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# The Implicit Logic of Plato's *Parmenides*[1]

Frege wanted to reduce all mathematics to a pure logic of predicates because he wanted to purge all the references to mathematical objects from mathematics. The logic of pure "laws of thought" should be truthlike and tautological. Therefore, it should refer to "nothing specific". In this way, it was possible to demonstrate that mathematics contains only analytic sentences. Logicism is a form of mathematical Platonism without ideal objects, i.e. it is a form of *objectivism*. It is rather interesting that logicism can be realized with the use of some very old ideas which emerge from ancient Platonian philosophy.

The paper is devoted to the reconstruction of the implicit logic of Plato's *Parmenides,* which suggests some new, hopefully interesting, ideas from the modern point of view. The reconstructed logic, ***F***, makes it possible to form a new semi-intuitionistic, non-extensional system of logic of predicates ***FN***. From ***FN*** follow axioms of Peano arithmetic ***PA***, as well as the existence of an infinite number of predicates. Therefore, ***FN*** can be viewed as an attempt at the realization of Frege's logicist program. The ontology of ***FN*** is very weak and probably would be accepted by Frege, since only the existence of some *positive* predicates is required (and that possibly of just one). Such ontology is in line with Frege's philosophy and is even weaker than the original ontology of the *laws of thoughts* — Frege accepted the existence of some sets (i.e. denotations of predicates), as distinct from the existence of the predicates themselves. Some other very strong systems can be seen as variants of ***FN***, e.g. Leśniewski's ontology, as it is well-known that the axioms ***PA*** are theorems

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of Leśniewski's ontology. On the other hand, Plato's logic, his way of thinking and the *protology* are very close to some modern intuitionistic ideas.

I will show also that the hypotheses from *Parmenides II* contain proofs of the existence of the two highest principles of Plato's *protology*, i.e. the One and the Dyad, their mutual relation, their relations to other things and that with regards to the mutual relation, the reasoning follows some exact formal rules. Therefore, I will also reconstruct the *implicit* logic of *Parmenides* and Plato's theory of the highest principles which creates the core of *agrapha dogmata*.

The main technical term in the *Parmenides* is *participation* (*methexis*), and some related, e.g. *participate in* (*meteho*) or *are participated in* (*methehontai*). The term *participation* was used in logic by Aristotle in the Platonic meaning as *being contained* (or *comprehended*) as genus or difference in species, cf. *Topics*, 132b 35. If "*A* participates in *B*", it means that *B* can be predicated *truly* on *A*. If something is "beautiful", "good", etc., it means that this something *participates in* (the ideas of) Beauty-Itself, Good-Itself, etc.

In the first part of *Parmenides*, Plato discusses some theories of the connections between ideas and things. The ideas cannot be radically *separated* from the plurality of the participated in things. The nature ("essential character") of every idea is determined by the relation of participation because not only things participate in the given idea but also the idea participates in more general ideas. The highest principles are at the top of this hierarchy. Therefore, every idea is *one over many* (*hen epi pollon*), i.e. every idea is both *one* and *many* (plurality) or every idea *participates* both in the One and in the Dyad.

The tools of dialectics show that if an assumption that *A* participates in *B* leads to a contradiction, it is possible that *B* participates in *A*, or if the latter is also inconsistent, it means that *A* and *B* are independent. The investigation demonstrates also that there are some *predicates* which can be predicated on everything except themselves. These are *the highest principles*: the One and the Dyad ("chaos"). It is even impossible to say that the One is "One" because the One would participate in "equality". In the same way, it is impossible for the One to participate in the Dyad and *vice versa*. Any such supposition terminates in a contradiction.

Thus the relation of participation fits the realm of everything with order, i.e. every idea and being has its own *rank*. Proclus explains also this point; see Proclus[2], p. 110, (734-735) or p. 112, (737-738).

It is not difficult to list the properties of the relation of participation. Even if the logic of participation is implicit, the reader can check that the reasoning in the hypothesis in *Parmenides II* follows the formal rules stated below in every case or that

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[2] Cf. Proclus, *Proclus' Commentary on Plato's* Parmenides, ed. & tr. by G. E. Morrow and J. M. Dillon with Introduction by J. M. Dillon, Princeton 1987, Princeton University Press, pp. 444-445 (in Greek: pp. 1098-1099; quoted below as "Proclus, pp. xx (in English translation), (pp. xx, in Greek edition)".

it terminates in a conclusion that is listed below. Some of these rules (e.g. **P.1.** and **P.2.** below) are not axioms in the modern sense but they are rather basic theorems or principles of Plato's *protology*. As we will see, the formal properties stated below are proved dialectically by Plato in the hypothesis of *Parmenides*.

There are two discerned objects ***O*** ("the One") and ***D*** ("the Dyad"). The expression "$A \uparrow B$" means "*A* participates in *B*". *A*, *B*, *X*, *Y* etc., are symbols of predicates representing every possible property which can be predicated on something and constructed without the use of the relation of participation "↑". There are also *three* different symbols for *negation*: "~" is, as usual in classical logic, a negation of a sentence (or of a well-formed formula), "¬" is a global negation of a predicate, and "ח" is a local negation of some predicates.

The distinction of these three types of negation is in line with many of Plato's texts and it is connected with the problem of the existence of the ideas of negative terms (or "negations").[3] Syrian considers the problem with reference to Hermodor, Plato's pupil, who linked negations of ideas with the "indeterminate dyad" (cf. the laws of global negation, "¬", below), and states that ideas correspond to universals but not every single universal must denote (something) one;[4] see also Simplicius *In Aristotelis Physica* 247, 30-248, 15 *Test. Pl.* 31.[5]

The finding that what is different from the given idea (i.e. the negation of the idea) means the finding of some antithesis (*antithemi*) corresponding to the given idea. This antithesis is not composed of the idea of "all that *x*'s that are not-*x*".[6] From this follows the idea of "local negation": not-*A* should be "something smaller" than "everything that does not participate in *A*"; cf. "ח" and the rules **P.5.**, **P.8.** and **P.9.** below. Therefore, it is not true that *every X* participates in *A* or it participates in not-*A*. However, the negation of the One is the Dyad and *vice versa*; cf. the global negation below. One can explain also that the negation "ח" is not the idea of *Difference* and the negation of a sentence is not any one negation of the predicate.

