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# Problem solving on absolute value – relevance of visualisation by means of TI-Nspire graphic calculator

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## Introduction

Mathematics is a subject, which is often regarded as the one which students have the biggest difficulty with. There are a number of reasons for that, but decreasing level of mathematical abilities is related to the fact, that traditional chalk and blackboard together with usually applied „giving" teaching style can be insufficient in teaching-learning process for a young man growing up in the environment of modern technology. Visualization is the most appropriate method to be applied in the contemporary World, having direct impact on all of students senses. Information being analyzed and presented in graphical form make up an extraordinarily relevant component enabling students to notice some facts, correlations and relationships and hence help him to solve the given problem. Thanks to visualization data and relationships between them can be presented in a way that reliably reflects the reality. It is usually impossible with regards of time to make a dozen or even several dozen of drawings illustrating the problem on a blackboard or in the notebooks. Besides, our drawings are often inexact and do not meet the generality conditions which can cause incorrect interpretation and finally mistaken problem solution. Visualization comprising of proper coloured objects and symbols as well as current values displays enables to underline and notice the most relevant properties of defined object.

## 1. Relevance of visualization in mathematical problem solving process

The article will cover some of those mathematical problem types where graphical analysis of solution plays a relevant role. Since problem solving favors development of basic mathematical activity and accomplishment of teaching objectives, we should be concerned to select them properly. Problems can be classified according to different criteria. A teacher should be aware of a wide variety of them when selecting them to be solved by students. Through problem solving one can introduce and form new definitions, discover and justify theorems, take advantage of gathered knowledge, verify data and complete them. While solving a problem student becomes a creator and a discoverer. Problem solving is what releases student's creativity in particular.

A teacher should organize teaching process in a way that concerns student's active attitude and enhance his orientation to action. A method based on this assumption is called by Z. Krygowska „functional method". Didactic measures applied in teaching -learning process play a relevant role in the above mentioned method.

Didactic measures facilitate activating methods use, help to master lasting knowledge and to take advantage of them in practice. Technology and latest information technology development offer more and more powerful didactic measures such as a computer, subject software, graphical calculator, multimedia projector or interactive table. We should remember that for young people growing up in a culture of pictures traditional chalk and blackboard and usually „giving" teaching style can turn out to be insufficient in teaching-learning process. In the article we will draw particular attention to the one of quite popular didactic measure, namely calculator. It is of quite common use in everyday life. Its application in mathematics raises still doubts, though the growing number of device adherents believes, that benefits of use it for a student and the whole mathematics education process can be fairly significant.

Currently available graphic calculators are truly advanced devices, so called „calculating and analytic machines". Product of Texas Instruments Nspire CAS CX is worth mentioning as it offers huge calculating possibilities and facilitates presentation of results in algebraic, graphical, geometrical, numerical and textual form. There can be analyzed up to four presentations on the colorful screen at the same time. It is a calculator of CAS (Computer Algebra System) type, so it is possible to make symbolic calculations as well.

The attention should be drawn to the fact, that a student with use of calculator can complete the deficiencies in making elementary operations, certainly only then, when they are not a lesson objective. In 2000 Bernhard Kutzler created a theory, which determined the role and significance of graphic calculator. He attributed the role of „mathematical scaffold" to it. In fact, while building a house each next floor can be build not before the previous one was successfully erected. While learning mathematics a student can master the following contents just after the previous contents had been mastered and assimilated. However, it is quite common that there is too few classes hours to form a solid foundations for further education. Thus construction of „mathematical home" by each student seems to be incomplete. Here the problematic question occurs: how to build the next floors of knowledge on the incomplete foundations i.e. „mathematical home"? Kutzler claimed that we can achieve it if calculator or computer become the scaffold enabling a student to reach the next level. Calculator will help to erect „mathematical home" within much shorter period, but it will require a student to master new abilities. Teachers are going to face new challenges as they are going to be responsible for proper and wise education with use of this measure. It obviously requires to modify objectives, curricula and teaching methods. When curricula is being „slimmed" constantly and requirements are being decreased, teaching process should be supported with new technology tools such as computer or calculator. The latter, because of its small size and low price can become a real student's friend. However, to be able to use calculator during the lessons one should get acquainted with technical instruction, basic functions, facilities and define its usefulness to accomplish specific teaching objectives. Besides, a teacher has to be able to include those lesson fragments, which are to be supported with graphical calculator use in teaching process and to establish appropriate forms of work with these medium with regards to possibility and situation. However, a calculator is still being used occasionally to support didactic process at school. Mathematics that can be presented to a student with use of calculator is though living mathematics being created before his eyes. Student can „touch" the problems that have been too hard or too complicated to understand to him. Use of computer in education supports traditional contents and forms of message, delivers new methods and measures which enable to perceive the way to communicate knowledge in a different way than before. Calculator can also be used to make complicated calculations, to confirm achieved results, to observe relationships, to make hypotheses, to draw function graphs, but also it helps to develop new abilities e.g. ability to experiment and to notice a problematic situation.

