Edward Nieznański, Agnieszka Burakowska

Formalized Proofs of the Existence of God

Collectanea Theologica 64/Fasciculus specialis, 109-122

1994

Collectanea Theologica
64 (1994) fasc. specialis

EDWARD NIEZNAŃSKI, WARSAW

FORMALIZED PROOFS OF THE EXISTENCE OF GOD[1]

Rev. Jan Salamucha (1903-1944) was the first to formalize in 1934 Aquinas' argument ex motu of the existence of God, inserted in *Summa contra Gentiles* (I,13). This first in the history of theodicy formalized proof for the existence of God was published exactly in *Collectanea Teologica* 15 (1934), 53-92. To commemorate the 50 th anniversary of the execution of Rev. Jan Salamucha by Nazis and the 60th anniversary of the publication of the mentioned above formalization giving birth to the new method of coming to God, let us try to estimate more important achievements, tools and ways to the absolute of the succusors of Rev. Jan Salamucha's idea.

Nowadays, years after the investigations around the formalization of theodicy we are ready to see the main sources of its complexity and also mistakes in the constant confusion of two different ways of thinking: logical and metaphysical. Since any qualifications and metaphysical relationships are in their nature modal – existential a philosopher has no way out from that maze of problems, even with the help of a formalization if he confuses the very metaphysical modalities with logical ones or if he refuses to acknowledge the empirical basis to any kinds of existential judgements.

Generally speaking, there are two possible attitudes of philosophers towards the treatment of the problem of the existence of the absolute: either – according to some – it is a purely linguistic issue and its solution is a simple consequence of terminological agreements or – according to the others – it is, precisely reverse, a factual problem and its solution does not comprise in only linguistic conventions. Detailed divergences are considerable and, not rarely, also extreme.

0.1. PROBLEM OF THE ABSOLUTE AND LINGUISTIC CONVENTIONS

First of all, some semantic conventions lead to contrasting solutions, when they themselves, *a priori*, set the logical value of existential sentences. To give an example, the so called semasiology which was practised by Stanisław Leśniewski in 1910-1915, claimed false all existential sentences. A sentence in its canonical form: subjectively-predicative – in accordance with the settlement of semasiology – states only the possesion by the object

[1] Translated by Agnieszka Burakowska

A the quality B. Meanwhile the „existence" is not a quality, so it cannot be possessed by any object. Therefore a sentence „The Absolute exists" is in this estimasion analytically false.

However, just an opposite metathesis to this proclaimed by Leśniewski about the logical valute of existential sentences, was brougt forward by Willard Van Orman Quine (1951)[2], to whom the thesis *Vx x exists is* trivially true, because to be is to be the value of a variable. Thus there is not any problem of existence of the absolute, but only the problem of its nature.

0.2. THE QUESTION OF THE MINIMUM OF ASSUMPTIONS

Both of these mutually exclusiwe options of logicians are, as a rule, rejected by a philosopher for whom the issue of the existence of the absolute is a factual problem and the thesis of the existence of God is neither trivially true nor trivially false and suggests itself with obviousness directly neither a priori nor a posteriori, since it is not the consequence of only linguistic conventions or a formally recorded sentence from experience. Thus its acceptance can be made only on the way of a proof and not otherwise than within the theory to which this proof belongs. But in the matter of proofs created in the scheme of philosophy the tendentions of a logician and metaphysician are also in general divergent. While a logican tends especially to the exposure of the formal correctness of deduction by formalizing it, a metaphysician practically does not attach any importance to formal considerations. And when, in turn, a metaphysican concentrates his whole effort on gaining other people's certitude multiplying unendingly axioms of a theory, a logician, on the contrary, aims at the minimalization of the number of original statements often having no regard to a sense or anybody's willingness to accept them. Also, in the same way, there misled to pragmatic emptiness, i.e. to the suspension of the judgement about the existence of the absolute both uformalized arguments for the thesis of the existence of God which do not give any possibility of the verification of the logical corollary from premises to a conclusion, as well as taken in isolation, i.e. placed beyond any theory, formalized proofs of the existence of the absolute based on some axioms pulled from the context of metaphysics. Hence, not easy to obtain, a compromise of aims and a golden means in theodicy requires probably such a formalization which accomplishes not one, but two minima at the same time:

1) deductively inevitale minimum of axioms without which the thesis of the existence of the absolute could be proved;

2) pragmatically inevitable minimum of axioms without which it is impossible to understand both simple and complex terms and to assent to oryginal and derivative propositions.

---

[2] The number (n) in brackets occuring at a surname means here, as well as in further places, a bibliograhical abbreviation of a gliven author's work from the year n, according to the list of bibliography placed at the end of the article.

At the same time the first minimum should be included in the second and not inversly.

1. Review of the accomplished formalizations of theodicy

All the attemts made so far of formalization of chosen fragments of theodicy were undertaken either within the theory of relations or under the basis of the logic of predicates.