It is also possible to consider the global negation of *every* predicate (not only for ***O*** and ***D***) and to compare it with its local negation. This comparison is useful for the explanation of some ancient discussions. However, it is inconsistent with the other axioms which would probably be accepted by Plato.

Negation (of a predicate) corresponds not to the absolute negation of the given term with infinite plurality of different things participating in, as — for instance in

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[3] W. D. Ross, *Plato's theory of ideas*, Oxford 1961, Oxford University Press, pp. 167-169.

[4] G. Kroll, Syrianus `In Metaphysica' , G. Kroll (Ed.), Berolini 1902, pp. 107-108; ; K. Gaiser, *Platons Ungeschriebene Lehre. Studien zur systematischen und geschichtlichen BegrÄundung der Wissenschaften in der Platonischen Schule. Appendix: Testimonia Platonica. Quellentexte zur Schule und mündlischen Lehre Platons*, IInd ed., Stuttgart 1968.

[5] W. D. Ross, *op. cit.*, pp. 167-169.

[6] Cf. also Ross, *op. cit.*, pp. 167-169 and K. M. Sayre, *Plato's Late Ontology. A Riddle Resolved*, Princeton 1983, Princeton University Press, pp. 229-238. Cf. also Aristotle, *Hermeneutics* 17b 3-22.

the case of “*not-ten-thousand*”, but rather determines just the relative negative term (*heteros*) in which the opposites (*enantios*) of the given term (designating the given idea) participate.[7] Therefore “not-big” means “small or equal”.

One can formally define two very important properties of Plato’s negation (“ח*X*”) in the context of the relation of participation as follows:

**PN.1** $\forall X \forall A.\ (A \upharpoonleft \boldsymbol{O} \wedge X \upharpoonleft A) \rightarrow (X \upharpoonleft A \wedge X \upharpoonleft \neg A)$;
**PN.2** $\forall A.\ A \upharpoonleft \boldsymbol{O} \rightarrow (A \upharpoonleft \neg A \wedge \neg A \upharpoonleft A)$.

The above formal properties of Plato’s negation follow from many fragments of *Parmenides* (cf. for example *Parmenides* 128e-129b) as well as from many parts of Proclus’ *Commentary on Plato’s Parmenides*; cf. for instance Proclus, *op. cit.*, pp. 125-126, (755-757) or p. 128, (759).

Thus, **PN.1** and **PN.2** are well established in the sources. However, in the formal codification of the logic of the *Parmenides*, I use a different symbol for Plato’s negation (**PN**), i.e. “¬” (not “ח”), because **PN.1** is not consistent with some other — and more important to Plato — properties of the relation of participation. As we will see, a consistent version of **PN** are the properties:

**PN.1’** $\forall X \forall A.\ \sim (X = \neg A \wedge A \upharpoonleft \boldsymbol{O}) \rightarrow X \upharpoonleft \neg A$;
**PN.2’** $\forall A.\ A \upharpoonleft \boldsymbol{O} \rightarrow (A \upharpoonleft \neg A)$.

**PN.1’** and **PN.2’** are provable properties of local negation in our system of the logic of the *Parmenides*. Therefore, the concept of *local negation* “ח” is a maximal possible noncontradictory formalization of **PN** in the system with *one* relation of local negation. However, there are other possibilities when one discerns more types of local negation, e.g. “upper”, “lower”, “global” and “absolute” negations; see below. This move seems to be in line with Plato as well as with Proclus, since the latter also speaks explicitly (in many places) about different types of negation; see Proclus, *op. cit.*, p. 426 (1073), 427 (1074).

Therefore, we can also formalize the concept of Plato’s negations using different symbols for different types of negation, i.e.:

**PN.1``** $\forall X \forall A.\ (A \upharpoonleft \boldsymbol{O} \wedge X \upharpoonleft A) \rightarrow (X \upharpoonleft A \wedge X \upharpoonleft \neg_1 A)$;
**PN.2``** $\forall A.\ A \upharpoonleft \boldsymbol{O} \rightarrow (A \upharpoonleft \neg_2 A \wedge \neg_3 A \upharpoonleft A)$.

I tried to formalize the concept of participation by avoiding any set theoretical context because the concept of participation is non-extensional (see below). However, one can try to introduce some part of a (non-classical) set theory in the following way.

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[7] Plato uses this term in this meaning in many places, e.g. in *Lysis* 215e, *Phedo* 103c, *Parmenides* 155a, *Laws* 899b. Cf. Sayre, *op. cit.*, pp. 230-231. Sayre establishes that the opposition designates *one* thing in every case, and that “that, what is different” designates many things which share in one common nature; cf. 257d 4.

Firstly, one can identify every given predicate $X$ with the set of its determinants, i.e. by introducing "the upper sets $< X >$": $\forall X \forall A.\ (A \uparrow \boldsymbol{O} \wedge X \uparrow A) \rightarrow\ < X >= [A : X \uparrow A]$. (When we accept also the "upper extensionality axiom" (see below), such a set is unique.) This last identification is similar to Russell's theory of description. Obviously, the relation between "$\in$" and "$\uparrow$" is: $\forall X \forall A.\ (A \uparrow \boldsymbol{O} \wedge X \uparrow A) \rightarrow (X \uparrow A \rightarrow A \in\ < X >)$. Next, the following identification is also reasonable:

**ID.** $\forall X \forall A.\ (X \uparrow \boldsymbol{O} \wedge A \uparrow \boldsymbol{O}) \rightarrow (X \uparrow A \leftrightarrow A \in\ < X >)$.