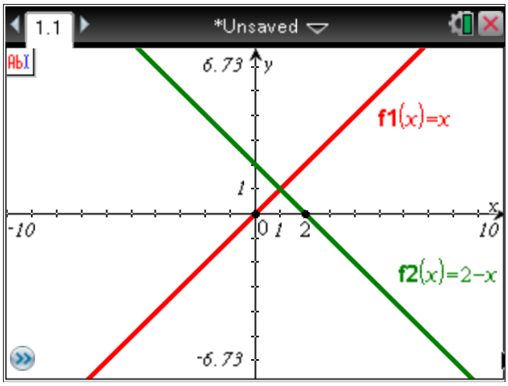
Visualization of geometrical, algebraic and other issues should help to establish problematic situation above all, should motivate a student to solve problems, to ask questions, to search for different solutions to the defined problem and last but not least to develop and shape student's imagination. Visualization of mathematical problems is also to speed up adoption of knowledge by a student.

## 2. Use of TI-Nspire calculator to solve problems in practice.

Below we will present different ways to use graphical calculator to solve problems on absolute value.

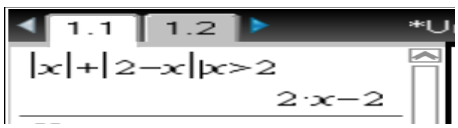
*Exercise 1.* Simplify the expression $|x|+|2-x|$ for $x>2$

Initially we should define values that functions $f_1(x)=x$ i $f_2(x)=2-x$ will take for $x>2$. We will take advantage of Pic. 1, which was drown with use of graphic calculator TI-Nspire application.




**Pic. 1.** $f_1(x) = x$ i $f_2(x) = 2 - x$ **functions graph**

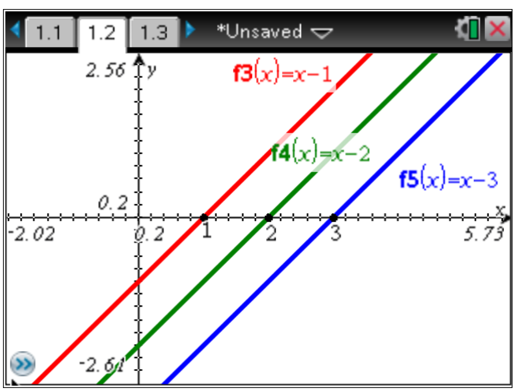
It is easy to note, that for $x > 2$, $|x| = x$ and $|2 - x| = x - 2$, so for $x > 2$ we will achieve $|x| + |2 - x| = x + x - 2 = 2x - 2$. Obviously we can use a ready to use option installed in calculator application and find the solution of our problem immediately (Pic. 2).




**Pic. 2. Simplification of expression** $|x| + |2 - x|$ **for** $x > 2$

*Exercise 2*. Solve the equation $|x-1| - 2|x-2| + 3|x-3| = 4$.

Using a calculator we are drawing graphs of the following functions $f_3(x) = x - 1$, $f_4(x) = x - 2$ and $f_5(x) = x - 3$



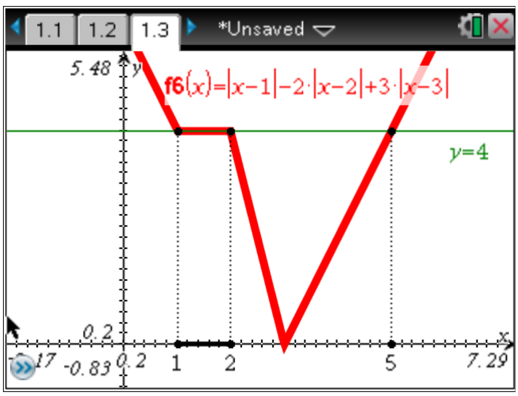

**Pic. 3. Graphs of functions** $f_3(x) = x - 1$, $f_4(x) = x - 2$, $f_5(x) = x - 3$

Axis was divided with points 1,2 and 3 in three subsets within which we will consider our equation.