1.1. FORMALIZATIONS WITHIN THE THEORY OF RELATIONS

Anthony Kenny (1969) noticed that numerous Thomist arguments come under the same formal scheme taken from the calculus of relations. E. Nieznański (1980) constructed the generally-logical theory of formal properties and extreme elements of binar relations for the use of the formalization of theodicy.

1.1.1. The application of the idea of linear orders

However, in the initial phase of the formalization of the proofs for the existence of God the idea of a chain predeminated indivisibly. J. Salamucha (1934), as well as J. Bocheński (1935), L. Koj (1954) and J. Bendiek (1956) treated the relationship of metaphysical movement as a relation arranging lineary a set of beings and assuming its finiteness, in the conclusion they were receiving a statement that there is the element first and minimal at the same time, i.e. *primum movens immobile*. These authors also thougt that they were formalizing the argument ex motu of St. Thomas Aquinas enclosed in *Summa contra Gentiles* I,13 and *Summa Theologica* I,q.2,a.3.

The idea of a chain comes back later again, first it was suggested by Peter Geach (1963) then in a certain version of a cosmological argument formalized by E. Nieznański (1992). This time however, not the chain of objects is a question, but a relation arranging lineary the family of a set of objects. By assuming the sign A_t to denote the set of all beings actual in the moment t, one designates the set of all „acual worlds", i.e. the family of sets $\{A_t\}_{t\in T}$ (where „T" means time continuum) and the relation arranging lineary this family of sets and then one assigns the conditions of unemptiness of the product of all actual worlds i.e. $\cap_{t\in T}A_t$.

1.1.2. Relations partially arranging all beings

Under the influence of criticism of the assumption about connectivity of the relation of movement as a thesis empirically false the criticism first carried out by F. Rivetti Barbò (1960) many philosophers engaged in the formalization of theodicy resigned from the idea of a chain in support of relations partially arranging the set of all real beings. And thus Korneliusz Policki (1975) presented for the first time a formalized argument ex motu using the lemma of Kuratowski-Zorn. In a similar way Zorn's lemma was employed by Reinhard Kleinknecht (1991) to the argument ex ratione causae efficientis.

The idea of a partital arrangement was also used by Kurt Gödel, where he employed the relation of inclusion in the family of all subsets of the set of real beings. Kurt Gödel's ontological proof for the existence of God (from 10th. Feb., 1970) in its philosophical contents approximates mostly the ontological argument of Leibniz from *Monadology* 41, 44 and 45, whereas in its formal layer it assimilates especially Ch. Hartshorne's (1941, 1961, 1962) formalized on the basis of modal logic S5 proof of St. Anselm for the existence of a maximal perfect being from Proslogion c. 2-3. In a simplified, not-modal version it was presented by Essler (1991).
There are assumed three axioms of positive features:

Ax1. $\forall F\ \forall G\ (F \subseteq G \wedge F\varepsilon Ps \rightarrow G\varepsilon Ps)$
„Oversets of positive classes are positive".

Ax2. $\forall F\ (-F\varepsilon Ps \rightleftarrows \neg\, F\varepsilon Ps)$
„Only a given class or its complement is positive".

Ax3. $\cap Ps\ \varepsilon\ Ps$
„The product of all positive classes is positive itself".

The conception of God is identified by Gödel with *summum bonum* according to the definition:

Df. $Gt = \cap Ps$, i.e.
$\forall x\ [x\varepsilon Gt \rightleftarrows \forall F\ (F\varepsilon Ps \rightarrow x\varepsilon F)]$

In other words axioms Ax1 and Ax3 state that the family of positive classes is a filter, while the axiom Ax2 additionally settles that this family is a maximal filter (ultrafilter), generated by *summum bonum* Gt. Three important theorems result from the mentioned axioms:

Th1. $\forall F\ [F\varepsilon Ps \rightarrow \neg\, (F = \emptyset)]$
„No positive class is empty".
The proof:

$F\varepsilon Ps, F = \emptyset \vdash \emptyset\varepsilon Ps, Ax1, F \subseteq U \vdash U\varepsilon Ps, Ax2 \vdash \neg(-U\varepsilon Ps) \vdash \neg\, (\emptyset\varepsilon Ps) \vdash$ contr.

Th2. $\neg\, (Gt = \emptyset$, because Df, Ax3 and Th1.
„*Summum bonum* exists'.

Th3. $\forall x\ \forall y\ (x\varepsilon Gt \rightarrow x = y)$
„At most one being is *summum bonum*".
The proof:

$x\varepsilon Gt, y\varepsilon Gt\ /\ \neg\, \{x\}\varepsilon Ps, Ax2 \vdash -\{x\}\varepsilon Ps, Df \vdash x\varepsilon - \{x\} \vdash \neg\, (x = x) \vdash$ contr. $/ \vdash$ $\{x\}\varepsilon Ps, \forall F\ (F\varepsilon Ps \rightarrow y\varepsilon F) \vdash y\varepsilon\{x\} \vdash x = y$

The theorems Th2 and Th3 state exactly that there is precisely one being which is summum bonum. However, it is obvious that only a formal principle, the idea with an ultrafilter is valuable in Gödel's proof, whereas the metaphysical contents are so extremaly poor in it that nothing interferes with giving in the above notation to the constant Ps the sense of a negative gualification and obtaining, as a result , summum malum – a one – element personification of evil.

