Kordula Świętorzecka

Towards Leibnizian Possibility : Formal Frame of Modal Theory of Individual Concepts

Studia Philosophiae Christianae 49/3, 71-84

2013

Studia Philosophiae Christianae
UKSW
49(2013)3

KORDULA ŚWIĘTORZECKA
Instytut Filozofii UKSW, Warszawa

TOWARDS LEIBNIZIAN POSSIBILITY. FORMAL FRAME OF MODAL THEORY OF INDIVIDUAL CONCEPTS[1*]

Abstract. In the presented analyses we propose a formal complement to a certain version of the semantics of possible worlds inspired by Leibniz's ideas and provide an adequate logic of it. As the starting point we take the approach of Benson Mates (*Leibniz on possible worlds*). Mates refers to Leibniz's philosophy, but also uses tools of contemporary semantics of possible worlds and elaborates on an original conception of predication due to which possible worlds can be identified with collections of certain concepts, and not individuals. We complete a fragmentary description given by Mates in order to analyze if his conception allows for the establishment of this specific idea of a possible world. Our first step is to define a notion of the individual concept and describe possible world semantics in which possible worlds consist of individual concepts of compossible individuals (s-worlds). Our second step is to choose some version of modal free logic with the identity (S5MFLID), which is complete in our reformulation of Mates' semantics. The connections between standard interpretation of S5MFLID and semantics inspired by Mates show that our logic does not distinguish s-worlds from i-worlds – counterparts of s-worlds that are collections of individuals.

Keywords: formal ontology, Leibniz, theory of concepts, possible world semantics, modal free logic

---

[1] The paper is a slightly modified translation of my Polish text *O możliwych światach pojęć jednostkowych. Formalna rekonstrukcja koncepcji B. Matesa* [*On possible worlds of individual concepts. A formal reconstruction of the idea of B. Mates*] in: *Nauka i język. (seria druga). Księga pamiątkowa Marianowi Przełęckiemu w darze na dziewięćdziesięciolecie urodzin [Science and Language (Series IInd). Commemorative book for Marian Przełęcki* as a gift of the *ninetieth anniversary of the birth]*, ed. by A. Brożek, J. Jadacki, Norbertinum, Lublin (in print), 125-137.

1. Introduction. 2. Possible s-worlds. 3. Languages and interpretation. 4. Logic. 4.1 System MS_{5C}. 4.2 System S5MFLID 5. From worlds of individual concepts to worlds of individuals.

1. INTRODUCTION

The following analyses raise the issue of modality, which is a compound of the subject and metatheoretical matters: we propose a formal complement to a certain version of the semantics of possible worlds inspired by Leibniz's ideas and provide a logic adequate to it.

Contemporary modal philosophical logics and their set-theoretical interpretations meet with skepticism among philosophers, who claim that such approaches do not capture the intended philosophical content of what is traditionally understood as modality. It is worth highlighting that sometimes such arguments are justified, however, in general, the matter is complicated enough because this philosophical content keeps escaping attempts to be satisfactorily precise. One such attempt was undertaken by B. Mates[2], whose conception is of interest here[3]. Mates' idea is interesting, because it refers to Leibniz's philosophy and also uses tools of contemporary semantics of possible worlds and elaborates on an original conception of predication, due to which possible worlds can be identified with collections of certain concepts and not individuals. Mates, however, provides only a fragmentary account of his idea, and the present work is aimed at completing it in order to analyze if Mates' conception allows for the establishment of this specific idea of a possible world.

2. POSSIBLE σ-WORLDS

The conception considered here is based on the assumption that possible worlds are determined by so called *individual concepts*. Accor-

---

[2] B. Mates, *Leibniz on possible worlds*, in: *Logic, Methodology and Philosophy of Science*, III, Studies in Logic and the Foundations of Mathematics, Logic, ed. B. van Rootselaar, J. F. Staal, Amsterdam 1968, 507–529.

[3] My inspiration comes from the paper of Prof. M. Przełęcki, *On Possibility and Possible Worlds*, Poznań Studies in the Philosophy of the Sciences and the Humanities 4(2010)1–4, 27–36.

ding to Mates, such an approach allows for the justification of certain key theses of Leibnizian theory of possible worlds together with the Leibnizian theory of individual concepts.[4]

Let us reconstruct the initial steps sketched by Mates.

We assume that $D=\{d_i\}_{i\in I}$ is a set of individual entities, which constitute the domain of *the real world*, and are the realizations of individual concepts. Mates assumes that D is infinite.