1. If $x \in (-\infty, 1\rangle$, then $|x-1| = -x+1,\ |x-2| = -x+2,\ |x-3| = -x+3$ and the equation takes a form $-x+1+2x-4-3x+9=4$, hence $x=1$.
2. If $x \in (1,2\rangle$, then $|x-1| = x-1,\ |x-2| = -x+2,\ |x-3| = -x+3$ and the equation takes the form $x-1+2x-4-3x+9=4$ that is $0=0$, so here we have every number form the range of $(1,2\rangle$ as a solution.
3. If $x \in (2,3\rangle$, then $|x-1| = x-1,\ |x-2| = x-2,\ |x-3| = -x+3$ and the equation takes the form $x-1-2x+4-3x+9=4$, and hence $x=2$, provided $2 \notin (2,3\rangle$.
4. If $x \in (3,+\infty)$, then $|x-1| = x-1,\ |x-2| = x-2,\ |x-3| = x-3$ and the equation takes the form $x-1-2x+4+3x-9=4$, hence $x=5$.

Considering points 1-4 we achieve $|x-1|-2|x-2|+3|x-3|=4$, if $x \in \langle 1,2\rangle \vee x=5$.

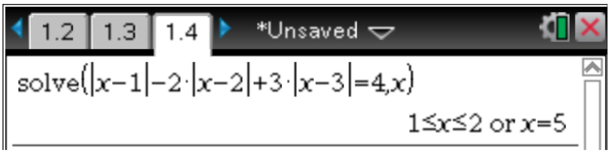
We can find a solution of our equation by means of calculator. Now we are going to make graphs of functions: $f_6(x) = |x-1|-2|x-2|+3|x-3|$ and $y=4$ in the common coordinate system. Abscissas of common points joining both functions graphs will be the solution of the equation (Pic. 4).




**Pic. 4. Graph of functions** $f_6(x) = |x-1|-2|x-2|+3|x-3|$ **and** $y=4$

It is easy to note, that $|x-1|-2|x-2|+3|x-3|=4$ then and only then, when $x \in \langle 1,2\rangle \vee x=5$. In this case mathematics will not be associated by students with boring and difficult calculations and transformations.

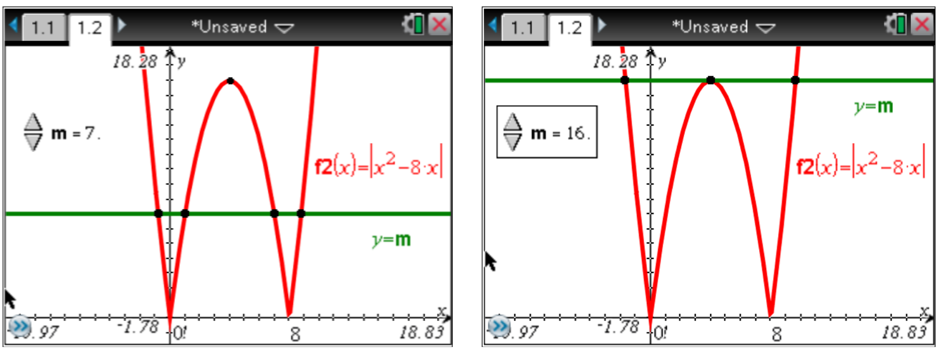
Raised problem visualization can be the pretext to start discussion on solutions of other equations regarding for example value of right side of starting equation. There are a number of possibilities to prolong or to modify the problem. We should remember that we can change graph's parameters and observe changes in the real time using „grab and move" function. It would be rather troublesome to make such modifications and changes in a student notebook. It is worth mentioning that we can solve our equation with help of the calculator and the option embedded in it, which takes less time but it will not favor to shape and develop elementary mathematical activities amongst students. However, if the lesson's objective is to achieve quick solution we can use „solve" function (Pic. 5).