1.1.3. Multiplicative quasihalfstructures

Still another set theory formulation besides the chain conception, partial arrangements and ultrafilters was the treatment of metaphysical relationships as the so called multiplicative quasihalfstructures, i.e. the relation R, which for each couple of elements x and y belonging to its field satisfies the condition: $x = y \vee xRy \vee yRx \vee \exists z\,(zRx \wedge zRy)$ The formalizations of F. Rivetti Barbò (1960, 1962, 1966, 1967), Ivo Thomas (1960) and W. K. Essler (1969) went exactly by this trail.

1.2. FORMALIZATIONS ON THE BASIS OF THE LOGIC OF PREDICATES

Fragments of theodicy formalized within the calculus of relations – it is to be emphasized – even if they avoided formal mistakes and made deduction – as to its logical form – verifiable, neglectes completely a pragmatic aspect shifting the whole weight of the implementation of its ontological assumptions to classical metaphysicians who do not occupy themselves with formalization. These, however, leave such expectations unnoticed refusing any value at all to formalizing endeavours, or only accusing them of a semantic deformation of metaphysical judgements. One should remember that the language of classical philosophy is a system of sings for objects and qualifications and not for sets and elements. So even if we successfully use set theory constructs to metatheoretical descriptions of semantic models of formalized theodicy, the very theory however should probably be created on the basis of the logic of predicates.

1.2.1. Modal formulations

In S5-modal calculus of predicates of an upper order are done (specially numerous) formalizations of the, so called, ontological argument and among them the most famous (mentioned above) formalizations of Ch. Harsthorne and K. Gödel. In as much as Hartshorne's calculi are entangled in the equivocation of logical and metaphysical modality, in Gödel's proof – as W. Essler showed (1991) – logical modalities occur inessentially. The often raised objection of paralogism[3] in relation to *tertia via* of St. Thomas Aquinas, particulary on the account of the thesis *Quod possibile est non esse, quandogue non est,* resembling an not-tautological formula in ordinary model systems $<>p \rightarrow p$, can be disproved by the application of a special extention of the system S5, in which the equivalence would be obligatory: $\Box p \rightleftarrows \langle\rangle p \rightleftarrows p$, i.e. when modalities are used inessentially and the modal calculus is generated by the semantic models of Kripke with identity as the equivalence relation of the accessibility od possible worlds.

In S5-modal logic of predicates of the first order Anthony Kenny (1969) and E. Nieznański (1991) formalized an argument *ex possibili et*

[3] See Kłósak, W poszukiwaniu Pierwszej Przyczyny, (Towards the First Cause), II, Warszawa 1957, p. 124.

necessario and also E. Nieznański (1991) formalized an argument *ex ratione sufficienti.*

1.2.2. Classical formulations

As it seems, the formalizations of theodicy carried out in the classical logic of predicates are least exposed to making semantic mistakes, since one avoids in them in a simple way – owing to the very nature of a language – confusing a quality with multiplicity and a logical modality with metaphysical one.

The argument ex motu of the existence of God unfolded by Leibniz in *Demonstratio Existentiae Dei ad Mathematicam Certitudinem Exacta* was formalized twice by Krystyna Błachowicz (1982, 1992). These formalizations are based, however, on a special postulate of Leibniz, which in K. Błachowicz's notation is in the following shape:

$\exists y\ \forall x\ (xPy \rightleftharpoons \varnothing(x))$

(where 'xPy' means 'x is a part of y') and which immediatelly leads in an open way to Russell's antinomy (when e.g. $\varnothing(x)$ is a formula $\neg xPx$).

Within the classical logic of predicates of the first order also formalized: Józef M. Bocheński (1989) – all *quinque viae* of St. Thomas, E. Nieznański (1977, 1979, 1980, 1981, 1982b) – classical arguments ex ratione sufficienti, E. Nieznański (1980, 1982a, 1984) – argument *ex ratione cause efficientis*, and H. Gentahaler and P. Simons (1987) – the cosmological argument of B. Bolzano.

2. Argument ex ratione sufficienti

The consideration of the existence and nature of the absolute must originate in some model of reality. Let us return, then, to $\{A_t\}_{t\in T}$ – the family of all „actual worlds".

2.1. THE NOTION OF A BEING

At the beginning, we assume the principle of the conservation of existence establishing that no actual world is an empty set, which in the language of the creating theory – in which individual variables x, y, z,... represent any uncontradictory individual objects, not only beings – we write in the axiom[4] „Something always exists":

A1. $\forall t\ \exists x\ Axt$ (where „Axt" means as much as „x is present at the moment t"). The object, which is at any time actual, we call a real being (we read „Bx": „x is a real being") according to the definition:

df.B: $Bx \rightleftharpoons \exists t\ Axt$.