In the same way, one can define "lower sets $> X <$": $\forall X \forall A.\ (X \uparrow \boldsymbol{O} \wedge A \uparrow \boldsymbol{O}) \rightarrow (A \uparrow X \rightarrow A \in\ >X<)$. Thus the local set-like Plato's negation (of a predicate) can be defined in two ways:

**PN.3** $\forall X \forall A \forall Y.\ A \uparrow \boldsymbol{O} \rightarrow (X \uparrow A \rightarrow (\daleth_1 A = Z \leftrightarrow \exists Z.\ Z \in\ [Y : X \uparrow Y \wedge \sim(A{=}Y)])$, i.e. $\daleth_1 A = (< X > - [A])$ where "$[A]$" is a one-element set and every such $Z$ is called an "upper local negation" of a predicate $A$ ($\daleth_1 A$), or

**PN.4** $\forall X \forall A \forall Y.\ A \uparrow \boldsymbol{O} \rightarrow (X \uparrow A \rightarrow \daleth_2 A = Z \leftrightarrow \exists Z.\ Z \in\ [Y : Y \uparrow A \wedge \sim(A{=}X)])$, i.e. $\daleth_2 A = (> X < - [A])$ where "$[A]$" is a one-element set and every such $Z$ is called a "lower local negation" of a predicate $A$ ($\daleth_2 A$).

The lower local negation is definable if one introduces "lower sets" corresponding to a predicate, i.e. $> X <$. The lower negation is consistent with some of Plato's and Proclus' texts, though it introduces a concept alien to Plato, that of a "lower set", and the concept of "lower set" has no clear conditions of identity because the "lower extensionality" (cf. below) is not consistent with our axioms **P.0.-P.9.** Thus every $Z$ satisfying **PN.4** is called a local negation $\daleth_2$ of a predicate X *if such Y exists*. However, every idea (predicate) is *one-over-many* and thus such $Y$ has to exist. (Also, empty predicates are excluded by Plato as belonging to sophistry which is "about nothing", i.e. it concerns non-existent subjects.)

With the use of "$\daleth_2$", one can obtain $\forall X \forall A.\ (A \uparrow \boldsymbol{O}) \rightarrow (X \uparrow A \rightarrow X \uparrow \daleth_2 A)$ (cf. **PN.1**). Therefore, one can introduce one more local negation (the "$\daleth_2$" besides "$\daleth$") into the system **P.0.** — **P.9.** below. One can use also one more negation (i.e. $\daleth_1$) with similar consequences for **PN**, as in the case of "$\daleth_2$".

An "absolute negation" (of a predicate $X$) can be introduced as follows. First, one has to define a set $< X` >$ containing all predicates $A$ which satisfy the condition "$A \uparrow \boldsymbol{O} \wedge \sim(X \uparrow A) \wedge \sim(A \uparrow X)$", i.e. $\forall X \forall A.\ < X` > = [A\!: A \uparrow \boldsymbol{O} \wedge \sim(X \uparrow A) \wedge \sim(A \uparrow X)]$. Every element $Z$ of a set $< X` >$ is called an "absolute negation of a predicate $X$", (in symbols: $Z = X`$ ):

**PN.5** $\forall X \forall A.\ (X \uparrow \boldsymbol{O} \wedge A \uparrow D) \rightarrow (Z = X` \leftrightarrow \exists Z.\ Z \in\ [A\!: A \uparrow \boldsymbol{O} \wedge \sim(X \uparrow A) \wedge \sim(A \uparrow X)])$.

"Absolute negation" is necessary to explain some of the rationale of *Parmenides* concerning terms of the kind "not-human" or "not-ten-thousand". The intuition behind the concept of absolute negation is evident: $X`$ is any predicate "disconnected" with $X$ which is not the One and which is not the Dyad.

The definitions **PN.3**, **PN.4** and **PN.5** are not formally correct in the frames of first-order logic (e.g. they are not sentences because they have a free variable) and they are nonconstructive and nonpredicative. However, they provide some intuitions. For instance, in first-order logic, it is necessary to accept, at first, the existence of, say, the non-empty set $<X`>$ and, secondly, a constant $Z \in \ < X` >$, or to postulate the existence of the $Z$ directly (see the axiom of choice or the axiom of infinity in ZFC; obviously, such a definition is possible also if the defined object is unique). The definitions **PN.3**, **PN.4** and **PN.5** "define" some objects which do not themselves behave in accordance with the laws of the relation of identity. For example, there are (intuitively) many possible objects representing $X`$ and thus it may happen that $\sim(X = X`)$ or that $A = X` \wedge (X` = B \wedge \sim(A = B))$. Therefore, if one would like to accept such definitions as those above of negations, one should resign from the classical laws of identity. This also creates one more possibility to operate with the non-predicative definitions.

It is also very easy to introduce into the system of Parmenidian logic the above set theoretic concepts,[8] as well as three types of negation (i.e. **PN.3**, **PN.4** and **PN.5**). This is interesting from a formal point of view. However, it is not necessary to do this in order to achieve our theoretical goal in the present paper. More important is to realize that our formal system, even enriched with some part of a set theory, cannot be an effective tool in intensional analyses. There exists a "third way" between strict formalization and intuitive informal reasoning, creating a formal system which *must* be completed with intensional analysis. Our formal system does not determine what exactly the elements of a lower and upper sets are corresponding to real intensional predicates as well as what participates and what is participated in the case of such full-blooded predicates, e.g. "good", "beautiful", "red". Therefore, in every case, one has to decide in an informal way which formal objects can represent the given intensional predicate.

I think that this is a real new alternative to the modern way of creating mathematics: to give the strict formal frames for some intensional content without any supposition that these two contexts are the same. Everyday mathematical practice agrees with this.

Very natural formal frames for the theory presented below are given within first-order logic with identity. (The concept of a well-formed formula is straightforward.) I decided to use only one relation of local negation (and not "upper" or "lower" one) because, in this way, it is possible to explain the logic of *Parmenides*. The local negation "ח" simply imposes the existence of some elements which can be elements of the sets $< X` >$, $< X >$ and $> X <$ without any explicit "decision" as to which of the

[8] Obviously, one has to discern between "lower" and "upper" sets.

sets they belong to exactly. Thus, this relation is so general that it can *formally* explain the underpinning rationale in *Parmenides*.