For each individual d_i, we define its individual concept:

$$\text{Def}\,(\sigma). \qquad \sigma(d_i) = \{X \subseteq D\colon d_i \in X\}.$$

CON is a set of all individual concepts:

$$\text{Def}\,(\text{CON}). \qquad \text{CON} = \{X\colon \exists_{i\in I}\,(X=\sigma(d_i))\}.$$

Of course, if D is infinite then also every concept of d_i is infinite as well as CON. Let us note that there are no concepts in CON which are *unrealized*.

Mates adopts Leibniz's idea of *compossibility* of existence of the objects which constitute possible worlds, and introduces, in the set of CON, the two-argument relation of *compossibility of individual concepts* $\Gamma \subset \text{CON}\times\text{CON}$.

Two individual concepts are in this relation just if they can be both realized. Following Leibiz's idea, two compossible individuals (in our case: individual concepts) are connected in so that they *mirror* each other and any local change in the universe of a possible world is a cause of global change. This idea is mentioned by Mates as a motivation to assume that:

(M Γ) Γ is reflexive, symmetric and transitive in CON

Γ divides CON into equivalence classes:

$[\sigma(d_i)] = \{\sigma(d_j)\colon \sigma(d_i)\ \Gamma\ \sigma(d_j)\}$, which will be called *possible worlds of individual concepts* w_i, w_j, ... (σ-*worlds*):

$$\text{Def}\,(\sigma\text{-worlds}). \quad w_i = [\sigma(d_i)]_\Gamma \qquad \text{for each } i\in I$$

What can be claimed about σ-worlds is:

[4] References to Leibniz's texts and their philosophical interpretations can be found in: B. Mates, B. Mates, *The philosophy of Leibniz. Metaphysics and Language*, Oxford UP, New York – Oxford 1986.

(1σ) $\forall_{wi}(w_i \neq \varnothing)$
(2σ) $\forall_{di}\exists_{wj}(\sigma(d_i) \in w_j)$
(3σ) $\forall_{di}\forall_{wj}\forall_{wk}(\sigma(d_i)\in w_j$ and $\sigma(d_i)\in w_k \Rightarrow w_j=w_k)$

According to (2σ), each individual has an individual concept, which is exemplified by this individual. (3σ) is a necessary condition for the realization of idea of mutually related individuals. Let us note that although no individual concept belongs to two different possible worlds, *attributes* of individuals alone can recur.

In consequence of Def(σ) we may say that:

(0σ) $\exists_X\forall_{dk}(X\in \sigma(d_k))$.

Introduced notions enable us to define *a frame of σ-worlds* as a triple:

Def (F). $F=\langle D, CON, \Gamma\rangle$.

3. LANGUAGE AND INTERPRETATION

Let us now consider the modal language of the first order $J_{\forall C\Box}$, the vocabulary of which consists of: variables (IV): x, y, z, ..., individual constants IC: c_1, c_2, ..., one-argument predicate letters (PRED): P, Q, R, ..., logical symbols: =, ¬, →, ∀, □.

(We use in metalanguage symbols: v, v', ... for individual variables and c, c',... for individual constants.)

The definitions of terms (NM), atomic formulae (AT) and formulae (FOR) are standard.

(For terms we use symbols: τ, τ', ..., and for formulae: A, B,...).

Notation: A(τ/τ') is used to speak about a substitution of τ by τ' in A on every place where τ occurs. If we substitute term τ by τ' in A at least one time we use notation: A(τ//τ').

The set of all free variables in A we name: FV(A) and the set of all constants in A: (A).

We say that A is a sentence iff $FV(A)=\varnothing$.

In our considerations we use also notion of universal closure and notion of modal universal closure of formula A:

Def (UC). Universal closure of a formula A is a formula obtained from A by prefixing it with any sequence of general quantifiers which bound all free variables in A.
If FV(A)=∅, then A is a general closure of itself (so the length of such a sequence may be 0).

Def (MUC). Modal universal closure of a formula A is a formula obtained from A by prefixing it with any sequence of general quantifiers and modal operators for necessity in any order to bind all free variables in A. (Quantity of it may be 0.)

(We will consider also fragments of $J_{\forall C\Box}$, language and names of sets of terms and formulae will be properly completed by indexes.)

To interpret $J_{\forall C\Box}$ language we take already defined the frame of σ-worlds F=<D, CON, Γ>.