[4] The axioms used in deduction are designated by a capital letter „A": A1,A2,...,A10; whereas the axioms which are not used as premises of any proof, but are added because of purely pragmatic reasons, are denoted by a small letter „a": a1, a2, a3, a4.

2.2. THE NOTION OF THE REASON OF EXISTENCE

The transmission of existence from some beings of the actual world into other beings of next worlds can be accomplished only through becoming or lasting, because either some things become other through a connection, disintegration, change or they last as identical in time, as unchangeable. The most important existential connection is the s.c. reason of existence. Rxy, i.e. x is a reason for the existence of y if and only if y cannot exist without x, when the existence (and the essence) of x is a necessary condition of existence (and essence) of y. And although the implication

a1. $Rxy \rightleftharpoons \neg <> (By \wedge \neg Bx)$

is universally important for all objects x and y, a reverse implication, however, is not always satisfied, because the metaphysical modality „y cannot exist without x" is a connection (richer) stronger than the connection „$\neg <> (By \wedge \neg Bx)$". The metaphysical sense of the notion „impossibility of existence without", i.e. „the reason of existence" is not suitable for logical analysis by means of simpler notion and because of this we treat them as an original notion. First we state that nothing can exist without itself, so:

A2. $\forall x\, Rxx$,

which means reflexivity of the relation R, and we also assent – secondly – to the impossibility of existence of y without x, while y cannot exist without z, and z without x, so:

a2. $\forall x\, \forall y\, \forall z\, (Rxz \wedge Rzy \rightarrow Rxy)$

which means then transitiveness of the relation R.

Very likely, the most important feature of the relation of the reason of existence is the fact that existence is hereditary because of the converse of this relation, and nonexistence – because of the very relation. Always, when y cannot exist without x, the existence of y guarentees the existence of x, and the nonexistence of x states also the nonexistence of y, so:

A3. $\forall x\, \forall y\, (Rxy \wedge By \rightarrow Bx)$.

The reason of existence is by no means a simple connection, on the contrary, it is a sum of many existential relations. First, the identity relation enters into its composition, which results directly from the axiom A2:

T1. $\forall x\, \forall y\, (x = y \rightarrow Rxy)$.

2.3. BECOMING AND CHANGE

Let us assume two abbreviations: „Sxy" for the predicate „x becomes y" and „Zx" for „x is changeable being". Becoming is a process, which happens only in beings: one being becomes to other and both are real beings, so the relation of becoming and its converse enter into the composition of the relation of the reason of existence.

A4. $\forall x\, \forall y\, [(Sxy \vee Syx) \rightarrow Rxy]$.

Becoming of a being is also reverse to its identity at time, hence the axiom is obligatory:

A5. $\forall x \forall y [Sxy \rightarrow \forall t (Axt \rightarrow \neg Ayt)]$,
from which the thesis of irreflexivity of the relation of becoming of a being results. It means that nothing becomes itself:
T2. $\forall x \forall y (Sxy \wedge By \rightarrow \neg x = y)$, since:
$Sxy, By, x=y, A5 \vdash \forall t (Axt \rightarrow \neg Axt), (p \rightarrow \neg p) \rightarrow \neg p \vdash \forall t \neg Axt \vdash \vdash \neg \exists t\, Axt$, df. $B \vdash \exists t\, Ayt \vdash \exists t\, Axt \vdash$ contradiction
It is also easy to prove that beings which are becoming are not constant entities:
T3. $\forall x \forall y (Sxy \wedge By \rightarrow \exists t \neg Axt \wedge \exists t \neg Ayt)$, since:
$Sxy, By \vdash Rxy, A3 \vdash Bx$, df. $B \vdash \exists t\, Axt, \exists t\, Ayt, A5 \vdash \forall t (Axt \rightarrow \neg Ayt), \forall t (Ayt \rightarrow \neg Axt) \vdash \exists t \neg Axt \wedge \exists t \neg Ayt$
Therefore each limit of the relation of becoming is called a changeable being according to the definition:
dt. Z: $Zx \rightleftharpoons \exists y [By \wedge (Sxy \vee Syx)]$
When x is a constant being, i.e. when $\forall t\, Axt$, x is always one and the same x, so it is an unchangeable being:
T4. $\forall x (\forall t\, Axt \rightarrow \neg Zx)$, since:
$\forall t\, Axt, Zx$, df. $Z \vdash Ba, Sxa \vee Sax, A5 \vdash \forall t (Axt \rightarrow \neg Aat) \vdash \forall t\, Axt \rightarrow \rightarrow \forall t \neg Aat \vdash \forall t \neg Aat, \exists t\, Aat \vdash$ contradiction.
Whereas a reverse implication is not valid, because one cannot a priori exclude the existence of beings unchangeable in a time, arising ex nihilo or undergoing annihilation. Instead, it is obvious that there are changeable beings:
a3. $\exists x\, Zx$.
One should distinguish contingency from changeability. The being x is contingent when it has at least one necessary condition of its existence ab alio, so when $\exists z (\neg z = x \wedge Rzx)$. It is clear, at the same time, that every changeable being is as well contingent, though a reverse connection does not have to happen:
T5. $\forall x [Zx \rightarrow \exists z (\neg z = x \wedge Rzx)]$, since:
Zx, df.$Z \vdash Ba, Sxa \vee Sax, A4 \vdash Rxa, Rax, A3 \vdash Bx, T2 \vdash \neg a = x \vdash \neg a = x \wedge \wedge Rax \vdash \exists z (\neg z = x \wedge Rzx)$.