**P.0.** $\sim(\boldsymbol{O} = \boldsymbol{D})$.
**P.1.** $\forall X.\ [\sim(X = \boldsymbol{O}) \wedge \sim(X = \boldsymbol{D})] \rightarrow (X \uparrow \boldsymbol{O})$.
**P.2.** $\forall X.\ [\sim(X = \boldsymbol{O}) \wedge \sim(X = \boldsymbol{D})] \rightarrow (X \uparrow \boldsymbol{D})$.
**P.3.** $\forall X \forall A \forall B.\ [\sim(X = \boldsymbol{O}) \wedge \sim(X = \boldsymbol{D})] \rightarrow [(X \uparrow A \wedge A \uparrow B \wedge \sim(B = X)) \rightarrow (X \uparrow B)]$.
**P.4.** $\forall A.\ [A \uparrow \boldsymbol{O} \rightarrow \exists \sqcap A.\ (\sqcap A \uparrow \boldsymbol{O} \wedge \sim(A = \sqcap A) \wedge \forall B\ (A = B \rightarrow \sqcap A = \sqcap B))]$.
**P.5.** $\forall X \forall A \forall C \forall D.\ [X \uparrow A \wedge \sim(X = \sqcap C) \wedge \sim(A = \sqcap D)] \rightarrow \sim(A \uparrow X)$.
**P.6.** $\boldsymbol{O} = \neg \boldsymbol{D}$.
**P.7A.** $\forall X.\ (X \uparrow \boldsymbol{O}) \rightarrow [\sim(X \uparrow \boldsymbol{O}) \rightarrow (X \uparrow \neg \boldsymbol{O})]$.
**P.7B.** $\forall X.\ (X \uparrow \boldsymbol{O}) \rightarrow [\sim(X \uparrow \boldsymbol{D}) \rightarrow (X \uparrow \neg \boldsymbol{D})]$.
**P.8.** $\forall A \forall B.\ A \uparrow B \rightarrow \sim(A = B)$.
**P.9.** $\forall X \forall A.\ (A \uparrow \boldsymbol{O} \wedge X \uparrow \boldsymbol{O}) \rightarrow [\sim(X = \sqcap A) \rightarrow [\sim(X \uparrow A) \rightarrow (X \uparrow \sqcap A)]]$.

Regarding the detailed explanation of some reasoning in *Parmenides* in all details, it is also useful to apply, in some places, the concept of absolute negation **PN.5** but defined correctly as explained above.[9]

One can also define some other objects, e.g. "individuals" ($\boldsymbol{Ind}(X)$) or "ideas" ($\boldsymbol{Id}(X)$):

**Def.1** $\forall X.\ \boldsymbol{Ind}(X) \leftrightarrow \sim(\exists A.\ A \uparrow X)$
**Def.2** $\forall X.\ \boldsymbol{Id}(X) \leftrightarrow (\exists A.\ A \uparrow X \wedge X \uparrow \boldsymbol{O})$ (i.e. to be an idea is to be "one over many" and to be different from the first principles).

Let us notice that, as a model for the **P.0.-P.9.**, we can use any two-element set containing two different objects "**O**" and "**D**" because the axioms do not impose any existence of other objects. Therefore, it is also necessary to accept one more axiom:

**P.0a.** $\exists X.\ X \uparrow \boldsymbol{O}$.

The existence of individuals $\boldsymbol{Ind}(X)$ is inconsistent with the system (see some considerations below). However, it seems that to Plato local negation is possible only for ideas $\boldsymbol{Id}(X)$. Otherwise, what would a local negation of an individual be if individuals could not be predicated about something other?[10] Thus we have to delimit the

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[9] In this case, it is more natural to define the (classical) negation of a sentence in our system as $\forall A \forall X.\ (A \uparrow \boldsymbol{O} \wedge X \uparrow \boldsymbol{O}) \rightarrow [\sim(A \uparrow X) \leftrightarrow \forall X`.\ X` \in \langle X` \rangle) \rightarrow (A \uparrow X`)]$, however, from the formal point of view, it is unnecessary. This possible addition needs some other complementary changes in our system.

[10] If one accepts instead of the **P.0a.** the following axiom: $\exists X \exists A.\ \boldsymbol{Ind}(X) \wedge \boldsymbol{Id}(A) \wedge (X \uparrow A) \wedge (\sqcap X = \sqcap A) \wedge \sim(\exists A`.[(A` \uparrow A) \wedge (X \uparrow A`) \wedge \sim(A` = \sqcap A) \wedge \sim(\exists Y.\ \boldsymbol{Ind}(Y) \wedge Y \uparrow A \wedge \sim(X = Y))]$, there is a possibility to operate with the homogeneous local negation which acts also on individuals. The axiom extends the concept of local negation to individuals: a local negation of a predicate is equal to the local negation of the "lowest" idea in which this individual participates.

axioms concerning local negation only to ideas,[11] and then one can introduce individuals:

**P.0b.** $\exists X \forall A. \sim (A \uparrow X)$ (or $\exists X. \boldsymbol{Ind}(X)$).

On the other hand, from the existence of individuals, the existence of ideas does not automatically follow. Therefore, we have to accept the next existential axiom (instead of the axiom **P.0a.** which is now superfluous):

**P.0c.** $\exists X. \boldsymbol{Id}(X)$.

In the above axioms, I added some formal conditions in order to avoid a contradiction. For instance, in the law of transitivity **P.3.**, it is necessary to add the condition “$\sim (B = X)$” if one wants to operate in one system both with transitivity of “$\uparrow$” and with local negation.

