a) \wedge (A \varepsilon B)]$

The name negation is read for example as *no man*. Sentences containing name and propositional negations have different meanings. For instance, in the sentence *A is not a man* the quality of not possessing the quality of being a man is stated, whereas in the sentence: *Not true that A is a man* negated is the attribution of possessing manhood to subject A.

D4. $\forall A \forall B[A \text{ om } B \leftrightarrow \forall a(a \varepsilon A \rightarrow a \varepsilon B) \wedge \exists a(a \varepsilon A)]$

D4 defines the expression *each ... is ...* . An expression formulated as *A om B* is a scheme of a strong universally confirmative sentence. The condition for the true character of this sentence is the existence of at least one designate for A. The functor *each ... is ...* is distinct in LO from the functor *any ... is ...* , the latter designated with the word *sub* and explained in the following definition:

D5. $\forall A \forall B[A \text{ sub } B \leftrightarrow \forall a(a \varepsilon A \rightarrow a \varepsilon B)$

The scheme of the sentence "*A* sub *B*" is a scheme of a general weak sentence. D5. defines the expression *each ... is ...* . D4 informs that the name found in the subject of the general–affirmative sentence cannot be empty.

D6. $\forall A \forall B[A = B \leftrightarrow \forall a(a \varepsilon A \rightarrow a \varepsilon B) \wedge \forall a(a \varepsilon B \rightarrow a \varepsilon A)]$

The above definition defines the meaning of the inter–name equivalence.

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[1] S. Leśniewski, *O podstawach matematyki* 1931, pp. 153–170.

[2] L. Borkowski, *Logika formalna*, p. 277.

[3] S. Leśniewski, *Über die Grundlagen der Ontologie*, p. 114.

[4] I present definitions of ontology according to T. Kotarbiński, *Elementy teorii poznania, logiki formalnej i metodologii nauk*, pp. 207–216.

D7. $\forall A \forall B\,[A \text{ i } B \leftrightarrow \exists a\,(a\,\varepsilon\,A \wedge a\,\varepsilon\,B)]$

The sign *and* is read as *some ... are ...* . The sentence *A* and *B* is therefore a sentence of detailed–confirmative character which is read *Some* A *are* B.

The term *it exists* which is a sentence–formative functor from one name argument is defined by:

D8. $\forall A(\text{ex } A \leftrightarrow \exists a\; a\,\varepsilon\,A)$

The definition states that exists what can be truly stated about. *A* exists if and only if a certain individuality is that *A*.

D9. $\forall A\;[\text{ob } A \leftrightarrow \exists a\;(A\,\varepsilon\,a)]$

It is a definition of an object. The symbol *ob* is therefore read as an *object*. Something is an object if and only if something can be stated as true in an individualised sentence with an *"A"* denote. What can also be said is that this definition claims that there exists at least one A.

The following definition defines the word *sol* which can be read as *at most one A exists*.

D10. $\forall A[\text{sol } A \leftrightarrow \forall a \forall b(a\,\varepsilon\,A \wedge b\,\varepsilon\,A \rightarrow a\,\varepsilon\,b)]$

D11. $\forall A \forall B(A \text{ id } B \leftrightarrow A\,\varepsilon\,B \wedge B\,\varepsilon\,A)$

The expression *id* is read as *identical with*. AO can be assumed on the grounds of AO and D8, D9, and D10. The above quoted definitions and theorems are an element of elementary ontology. In the non–elementary ontology apart from name variables there are also functor variables representing sentence–formative or name–formative functors. They are introduced by means of a rule of joining higher syntax categories to the definition system. Apart from that, rules of extensionality for different semantic categories are assumed.

2. 5. Mereology

Mereology is not a logical theory but it is formal theory. It was built on systems of protothetics and ontology. A conception of a set was the starting point of research and it led to formulating the conception of a collective set, which enabled claiming of any set of objects that it consists just of these objects[1]. As far as the idea of a distributive set is concerned, an essential difference is that the Leśniewski's set exists physically, as well as its elements. If, for example, an *AB* segment exists, which consists of an *AC* segment and a *CB* segment, both segments the *AC* and the *AB* exist in the same way, and the *AC* segment is a part of the *AB* segment. In a similar way we can talk of an apple, which consists of flesh, peel and pips. Both flesh and apple exist in reality.

Mereology is a part of a collective set. Such a set is different in an essential way with its formal properties from a set in its distributive sense[2]. It is defined mostly by axioms and mereology definitions. Similarly as at protothetics and ontology, mereology may have different equivalent axiomatics. Mereology is based on a primitive term *is a part of*.

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[1] S. Leśniewski, *O podstawach matematyki* (1927), p. 190.