2.4. THE NOTION AND PRINCIPLE OF A SUFFICIENT REASON OF BEING

One special case of the reason of existence is sufficient reason of a being. The notion of „a sufficient reason, which does not need any other reason" was introduced by G.W. Leibniz, who cut the left domain of the relation of reason to the set of beings which do not have the reason ab alio. By assuming the abbreviation „Dxy" for the predicate „x is a sufficient reason of existence of y" we define:
df. D: $Dxy \rightleftharpoons Rxy \wedge \neg \exists z (\neg z = x \wedge Rzx)$.
Classical philosophy coped in three different though equivallent ways with the main metaphysical problem: why rather is there something than nothing? St. Thomas made use of the rule: non est procedere in infinitum. Namely, if we take into account conected underrelations of the relation R and we will call the maximal of them series (even when they make cycles), a question arises, whether a regress in infinitum is possible in them

what it means. J. Bendiek (1956) accurately noticed that: „das Verbot eines regressus in infinitum nicht auf das Unendliche, sondern auf die Anfanglosigkeit bezieht" (p.10). Thus St. Thomas' impossibility of regress in infinitum points out not its finiteness (as it was assumed in the first formalizations of theodicy), but possessing a beginning. Conditioning reasons by reasons cannot be beginningless, even though it was infinite. St. Thomas' justification of the impossibility of beginningless series by the fact that the nonexistense of a beginning would involve the nonexistence of the whole series, J. Bendiek estimated as petitio principii, a vicious circle: the beginning of a series is, because a beginningless series would be impossible, while a series cannot be beginningless, since it would not exist at all without a beginnig. However one could not, in our opinion, impute to such a mature thinker as St. Thomas as primitive mistake as all that vicious circle. If St. Thomas in his ways to the absolute was reality determining something more than only the obvious equivalence of the impossibility of beginninglessness with the necessity of the beginning, he could only yield to a difficult to discover equivocation of the word „exist": once in a sense of the existential quantifier and for the second time, as a predicate. For when speaking about the beginninglessness of a series we have in mind the fact that $\neg \exists x \forall z (Rzx \rightarrow z = x)$, when we, however, further say that if there is not the beginning a, so there are not next elements of the series, either since $\forall y (Ray \wedge \neg Ba \rightarrow \neg By)$ – in accordance with the axion A3 – so there is made an equivocation because of the ambiguity of „existence", and moreover – in forbidden way – the notation „a" is attributed to a contradictory thing. In this situation we understandt finally that St. Thomas adopted axiomatically the theorem: $\forall y \{By \rightarrow \exists x [Rxy \wedge \wedge \forall z (Rzx \rightarrow z = x)\}$, though the added commentary for the reinforcement of the readiness to accept this axiom is not efficient and misses the aim.

The other approach to the attempt of the final solution of the problem of existence in the Thomist axiom of the impossibility of the existence of series consisiting only of contingent beings: $\neg \forall y [By \rightarrow \exists x (\neg x = y \wedge Rxy)]$. However the commentaries appearing sometimes which are to confirm the conviction about the rightness of this axiom because of the fact that series of contingent beings (as it is claimed then) are in themselves contingent beings – make fallatium compositionis[5].

A particularly significant way of solving the problem of a genesis of existence is the application of Leibniz's principle of a sufficient reason:

A6. $\forall y (By \rightarrow \exists x Dxy)$

Contingency, i.e. conditioning of beings in existence ab alio, even though it reached infinity, does not account for the fact of existence; the beginninglessness of the series of reasons does not give an explantion but only suspends it and pushes into infinity and begininnglessness, i.e. to nowhere.

We can notice straight away that there are two, at the utmost, possible attitudes towards the nature of existence: either universal variabilism which

[5] See e.g. K. Kłósak, W poszukiwaniu... (Towards the First Cause), p. 118.

excludes the principle of a sufficient reason is obligatory or – on the contrary - there is valid the principle of a sufficient reason, which eliminates the possibility of universal variabilism:

T6. $\forall y\,(By \rightarrow \exists x\, Dxy) \rightarrow \neg\, \forall x\,(Bx \rightarrow Zx)$, since:
$\forall y\,(By \rightarrow \exists x\, Dxy)$, $\forall x\,(Bx \rightarrow Zx)$, A1, df. B $\vdash \exists x\, Dxa \vdash Dba$, df. D$\vdash$ $\vdash Rba$, $\forall z\,(Rzb \rightarrow z=b)$, A3 $\vdash Bb \vdash Zb$, df. Z $\vdash Scb \vee Sbc$, T2, A4 $\vdash \neg c=b$, $Rcb \vdash c=b \vdash$ contradiction.