Obviously, Plato does not speak about such conditions because he speaks about the properties of the relation of participation, such as transitivity, and about the properties of a local negation separately. Therefore, the explicit reconstruction of **P.3.** based on Plato’s text should be “$\forall X \forall A \forall B. (\sim (X = \boldsymbol{O}) \wedge \sim (X = \boldsymbol{D})) \rightarrow ((X \uparrow A) \wedge (A \uparrow B) \rightarrow (X \uparrow B))$”. However, the last axiom is inconsistent with the axioms concerning local negation “$\sqcap$”.

The same applies to the axiom **P.6.** because the properties of local negation impose the symmetry of the relation of participation, and — in other places — Plato speaks about the antisymmetry of the relation. Therefore it is necessary to limit the antisymmetry to *positive* predicates, i.e. the predicates which are constructed without local negation.

The global negation (**P.0**, **P.6**, **P.7A**, **P.7B**) determines the global double negation law (i.e. $\neg\neg\boldsymbol{O} = \boldsymbol{O}$ and $\neg\neg\boldsymbol{D} = \boldsymbol{D}$).[12]

It is easy to prove some other properties, such as:

**L.1** $\forall X. \sim (\boldsymbol{O} \uparrow X)$.
**L.2** $\forall X. \sim (\boldsymbol{D} \uparrow X)$.
**L.3** $\forall X. \sim (X \uparrow X)$.

The classical logic of the system, i.e. the law of excluded middle (**LEM**), imposes some very strong non-constructive conditions on the universe of properties. From the **LEM**, it follows that for every two predicates $X$, $A$ (and $A$ is an idea), we have $X \uparrow A$ or $\sim (X \uparrow A)$. In both situations we obtain the sentence:

**L.4** $\forall X \forall A. (\boldsymbol{Id}(A) \wedge \sim (X = \sqcap A)) \rightarrow X \uparrow \sqcap A$ (or $\forall X \forall A. (\boldsymbol{Id}(A) \wedge \sim (X \uparrow \sqcap A)) \rightarrow (X = \sqcap A)$).

[11] For instance, the axiom **P.4.** is now: $\forall A. Id(A) \rightarrow \exists \sqcap A. [\sqcap A \uparrow \boldsymbol{O} \wedge \sim (A = \sqcap A) \wedge \forall B (A = B \rightarrow (\sqcap A = \sqcap B))]$.

[12] I explain below that the first-order conditions of identity are used by Plato in *Parmenides*.

From the above, we obtain (**L.5**) if $X \uparrow A$, $X \uparrow B$ and $\sim (A = B)$ then $X \uparrow \sqcap A$, $X \uparrow \sqcap B$, $\sqcap A \uparrow \sqcap B$ and $\sqcap B \uparrow \sqcap A$.

It is also very easy to demonstrate the connection between the classical negation "$\sim$" and the local negation "$\sqcap$" (cf. also **P.9.**):

**L.6** $\forall A.\ [\boldsymbol{Id}(A) \rightarrow (\sim (A \uparrow A) \rightarrow (A \uparrow \sqcap A))]$.

If for all $A$'s we accept that $\sqcap\sqcap A = A$ then we have also:

**L.6'** $\forall A.\ [\boldsymbol{Id}(A) \rightarrow (\sim (\sqcap A \uparrow \sqcap A) \rightarrow (\sqcap A \uparrow A))]$ (therefore the existence of individuals is inconsistent with a system with a local negation of individuals). Then we obtain a version of **PN.1.**, and **PN.2.** It seems that the law of excluded middle and dichotomy principle were somehow withheld and so the negation of terms was used in an almost intuitionist manner.

Plato would also reject a theory of arbitrary multitude of whatever nature, treated as one. This would match his non-propositional concept of truth for which the sentences containing names that do not denote ideas can be neither true nor false. Aristotle's syllogistic can be seen as a theory stemming from that non-propositional concept of truth in an attempt to reject it. On intuitionistic character of the local negation "$\sqcap$" indicates the possible violation of double negation because our axioms do not decide if "$(\sqcap\sqcap X = X)$".[13]

Therefore, there are two other possibilities. If one accepts in **P.9.** that

**P.9'.** $\forall X \forall A.\ (A \uparrow \boldsymbol{O} \wedge X \uparrow \boldsymbol{O} \wedge \boldsymbol{Id}(A)) \rightarrow [\sim (X \uparrow A) \leftrightarrow (X \uparrow \sqcap A)]]$

then we will get "$(\sqcap\sqcap A = A)$", i.e. "$[(X \uparrow \sqcap A) \rightarrow \sim (X \uparrow A)] \rightarrow (\sqcap\sqcap A = A)$".

By the transposition of the last sentence, we obtain: "$\sim (\sqcap\sqcap A = A) \rightarrow \sim [(X \uparrow \sqcap A) \rightarrow \sim(X \uparrow A)]$".

In the case when "$(\sqcap\sqcap A = A)$", the resulting system is very simple (**PN.1.** is also valid) and there are possible finite "models" of **P.0.-P.9.** (even with the **P.0a.**).[14] We have that: $\forall X \forall A.\ [\boldsymbol{Id}(A) \wedge (\sqcap\sqcap A = A)] \rightarrow (X = \sqcap A) \vee (X = A)$.

There is also one more possibility: if in the **P.4.** we resign from the explicit condition "$\sim (\sqcap\sqcap A = A)$", we can interpret the relation of participation as the relation of difference, i.e. $A \uparrow B \leftrightarrow \sim (A = B)$.

However, it seems that Plato cannot accept such a simple universe of discourse. In particular, in the second hypothesis, he tries to construct natural numbers in his system. Therefore, it is necessary to accept the old version of **P.9.** and two more axioms:

**P.0d.** $\forall A.\ [\boldsymbol{Id}(A) \rightarrow \sim (\sqcap\sqcap A = A)]$

**P.0e.** $\exists A \forall X.\ \sim (A = \sqcap X)$; i.e. there exists a "positive property".

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[13] The addition of **P.10.** and **P.11.** also cannot decide this point.

[14] For instance, there is a four-elements model.

The last axiom (**P.0e.**) is inconsistent with the **L.6’**, and then also with “~ (*A* = ח*A*)”.