[2] Sets are not exist in the distributive sense, contrary to collective sense. S. Leśniewski, *O podstawach matematyki* 1927, pp. 203–204. wrote (...) *in classes of Whitehead and Russell feeling* (...) *smelling of mythical specimen from abundant gallery of* invented *objects I can not dispose propensity for solidarizing* on credit *with Authors' dubts with reference to that these objects exist in the world.*

The first axiom points out some areflexivity of *being a part of* relation. Nothing can be a part of itself. The *being a part of* relation is expressed by means of a symbol *par*[1]:

A 1. $P \varepsilon \text{ par } Q \rightarrow \sim(Q \varepsilon \text{ par } P)$

As opposed to the relation of belonging to a distributive set, the *being a part of* relation is transitive one:

A 2. $P \varepsilon \text{ par } Q \wedge Q \varepsilon \text{ par } R \rightarrow P \varepsilon \text{ par } R$

The ingrediens definition, symbolically *ing*:

D 1. $P \varepsilon \text{ ing } Q \leftrightarrow P \equiv Q \vee P \varepsilon \text{ par } Q$

The definition of a class in its collective sense (symbolically *kl*)[2]:

D2. $P \varepsilon \text{ kl } (a) \leftrightarrow \exists Q\, (Q \varepsilon a) \wedge \forall Q\, (Q \varepsilon a \rightarrow Q \varepsilon \text{ ing } P) \wedge \forall Q\, [Q \varepsilon \text{ ing } P \rightarrow \exists C\, \exists D (C \varepsilon a \wedge Q \varepsilon \text{ ing} C \wedge D \varepsilon \text{ ing } Q]$

D2 shows symbolically three conditions, which were introduced by Leśniewski. *P* is a class of objects *a*, if and only if, if (1) *P* is an object, (2) each *a* is an ingrediens of an object *P*, (3) with every kind of *Q*, if *Q* is an ingrediens of some *a*.

Leśniewski understands the class in its collective sense as a set of all such objects *a*, which means an object *P*, whose each ingredient includes an ingredient *a* within itself, which is characterized by the fact that whatever is *a*, it is *P* as well[3]. Having defined the definition of a mereological class, Leśniewski introduces next axioms:

A 3. $P \varepsilon \text{ kl } (a) \wedge Q \varepsilon \text{ kl } (a) \rightarrow P \varepsilon Q$

If *P* is a class of objects *a* and *Q* is a class of objects *a*, *P* and *Q* are the same object.

A 4. $\exists S\, [S \varepsilon a \rightarrow S \varepsilon \text{ kl}(a)]$

If an object is *a*, an object is a class of objects *a*.

In his theory Leśniewski decidedly opposes to the fact that empty classes exist. In other words, at mereology it cannot be said of any classes, which don't consist of elements. For the reason that sets are some entities in their collective sense, a formal theory of a set may be interpreted in a physical and to–become–realized way. Any parts of concrete objects are their physical parts. We cannot identify the formal theory with its physical and to–become–realized interpretation[4].

Mereology is a theory which concerns its relations and these relations characterize objects consisting of parts, irrespective of material objects and their parts' nature. Mereology was used to describe the structure of expressions understood as physical objects (entities) consisting of parts. The language of metalogics of protothetics and ontology systems, which is built in compliance with semantic rules of parts–and–collective–sets theories[5].

[1] Write down below axioms and main definitions of mereology are placed in: S. Leśniewski, *O podstawach matematyki* (1928), pp. 263–265. Symbolics of axioms used in this papper refer to ontology's symbolics. Leśniewski uses in original text the natural language and name variables *P*, *Q*, *a* etc.

[2] Formalization of this axiom dates from B. Sobociński, *Studies in Leśniewski's Mereology*, p. 219.

[3] T. Kotarbiński, *Elementy teorii poznania, logiki formalnej i metodologii nauk*, p. 18.

[4] J. A. Stuchliński, *Definicja zdania prawdziwego w języku logiki i językach opartych na logice*, p. 40.

[5] J. A. Stuchliński, *Definicja zdania prawdziwego w języku logiki i językach opartych na logice*, p. 39.

It should be added that on the basis of mereological conception of a set, there does not exist any problem of antonymy. The conception of a set proposed by Leśniewski does not allow to claim that a set, which means an entity, is its own element, which means its own part. It is already defined by A1. In this context Woleński points out that only mereology expresses nominalistic Leśniewski's convictions. Leśniewski treated everything as individuals. It may be stated that, in his opinion, mereology is a nominalistic theory of plurality in its own kind[1].

Conclusion

This presentation of theories of one of the most remarkable representatives of Lvov–Warsaw school is of necessity incomplete. Leśniewski's achievements were presented in broad outline. Leśniewski's intention was to build up a system, which would become the basis for Maths in an analogical way like *Principia Mathematica.* Leśniewski's body of work did not generate interest among logicians, mathematicians and philosophers after his death. It happened in this way decidedly because of formalism, which is complicated and does not go with intuitions of colloquial language. It does not mean that the great Polish logician's work did not generate any interest at all within the scope of the worldly logistics. Surely, Leśniewski's work is outstandingly unconventional and other logical systems do not equal it as far as accuracy is concerned. Woleński notices that Leśniewski is an author of a formal paradigm, which orders absolute exactness of utterance. Some formal dissertations famous all over the world of such Polish logicians like Tarski or Łukaszewicz, were stimulated mainly by Leśniewski and not only as far as the formalism itself is concerned[2].

Protothetics is the Porpositional Calculus, which contains all possible to be thought up semantic categories deriving from sentences; ontology includes all possible to be thought up semantic categories deriving from names. Surely, mereology is the weakest formal theory. Grzegorczyk made obligations to pragmatic nature of Leśniewski's systems. Grzegorczyk thinks that opulence of these systems is not necessarily needed within the scope of Maths' practice. In his opinion, ontology is sheer Boole's algebra and mereology is Boole's algebra without a zero element.

Leśniewski's systems played some role at Lvov–Warsaw school, whose the most important backlash was Kotarbinki's ideas. Kotarbiński, in his reism, accepted only one ontological category – objects. All objects are individuals and the conception went with Leśniewski's nominalistic point of view.

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