2.5. THE NOTION OF THE ABSOLUTE

Let „αx" be the abbreviation for the predicate „x is the absolute".

df.α: $\alpha x \rightleftarrows Bx \wedge \forall z\,(Rzx \rightleftarrows z=x)$.

The absolute, therefore, is called the being which is itself the only and exclusive reason of its existence. Now we can prove the existence of the absolute:

T7. $\exists x\, \alpha x$, since:
A1, df. B $\vdash \exists x\, Bx \vdash Ba$, A6 $\vdash \exists x\, Dxa \vdash Dba$, df. D $\vdash Rba$, $\neg \exists z\,(\neg z=b \wedge Rzb) \vdash \forall z\,(Rzb \rightarrow z=b)$, A3, T1 $\vdash Bb$, $\forall z\,(Rzb \rightleftarrows z=b)$, df. $\alpha \vdash \alpha b \vdash \exists x\, \alpha x$.
The thesis about uniqueness of the absolute: $\forall x\, \forall y\,(\alpha x \wedge \alpha y \rightarrow x=y)$ is not, however, easy to prove or reject in the Thomism, which – although it is philosophy – nevertheless knows the theology De Trinitate. On the basis of the definition of the absolute it is only obvious that $\forall x\, \forall y\,(\alpha x \wedge \alpha y \wedge Rxy \rightarrow x=y)$, which means that if there were more absolutes than one they could not be in any existential connections with one another.

The existence and uniqueness of the absolute are, however, apparently protected by every possible materialism. It assumes that matter is the absolute, matter which conditions the existence of every being and, at the same time, is the being, which all the necessary conditions to its existence has only in itself: $\exists_1 x\,[\forall y\, Rxy \wedge \forall z\,(Rzx \rightarrow z=x)]$, which results directly from the – stronger than the principle of a sufficient reason – postulate of the final reason: $\exists x\,[\forall y\, Rxy \wedge \forall z\,(Rzx \rightarrow z=x)]$, on which materialism bases unconsciously. In this way, using an apparent name (onomatoid) „matter" materialists – due to the confusion of the category of existence – find the final justification of the fact of existence, the final reason of a being in the abstract idea of matter undergone ad hoc reification.

2.6. INTRINSICNESS OF THE ABSOLUTE

Since the essence and existence of a being are completely fixed by the factors determinating *a se* or *ab alio*, then the being which would comprise all the reasons of its existence in itself and only in itself, would not undergo any determinations from the aoutside and under the influence of its own reasons – according to the theorem T4 – it would not undergo any changes, which just means that in its nature it would be a constant being:

A7. $\forall t\, Axt \rightleftarrows Bx \wedge \forall z\,(Rzx \rightleftarrows z=x)$.

In this way we come to the conclusion that the absolute and a constant being are one and the same being:

T8. $\forall x\,(\alpha x \rightleftarrows \forall t\, Axt)$, since df. α and A7.
From the theorems T4 and T8 it is also results in a simple way that the absolute is an unchangeable being:

T9. $\forall x\,(\alpha x \rightarrow \neg Zx)$.
And since we generally acknowledge that every material being (every physical body) is changeable being:

A8. $\forall x\,(Mx \rightarrow Zx)$,
where „Mx" is the abbreviation of the predicate „x is a material being", we must also assume that the absolute in an immaterial being:

T10. $\forall x\,(\alpha x \rightarrow \neg Mx)$, since T9 and A8.

Let us complete axiomatically our theory by the notion of a proper part as the next existential connection which makes up the relation of the reason of a being. Let the notation „$x<y$" be the abbreviation of the predicate „x is the part of y":

A9. $\forall x\,\forall y\,(x<y \rightarrow \neg x=y)$,

A10. $\forall x\,\forall y\,[(x< \vee y<x) \rightarrow Rxy]$.

It is also obvious that the connection of a part to a whole is a transitive relation (although it is not a chain):

a4. $\forall x\,\forall y\,\forall z\,(x<y \wedge y<z \rightarrow x<z)$.

On the basis of the theory of a part we can now introduce the notion of a simple being („Px" means „x is a simple being") and complete being („Ux" – „x is a complete being"):

df.P: $Px \rightleftarrows Bx \wedge \neg\, \exists y\; y<x$,

df.U: $Ux \rightleftarrows Bx \wedge \neg\, \exists y\; x<y$.