Obviously, such a positive property can play the role of “0” in our system, in which it is possible now to define natural numbers partially. There are not finite “models” of **P.0.-P.11.** + **P.0b.** + **P.0c.** + **P.0d.** + **P.0e.** One can demonstrate also that the first four Peano axioms of arithmetic are provable sentences in this system (with the change of the axioms **P.4.**, **P.7.** and **P.9.**, concerning local negation “ח” which must act only on the ideas ***Id***(*X*). “0” is our “*A*” in **P.0e.**, “*S*(*n*)” is “ח… ח*A*” in **P.0e.**, “1” is interpreted by “ח*A*” in **P.0e**. One can obtain also a (little strange) version of the axiom of induction (the proof is similar to the usual proof of induction theorem in ***ZF***).

The axioms **P.0.-P.9.** decide some other hypothesis as being inconsistent with them. First of all, the relation of participation “↑” is non-extensional because the sentence: ∀*A*, *B*, *C*. [(*A* ↑ *B* ↔ *A* ↑ *C*) → *B* = *C*] is inconsistent. However, the local extensionality (or “upper extensionality”), i.e. ∀*X*, *A*, *B*, *Y*. [(*A* ↑ ***O*** ∧ *B* ↑ ***O***) → (*X* ↑ *A* ↔ *X* ↑ *B*) ∧ (*A* ↑ *Y* ↔ *B* ↑ *Y* ) → *A* = *B*] (“the Leibniz condition of identity”), even though consistent, would be not accepted by Plato (because of ancient *horror infiniti*). Nevertheless, in some places, Plato seems to be aware of the local extensionality, cf. *Parmenides* 139d.[15] The lack of the global extensionality indicates that some possible models of the theory **P.0.-P.9.** are out of Zermelo-Fraenkel set theory with the axiom of extensionality (the relation “↑” is not a function and is not even an extensional relation).[16] The idea of a global negation is also inconsistent. In particular, the following sentence is inconsistent: ∀*A*. *A* ↑ ***O*** → ח*A* = ***D***.

In **P.0.-P.9**, we deduce (**L.8**) ∀*A*. ***Id***(*A*) → ∃*B*. (*B* ↑ ***O*** ∧ ח*A* ↑ *B*) (let us take, for instance, *B* = חח*A*). Thus, there is an infinite number of ideas between the given idea *A* and ***O***. In many places, (e.g. in the *Sophist* and also in the *Parmenides*) Plato argues for the **L.8**.[17] Moreover, the dissection of unity, (see below), leads directly to the **L.8**. Therefore, we accept **L.8**.

Now, it is possible to compare this system with Leśniewski’s ontology. The latter, as, for example, consistent with global transitivity, is inconsistent with **P.0.** — **P.9.b.**;[18] cf. the axiom of Leśniewski’s ontology (**LOA.**), expressed with the use of the participation symbol “↑”:

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[15] “But if the one and the same were identical, whenever anything became the same it would always become one, and when it became one, the same.”; cf. H. N. Fowler, *Plato*. *Plato in Twelve Volumes*, *Vol. 9*, tr. by H. N. Fowler, London 1925, Harvard University Press, Cambridge, MA; London 1925, William Heinemann Ltd.; retrieved 2010 from the Perseus Digital Library, http://www.perseus.tufts.edu/hopper/searchre-sults?q=parmenides+plato.

[16] I introduce below also the upper extensionality *axiom*, cf. **P.11**.

[17] Proclus accepts also **L.8**. Cf. for instance Proclus, p. 128, (759-760).

[18] However, one can define one more kind of individuals (i.e. Leśniewski’s individuals), and delimit the concept of local negation, which enables to see Leśniewski’s ontology as a part of such a changed system. However, such a change — contrary to Plato — introduces some objects which are not any *one over many* because *B* is an object in which participates exactly one other object.

**LOA.** $X \upharpoonleft B \leftrightarrow \exists C.\ C \upharpoonleft X \wedge \forall C \forall D.\ ((C \upharpoonleft X \wedge D \upharpoonleft X) \rightarrow C \upharpoonleft D) \wedge \forall C.\ (C \upharpoonleft X \rightarrow C \upharpoonleft B)$.

With the use of the local extensionality axiom (see above), one can reformulate the theory **P.0.-P.9.** (eventually with **P.10.** and **P.11.** below) with the use of two different relations "$\upharpoonleft$" which allows us to consider this resulting theory as a kind of set theory with two different relations "$\in$".[19]

From the above, it is also easy to see that the most natural logic of Plato's *Parmenides* is intuitionistic logic without the **LEM**. In this last case, it is impossible to infer the **L.4** and **L.5.** Moreover: some axioms are simpler. The system ***F*** is "semi-intuitionistic" because of some properties of local negation, e.g. "$\sim (\sqcap\sqcap A = A)$"; in classical syllogistics the classical negation of a predicate is definable with the use of classical propositional negation. In the case when it appears that the system ***F*** with classical logic is inconsistent, it is reasonable to try to reformulate it with the use of intuitionistic logic. However, I will save a detailed consideration of this case for a later work.

In order to compare the purpose of Plato with modern first-order frames, one can introduce one more axiom ("comparative axiom"):

**CA** $\forall X \forall A.\ X \upharpoonleft A \rightarrow A(X)$.

Though this axiom is not "Platonic", it is possible to add this axiom (obviously, we have to define strictly, what "$A(X)$" means) because it enables the comparison of Plato's logic with modern logic of predicates, as well as makes it possible to consider the modern problem of some antinomies, e.g. the Russell's antinomy. The Russell's antinomy is not a danger to our system of Plato's logic. In modern logic, it seems to be obvious and natural that "$X \upharpoonleft A$" is equivalent to "$A(X)$". However, it is necessary to differentiate between them because — according to Plato — the relation of participation must be rooted in something real. In sophistry, we can speak about everything and our words that "*X* is good" does not imply that *X* is really good. On the other hand, if "*X* is Good", one can truly say that "*X* is good". Plato is, on this point, in opposition to modern analytical philosophy which concentrates on the inquiry of what follows from the verbal supposition that "$A(X)$", in a language which has enough formal tools to express the given sentence. This axiom indicates also that the **LEM** is not a natural "law of thought" in Plato's *protology*.