**Pic. 5. Solution of the equation** $|x-1|-2|x-2|+3|x-3|=4$

*Excercise 3.* For which value of $m$ parameter the equation $|x^2-8x|=m$ will achieve exactly three roots?

We will draw in the common coordinate system graphs of functions: $f_2(x)=x^2-8x$ and $y=m$. Using „slide" option, in case of the latter function, we can in a simple way give numeral value to $m$ parameter and observe changes on the graph in the real time. It will enable us to easily find the answer to the raised question.

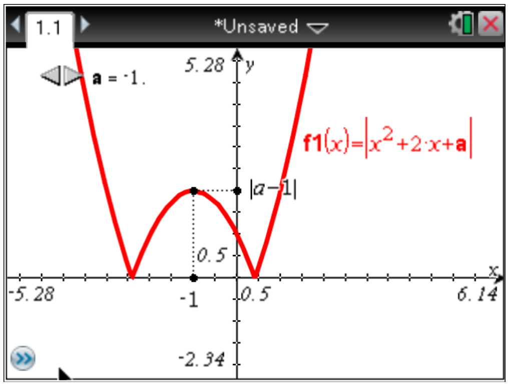



**Pic. 6a, 6b. Mutual position of** $f_2(x)=x^2-8x$ **and** $y=m$ **function graphs**

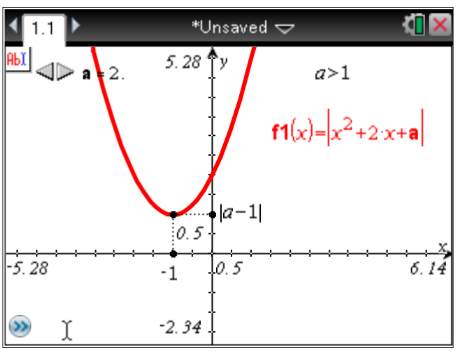
It should be noticed that equation $|x^2-8x|=m$ has exactly three roots (Pic. 6b) then and only then, when graphs of the function $f_2(x)$ and $y$ intersect each other in exactly three points. When selecting adequate value of $m$ („slide" – choosing upper triangle we increase parameter value, and choosing lower triangle we decrease value of $m$) it turns out that this situation takes place if $m=16$.

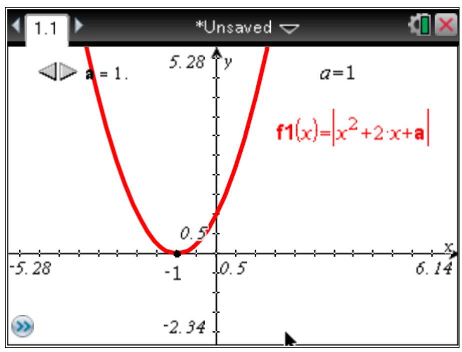
*Exercise 4.* For which value of $a$ parameter equation $|x^2+2x+a|=2$ will achieve four different roots?

Suppose $f_1(x)=|x^2+2x+a|$. Let's study the position and shape of above mentioned function in relation to $a$ parameter. We will use „slide" option once again. It should be noticed, that for $x=-1$, $f_1(-1)=|a-1|$. When manipulating the value of $a$ parameter it is not difficult to notice, that the following, specific positions of $f_1$ function graphs are possible (Pic. 7a, 7b, 7c).