Hence we receive two succeeding conclusions: that the absolute is a simple being and that it is a complete being:

T11. $\forall x\,(\alpha x \rightarrow Px)$, since:
$\alpha x, \neg Px$, df. α, df. $P \vdash Bx$, $\forall z\,(Rzx \rightleftarrows z=x)$, $\exists y\; y<x \vdash a<x$, A10, A9 $\vdash$ $a=x, \neg a=x \vdash$ contradiction.

T12. $\forall x\,(\alpha x \rightarrow Ux)$, since:
$\alpha x, \neg Ux$, df. α, df. $U \vdash Bx$, $\forall z\,(Rzx \rightleftarrows z=x)$, $\exists\, y\; x<y \vdash x<a$, A10, A9 $\vdash$ $\vdash Rax \vdash a=x, \neg a=x \vdash$ contradiction.

In this way we come to the conclusion that there exists the absolute which is a simple, complete, constant, unchangeable, necessary, autonomous and immaterial being.

2.7. OMNIPOTENCE OF THE ABSOLUTE

Since it is even impossible to conceive such a situation, in which there exist absolute beings unrelated to one another in any existential way, we could assume that $\forall x\,\forall y\,(\alpha x \wedge \alpha y \rightarrow Rxy)$, which – considerating df.α – would lead to the acceptance of the uniqueness of the absolute and the conclusion that $\forall x\,[\alpha x \rightarrow \forall y\,(By \rightarrow Rxy)]$ i.e. to thetheorem that the absolute is also the first being. Adding the conception of the causative reason (Cxy), by means of axioms: $\forall x\,\forall y\,(Cxy \rightarrow \neg x=y)$, $\forall x\,\forall y\,\forall z\,(Cxy \wedge Cyz \rightarrow Cxz)$, $\forall x\,\forall y\,(Cxy \rightarrow Rxy)$ and $\forall y\,(By \wedge \exists\, t \dashv Ayt \rightarrow \exists\, x\, Cxy)$, we would have to notice that the immaterial absolute cannot exert physical forcel on material beings, hence its impact on the real world is of a special nature. Now that it has been credited an immaterial character we could follow further the formalizations of Kurt Christian (1957) and Paul Weingartner (1974, 1979), assuming in the first place that the absolute has a will: $\forall x\,(\alpha x \rightarrow \exists p\ WLxp)$ – where „p" is a sentence variable, and the function „WLxp" means the same as „x wants p to be" – and, secondly, that the absolute is omnipotent: $\forall x\,[\alpha x \rightarrow \forall p\,(WLxp \rightarrow p)]$, i.e. that everything which it wants – is. Thereby philosophical ways would adjoin theological ways: because it turns out that the absolute – contrary to the rest of existence which is becoming – „is the one who is" and also is the one „to whom the heaven and earth are obedient".

Bibliography

Bendiek, J. (1956), „Zur logischen Struktur der Gottesbeweise", Franciskanische Studien 38: 1-38, 296-321.
Błachowicz, K. (1982), „Leibniz's 'demonstratio existentiae Dei ad mathematicam certitudinem exacta'. A Logical Analysis", [in:] J. Kopania (Ed.), Studies in Logic, Grammar and Rethoric II, Białystok: Papers of Warsaw University, 43-55.
Błachowicz, K. (1992), Leibniz, wczesne pojęcie substancji, (Leibniz, An Early Conception of Substance) Białystok: Idea.
Bocheński, J. I. M. (1935), „Compte rendu nr 936", Bulletin Thomiste 12: 601-603.
Bocheński, J. I. M. (1989), „Die fünf Wege", Freiburger Zeitschrift für Philosophie und Theologie 36: 235-265.
Christian, C. (1957), „Eine Note zum Gottesbegriff", Religion-Wissenschaft-Kultur 8: 227-228.
Essler, W. K. (1969), Einführung in die Logik, Stuttgart.
Essler, W. K. (1991), „Gödels Beweis", [w:] F. Ricken (ed.), Klassische Gottesbeweise in der Sicht der gegenwärtin Logik und Wissenschaftstgenheorie, Stuttgart: Kohlhammer, 140-152.
Ganthaler, H., Simons, P. (1987), „Bernard Bolzanos kosmologischer Gottesbeweis", Philosophia Naturalis 24: 469-475.
Geach, P.T. (1963), „Aquinas", [w:] G.E.M. Anscombe, P. T. Geach Three Philosophers, Oxford: Basil Blackwell & Mott Ltd.
Hartshorne, C. (1941), Man's Vision of God and the Logic of Theism, New York.
Hartshorne, C. (1961), „The Logic of the Ontological Argument", Journal of Philosophy 58: 471-473.
Hartshorne, C. (1962), The Logic of Perfection, La Salle.
Kenny, A. (1969), The Five Ways. St. Thomas Aquinas' Proofs of God's Existence, London.
Kleinknecht, R. (1991), „Auswahlaxiom und causa prima", [in:] F. Ricken (ed.), Klassische Gottesbeweise in der Sicht der gegenwärtigen Logik und Wissenschaftstaheorie, Stuttgart: Kohlhammer, 111-123.