From system ***F***, it is possible to form a simpler system of the foundations of mathematics, ***FN***, because one can generate the numbers without any acceptance of

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[19] Z. Król, *Uwagi o stylu historycznym matematyki i rozwoju matematyki*, (in Eng.: *Remarks on `historical style of mathematics' and on the development of mathematics*). [In:] *Światy matematyki: tworzenie czy odkrywanie?*, ed. I. Bondecka-Krzykowska, J. Pogonowski, Poznań 2010, Wydawnictwo Naukowe UAM, pp. 203-234.

two different objects: the One and the Dyad. The minimal set of axioms necessary for the (partial) definition of natural numbers is as follows:[20]

**FN.0** $\exists N \forall X.\ \sim (X = N) \rightarrow X \uparrow N$
**FN.1** $\forall X \forall A.\ X \uparrow A \rightarrow \sim (X = A)$
**FN.2** $\forall X \forall A.\ [(\sim (X = N) \rightarrow (X \uparrow A) \wedge (A \uparrow B) \wedge \sim (B = X)] \rightarrow (X \uparrow B)]$
**FN.3** $\forall A.\ [A \uparrow N \rightarrow \exists\, \text{ה}A.\ (\text{ה}A \uparrow N \wedge \forall B.\ (A = B \leftrightarrow \text{ה}A = \text{ה}B))]$
**FN.4** $\exists A \forall X.\ \sim (A = \text{ה}X)$
**FN.5** $\forall X \forall A \forall C \forall D.\ (X \uparrow A \wedge \sim (X = \text{ה}C) \wedge \sim (A = \text{ה}D) \rightarrow \sim (A \uparrow X)$

The presented formalization ***F*** creates a formal reconstruction ("formal skeleton") of *implicit logic* in *Parmenides*. Such a reconstruction indicates that Plato's purpose in the dialogue is strictly defined. Next, we will see how Plato argues for the above "axioms" (i.e. for the ***F***) in some places of the *Parmenides*, only because it is impossible to discuss this point in full in the present paper.

***The first hypothesis*** **(137*c* 4-142*a* 8).** In the first hypothesis, Plato demonstrates that the One does not participate in anything, i.e. there is no predicate which can be predicated on the One; cf. the **L.1.** and **L.3.** above. It is impossible to say even that the One is "one" ($\boldsymbol{O} \uparrow \boldsymbol{O}$); cf. 141e 10-11. The main supposition of the first hypothesis is "if the One exists" in a specific sense, i.e. if the One is *itself* . The One *itself* is not anything other, i.e. one cannot say that it participates in anything and that it is something other than the One itself. We can characterize the One by saying what it is *not*. The reasoning in this hypothesis starts from the intuitively obvious evidence that the One *itself* does not participate in the Dyad; i.e. in "multiplicity". (This last statement is formally derivable from **P.0.-P.9.**) Therefore, the One is, in a sense, separated from other beings, ideas and the Dyad. Plato shows that any kind of predication of something about the One leads to contradiction, for if there is even a single predicate about the One, one can state everything about the One as a consequence of this, i.e. even every pair of contradictory properties.

Plato successively demonstrates that the One does not participate in "many", "part", "whole", "magnitude", "place", "motion", etc.; cf. for instance 137c.

Each of the considered properties participates in spatiality, i.e. in the primordial spatial Dyad. (If something participates in "spatiality" (which is potentially divided into many parts), it participates in the Dyad, and the One does not participate in everything that participates in the Dyad; cf. $\forall X.\ [X \uparrow \boldsymbol{D} \rightarrow \sim (\boldsymbol{O} \uparrow X)]$.)[21] Thus there is no *place* in which the One "is". In particular, the One does not exist in "mind". Therefore, it is very informative to know in what the One *does not* participate. In the same way, it is impossible that the One is *different* from something, the *same* as something (even itself) or *similar* to something.

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[20] With some changes, it is also possible to introduce one more axiom concerning "upper extensionality": **FN.6**, $\forall A \forall B \forall X.\ [A = B \rightarrow (A \uparrow X \leftrightarrow B \uparrow X)]$.

[21] This sentence, in an obvious way, follows from our axioms.

The second part of the first hypothesis considers the problem of the participation of the One in time. "Time", as "spatiality" before, also participates in the Dyad. Plato shows that the One itself does not participate in "time", nor in any predicate which participates in "time" as well as — *a fortiori* — in the Dyad. "*It* [i.e. the One — Z.K.] *has nothing to do with time, and does not exist in time*" (*ibidem*, 141d).

We have no predicate to explain and to characterize what the One itself is. From this point of view, the One is indeterminate and unlimited (*apeiron*). The One itself cannot explain anything, even only itself, because to say "what the One is not" needs the use of some properties which participate in the Dyad (see the concept of the global negation above).

***The second hypothesis*** **(142*b*1-157*b*5).** The assumption of this hypothesis is that "the One is", i.e. that "the One participates in *Being*". Therefore, one has to enquire as to the consequences of the existence of a predicate which can be predicated on the One. The assumption that the One partakes in *Being* leads immediately to the acceptance of the properties of the One which are in contradiction with the nature of the One, i.e. with the One taken in itself. From the formal point of view, the acceptance of an assumption which is in contradiction with a previously accepted sentence (e.g. with the sentence "$\sim (\exists X.\ \boldsymbol{O} \uparrow X)$") leads immediately to a contradiction.