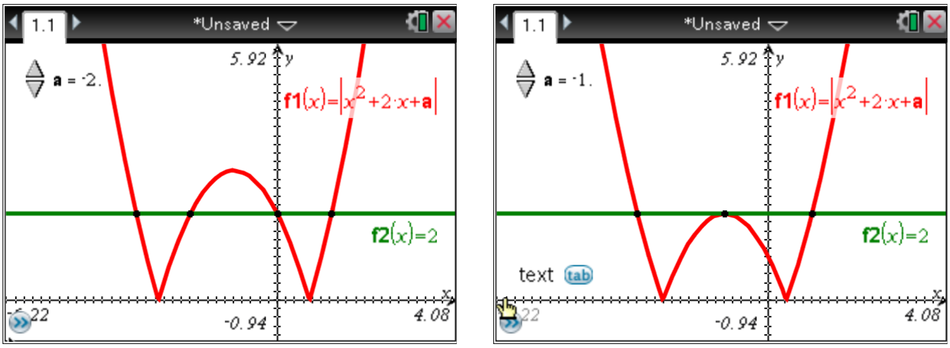

**Pic. 7a. Graph of function $f_1$ for $a<1$**




**Pic. 7b, 7c. Graph of function $f_1$ for $a>1$ and $a=1$**

Equation $|x^2+2x+a|=2$ has 4 solutions only then, when function graphs $f_1(x)=|x^2+2x+a|$ and $f_2(x)=2$ intersect in exactly four points. When changing value of $a$ parameter students will surely notice, after analysing the situation presented on Pic. 8a and 8b, that equation $|x^2+2x+a|=2$ has 4 solutions then and only then, when $a \in (-\infty, -1)$. Considerably the defined equation has 4 roots, when $|a-1| > 2 \ \wedge \ a < 1$, hence $-a+1 > 2$ and finally $a < -1$.



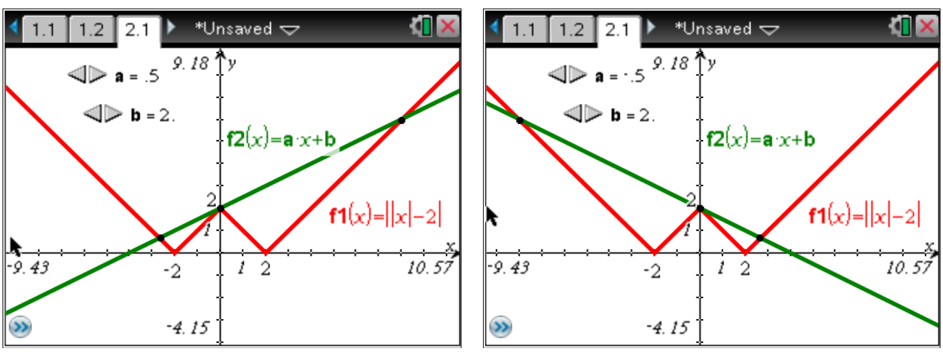

**Pic. 8a, 8b. Mutual postion of $f_1$ and $f_2$ function graphs**

*Excercise 5.* Find the minimum value of $a^2 + (b-1)^2$ expression for those value of *a* and *b*, for which the equation $||z-4|-2| - az + 4a - b = 0$ achieves 3 different roots.

Initially we should note, that equation $||z-4|-2| - az + 4a - b = 0$ is equivalent to equation $||z-4|-2| = a(z-4) + b$, which after substitution $x = z - 4$ finally takes form $||x|-2| = ax + b$. Suppose $f_1(x) = ||x|-2|$ i $f_2(x) = ax + b$.

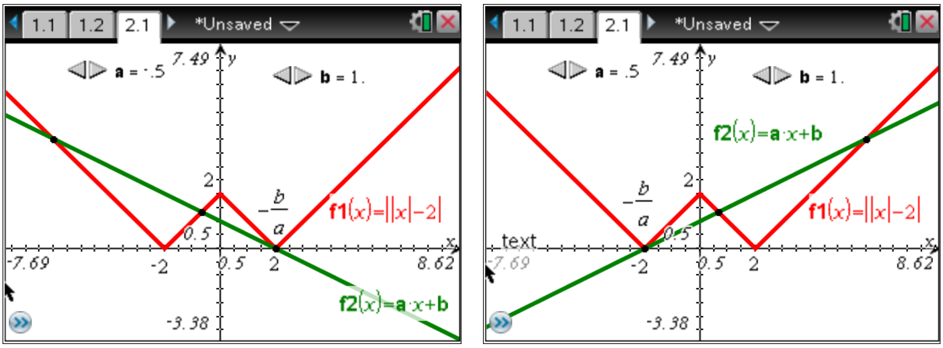
This time position of function graph $f_2$ depends on two parameters a and b. We will use „slide" option twice – for parameter a and b.

We will study mutual position of both function graphs by manipulating parameters value. Selecting right values of a and b parameters it turns out that equation $||x|-2| = ax + b$ has 3 solutions exclusively in three cases shown below.




**Pic. 9a, 9b. Mutual position of $f_1$ and $f_2$ function graphs**

as well if function graph denoting $f_2$ intersect point $(2,0)$ or $(-2,0)$ (Pic. 10a and 10b).