Koj, L. (1954), Poglądy ks. Salamuchy na uściślenie filozofii, (Rev. Salamucha's Views on the Specification of Philosophy) typescript of M.A. thesis, Catholic University of Lublin.
Nieznański, E. (1977), „Logistyczny przyczynek do analizy pojęcia 'istoty, do której należy istnienie'" („A Logical Contribution, to the Analysis of the Notion of 'Essence to which Existence Belongs'"), Studia Philosophiae Christianae 13: 139-156.
Nieznański, E. (1979), „Formalizacja tomistycznych podstaw dowodu na istnienie koniecznego bytu pierwszego" („A Formalization of Thomistic Foundations of a Proof for the Existence of the Necessary First Being"), Studia Philosophiae Christianae 15: 163-180.
Nieznański, E. (1980), „Formalizacyjne próby ustalenia logiko-formalnych podstaw stwierdzania pierwszych elementów rozważanych w tomistycznej teodycei", („Formalized Attempts to Set Forma-logic Bases to State First Elements of Relations Considered in Thomistic Theodicy") [in:] E. Nieznański (ed.). W kierunku formalizacji tomistycznej teodycei, Miscellanea Logica, vol. 1, Warszawa: Academy of Catholic Theology, 7-194.
Nieznański, E. (1981), „Ein formalisierter Beweis für die Existenz eines ersten notwendigen Seienden", [in:] E. Morscher, O. Neumaier, G. Zecha (eds.) Philosophie als Wissneschaft, Bad Reichenhall: Commes, 379-389.
Nieznański, E. (1982a), „W poszukiwaniu pierwszej przyczyny z pomoca logiki formalnej" („Towards the First Cause with Help of Formal Logic"), Analecta Cracoviensia 14: 51-60.
Nieznański, E. (1982b), „Sformalizowany dowód istnienia koniecznego bytu pierwszego" („Formalized Proof for the Existence of the Necessary First Being"), [in:] B. Bejze (Ed.), W kierunku Boga, Warszawa: Academy of Catholic Theology, 405-409.
Nieznański, E. (1984), „Formalisierung des Gottesbeweises 'ex ratione causae efficientis' , Salzburger Jahrbuch für Philosophie 27/29: 79-84.
Nieznański, E. (1991), „Gründe, zureichende Gründe und Gottesbeweise 'ex contingentia mundi'", [in:] F. Ricken (ed.), Klassische Gottesbeweise in der Sicht der gegenwärtigen Logik und Wissenschaftstheorie, Stuttgart: Kohlhammer, 124-139.
Nieznański, E. (1992), „Mit o mitach" (A „Myth about Myths"), [in:] B., Bejze (Ed.), Studia z filozofii Boga, O filozoficznym poznaniu Boga dziś, Warszawa: Academy of Catholic Theology, 250-261.
Policki, K. (1975), „W sprawie formalizacji dowodu 'ex motu' na istnienie Boga" („On the Formalization of the Proof 'ex motu' for the Existence of God"), Roczniki Filozoficzne 23: 19-30.
Quine, W.V.O. (1951), Mathematical Logic, Cambridge, translated into Polish by L. Koj.
Rivetti-Barbò, F. (1960), „La struttura logica della prima via per provare l'esistenza di Dio. Applicazioni di logica simbolica e nessi di contenuti", Rivista di Filosofia Neoscolastica 52: 241-320.

Rivetti-Barbò, F. (1962), „Ancora sulla prima via per provare l'esistenza di Dio", Rivista di Filosofia Neoscolastica 54: 596-616.
Rivetti-Barbò, F. (1966), Logica simbolica e prove dell' esistenza di Dio. Rilievi metodologici, [in:] F. Selvaggi (ed.), De Deo in philosophia S. Thomae et in hodierna philosophia, vol.2, Roma, 359-363.
Rivetti-Barbò, F. (1967), „La formalizzazione e la strutture propria delle 'vie' diascesa a Dio. Considerazioni metodologiche", Rivista di Filosofia Neoscolastica 59: 161-177.
Salamucha, J. (1934), „Dowód na istnienie Boga. Analiza logiczna argumentacji św. Tomasza z Akwinu" („The Proof for the Existence of God. Logical Analysis of St. Thomas' Argument"), Collectanea Theologica 15: 53-92.
Thomas, I. (1960), Rewiew of the Rivetti (1960), Journal of Symbolic Logic 25: 347-348.
Weingartner, P. (1974), „Religiöser Fatalismus und das Problem des Übels, [in:] E. Weinziel (ed.) Der Modernismus. Beiträge zu seiner Erfoschung, Festschrift für Th. Michels Graz: Styria, 369-409.
Weingartner, P. (1979), „Kommentar zu Thomas von Aquin, Summa Theologica I, 19, 1, über den Willen Gottes", [in:] E. Weinzierl (ed.), Kirche und Gesellschaft, Wien: Geyer, 167-187.