From the fragment 142b-c. of the *Parmenides*, it follows that Plato accepts the formal rule **L.3** $\forall X.\ \sim(X \uparrow X)$ because the relation of participation concerns only the different predicates (i.e. the predicates of different meanings). In the same way, the above words seem to indicate the implicit acceptance of local extensionality and, on the acceptance of the formal rules of replacement, accepted in first-order logic; cf. e.g. the fragment: "*Then the being of one will exist, but will not be identical with one; for if it were identical with one, it would not be the being of one, it would not be the being of one, nor would one partake of it, but the statement that one is would be equivalent to the statement that one is one* …". The above reasoning is in accordance with one of the first-order axioms of identity $X = Y \rightarrow \varphi(A_0, \ldots, A_{i-1}, X, A_{i+1}, \ldots, A_n)$ — $\varphi(A_0, \ldots, A_{i-1}, Y, A_{i+1}, \ldots, A_n)$ (I omitted the quantifiers).

Because of **L.3**, the One participating in *Being* is different than *Being*, i.e. the One (and the *Being*) participates in *Difference*. Therefore, from these two, we obtain three predicates: "one", "being" and "different", and next — four, five etc., i.e. any number of predicates. Thus we also have "odd" and "even". From the One follows "number" which, in an obvious way, participates in "multiplicity" and in the Dyad. This reasoning is based on two possible formal rules. The first is:

**N.1** $\forall A,B\ \exists P \uparrow \boldsymbol{O}.\ [(\sim (A=B) \wedge A \uparrow \boldsymbol{O} \wedge B \uparrow \boldsymbol{O} \wedge A \uparrow \text{`}Exists\text{`} \wedge B \uparrow \text{`}Exists\text{`}) \rightarrow (A \uparrow P \wedge B \uparrow P)$, and "$P$" means a "Pair".

Thus, everything that participates in *Difference* (i.e. in a local negation) participates in "number". This process Plato calls "the division of the One". In the case where $A$ and $B$ in **N.1** are ideas, the process generates numbers; cf. 143b-144a.

However, the next part (which starts at 144a) extends the division of the One to the case where $A$ and $B$ are not only ideas but are something existent (as in the **P.11.**) and the division terminates in indeterminate multitude: not only the existent One is divided infinitely but also the One itself is divisible *ad infinitum*:[22]

The division (or dissection) of the One is a kind of Third Man Argument. The crux of the argument rests on the assumption that the idea is not *one-over-many* but it is a thing of the same kind as the things participating in it. However, it is different from them, i.e. the idea of something, together with this "something", participates in the *Difference*.

One can establish a (partial) connection with extensional set theories by the identification of "$P$" and the two-element set "$[A;B]$". In order to do this in a more formal way, one can add two axioms which seem to be acceptable to Plato:

**P.10.** $\forall A, B, X.\ [(A \uparrow \boldsymbol{O} \wedge B \uparrow \boldsymbol{O}) \rightarrow (A{=}B \leftrightarrow (A \uparrow X \leftrightarrow B \uparrow X))]$, "upper extensionality axiom",[23]

**P.11.** $\forall A, X.\ (A \uparrow \boldsymbol{O} \rightarrow A = [X : A \uparrow X])$, i.e. $A$ can be identified with the set of all its properties.

In our system $\boldsymbol{F}$, it is unnecessary to accept the **N.1.** because we can formally define (natural) numbers together with the mechanism of the dissection of the One. Coming back to $\boldsymbol{F}$ + **P.10.** + **P.11.**, **P.10.** and **P.11.** in conjunction with **L.8**, entail that the sets $A = [X : A \uparrow X]$ are infinite because, with every *positive* property $A$, they also contain the infinite series $A$, $\sqcap A$, $\sqcap\sqcap A$, … . Thus, we define a *positive* property formally as every property different from a property which is defined with the use of the local negation "$\sqcap$" in front of the defining formula. Furthermore, we can identify the property $A$ with the set of all positive predicates in which this property participates.[24]

The next section of hypothesis II is concerned with some demonstrations that if the One exists and if it is divided by *Being*, it must be a whole consisting of parts, that the One participates in "limit", in "beginning", "end", "middle", (in different kinds of) "shape", "place" (it is in itself or it is in something other), it is the same as itself and it is different from itself, etc. Plato shows that, under the assumption of the second hypothesis, the One participates in all of these contradictory predicates.

The connection with the Dyad is hidden behind the categories "from the greatest to the smallest". The participation of the One in the Dyad and in "$\boldsymbol{Ind}(X)$" necessi-

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[22] Let us notice that "being" is conceived as an idea (of *Being*). Therefore not every idea exists, i.e. not every idea is something existent or it participates in being.

[23] The axiom decides some strong properties of the universum of properties. However, some of them seems to be undesirable. Obviously, from **P.10.** follows a very useful criterion of difference: $\forall A, B, X.\ [(A \uparrow \boldsymbol{O} \wedge B \uparrow \boldsymbol{O}) \rightarrow (\sim (A{=}B) \leftrightarrow \exists X.\ (A \uparrow X \wedge \sim(B \uparrow X))]$.

[24] One can also compare the resulting system with the *positive set theories*; cf. for instance R. Hinnion, *Intensional Positive Set Theory*, "Reports on Mathematical Logic", 2006 nr 40, pp. 107-125; cf. also R. Hinnion, *About the coexistence of `classical sets' with `non-classical' ones: a survey*, "Logic and Logical Philosophy", 2003 nr 11, pp. 79-90.

tates participation in "spatiality", and — in particular — in "limit(s)", "shape", "whole", "part". The spatial One starts to participate in "something other" and in "itself" (understood in a spatial way, i.e. as a "place"). Next, the One participates in "rest" as well as in "motion", in "identity" and in "difference", etc. From 147a, Plato demonstrates that the Dyad does not participate in the One itself (cf. **L.2**), and that the (global) negation of the One itself is the Dyad (and *vice versa*; cf. **P.5.**).

Thus, the goal of the reasoning presented by Plato in the second hypothesis can be summarized as follows. The One is the highest principle because it is at the top of the hierarchy of the relation of participation: there is no predicate which can be predicated on the One. I leave it to the reader to check the details of how the logic **P.0.-P.11.** is used in many other places in the second hypothesis, in the *Parmenides* as well as in other dialogues.[25]

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