**Pic. 10a, 10b. Mutual position of function graph denoting $f_1$ and $f_2$**

We can solve the follwing cases, where equation $||x|-2| = ax + b$ has three roots:

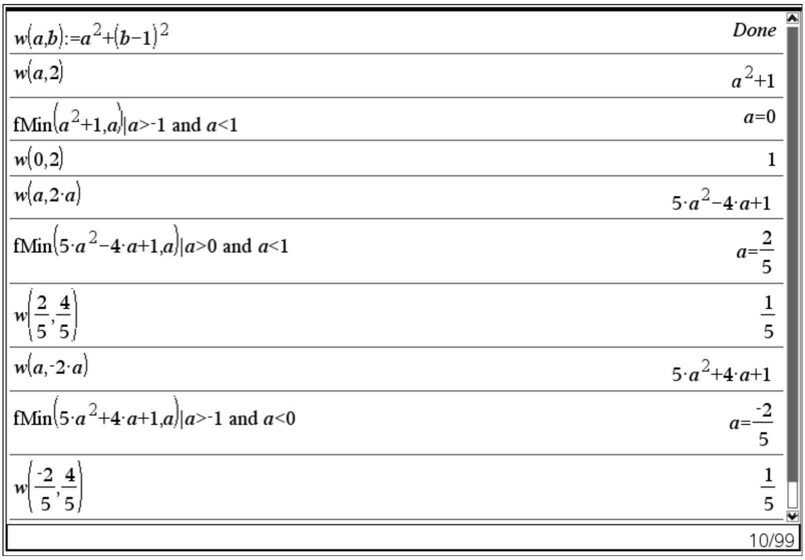
a) $b = 2$ i $a \in (-1,1)$ (Pic. 9a i 9b),

b) $-\dfrac{b}{a} = -2$ or $b = 2a$ i $a \in (0,1)$ (Pic. 10a),

c) $-\dfrac{b}{a} = 2$ or $b = -2a$ i $a \in (-1,0)$ (Pic. 10b).

If we substitute the expression $a^2 + (b-1)^2$ with $b = 2$ we will obtain $a^2 + 1$, which reaches a minimum equals 1 for $a = 0$. In case $b = 2a$ expression $a^2 + (b-1)^2$ takes a form and reaches minimum equals $\dfrac{1}{5}$ for $a = \dfrac{2}{5}\left(b = \dfrac{4}{5}\right)$. Whereas if $b = -2a$ our expression can be noted as $a^2 + (-2a-1)^2 = 5a^2 + 4a + 1$. Then the minimum is reached for $a = -\dfrac{2}{5}\left(b = \dfrac{4}{5}\right)$ and equals $\dfrac{1}{5}$.

Thus expression $a^2 + (b-1)^2$ takes the lowest value being equal $\dfrac{1}{5}$ for $a = \pm\dfrac{2}{5},\ b = \dfrac{4}{5}$ (Pic. 11).




**Pic. 11. Calculating the lowest value of $a^2+(b-1)^2$ with use of TI-Nspire calculator**

## 4. Conlcusions

When using a calculator we are able to constantly modify the picture, we can experiment and observe the dynamics of mathematical objects changes.

The dynamics of objects transformations on the calculator display is possible, easy and fast to be observed which seem to have though enormous weight in teaching process. Picture that is being formed is dynamic – and thus considerably more assimilable – easer to be accepted and remembered by a student. While working with calculator a student becomes a discoverer. Facts, relationships or theorems that he discovers are kept in his mind for a long time.

## Literature

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**Abstract**

In the article authors present concise description of visualization being used as a tool in problem solution process. They emphasize the role of visualization while solving problems regarding absolute value.

**Key words:** data visualization, teaching mathematics, graphing calculator.

## Znaczenie wizualizacji procesu rozwiązywania zadań z wartością bezwzględną za pomocą kalkulatora graficznego TI-Nspire

**Streszczenie**

W artykule przedstawiono zwięzłą charakterystykę wizualizacji jako narzędzia przydatnego w procesie rozwiązywania zadań. Podkreślono rolę wizualizacji przy rozwiązywaniu zadań z wartością bezwzględną.

**Słowa kluczowe:** wizualizacja danych, nauczanie matematyki, kalkulator graficzny.