Mates chooses a constant function of interpretation and we consider any interpretation g specified by Mates. A function γ is characterized as follows:

(1γ) γ: IC $\rightarrow$ CON
(2γ) γ: PRED $\rightarrow 2^D$
(3γ) $\gamma(=) = \{<\sigma(d), \sigma(d)>: d \in D\}$

We define the validity of formula A in the possible world w_i of the frame F:

(i⊨) $(F,w_i) \models^{\gamma} P(c)$ iff $\gamma(P) \in \gamma(c)$ and $\gamma(c) \in w_i$
(ii⊨) $(F,w_i) \models^{\gamma} c=c'$ iff $<\gamma(c), \gamma(c')> \in \gamma(=)$ and $\gamma(c) \in w_i$
(iii⊨) , (iv⊨) for ¬A and (A→B) are standard
(v⊨) $(F,w_i) \models^{\gamma} \forall vA$ iff $(F,w_i) \models^{\gamma'} A(^v/_c)$,
for $c \notin NM(A)$ and for every $\gamma' \in$ where $\gamma'(c) \in w_i$

(Set $V(\gamma$,c) is the set of all c-variants of γ, i.e. interpretation with all the same values like g with the possible exception for c.)

(vi⊨) $(F,w_i) \models^{\gamma} \Box A$ iff $(F,w_j) \models^{\gamma} A$ for every $w_j \in CON_{\Gamma}$

Instead of speaking about *necessary* sentences – i.e. sentences, which are true in every possible world, we will speak about *valid* formulae in the following sense:

Def (validity). Formula A is valid in (F, γ) iff for every formula B∈MUC(A):

$$(F,w_i) \vDash^{\gamma} B \quad \text{for every } w_i \in CON_{\Gamma}$$

Mates stops at formulating the notion of the necessary sentence. In our formalization we do not consider a constant interpretation, but any function γ and so we can also formulate the notion of *logical validity*:

Def (l-validity) Formula A is l-valid iff A is valid for every valuation γ.

Let us summarize the key ideas of Mates' semantics.
(1) Considered σ-worlds have exclusive universes – Mates' model is the case of the so called *semantics with world-relative domains*.
(2) Individual constants are interpreted globally – they are *rigid terms*.
(3) Extensions of predicates are subsets of the set D, however their interpretation is local due to the condition (i⊨). A local interpretation also concerns = - (ii⊨).
(4) The condition (i⊨) says that atomic sentences describe the relation, which is the reverse of the predication[5].
(5) Quantifiers have actual meaning – they are relativized to a possible world.

4. LOGIC

A formal account of the semantics discussed here will be completed by the suggestion of an appropriate logic. Mates does not axiomatize

[5] Mates' conception is similar to the basic idea of Meinong's semantics elaborated by J. Paśniczek in *Predykacja. Elementy ontologii formalnej przedmiotów, własności i sytuacji* [*Predication. Elements of formal ontology of objects, attributes and situations*], Copernicus Center Press, Kraków (in print). In this case, the interpretation function assigns sets of attributes to name constants, and attributes to predicates, but elements of semantic correlates of name constants need not identify one individual (and, in general, need not be realizations of any individual, since unreal objects are also considered). I owe these suggestions to the Author.

the presented structure of σ-worlds and cannot raise the issue of establishing logic, which could serve as a formal basis for some theory (Mates' semantics operates on the constant function of interpretation, and does not include the notion of logical validity for formulae).

4.1 System $M\Sigma_{5C}$

Mates sees that the logic he prefers should be a modal (S5) extension of first-order monadic quantifier calculus with identity Σ_5 in the version proposed by Kalish and Montague.[6]
System Σ_{5C} is expressed in the $J_{\forall}$ - fragment of $J_{\forall C\Box}$ language without individual constants and modalities. It is characterized by all universal closures of the following shapes:

(01) $(A \to B) \to ((B \to C) \to (A \to C))$
(02) $(\neg A \to A) \to A$
(03) $A \to (\neg A \to B)$
(Q1) $\forall v(A \to B) \to (\forall vA \to \forall vB)$
(Q2) $A \to \forall vA$, where $v \notin FV(A)$
(ID1) $\neg\forall v\neg (v=v')$
(ID2) $v=v' \to (A \to A({}^{v}/_{v'}))$, and $A \in AT_{J\forall}$

The only one primitive rule is *modus ponens*: (MP) $A \to B, A \vdash B$

Following Kalish and Montague we note that a standard interpretation for Σ_5 excludes models with empty domain. System Σ_5 may be weakened to Σ_6 which axiomatics is different from Σ_5 because of the restriction on ID1 – in Σ_6 it is assumed that v and v' should not be of the same shape and this restriction enables the interpretation of Σ_6 also in models with empty domain. It is of course understandable that Mates excludes in his semantics the possibility of speaking about empty possible worlds (cf. (1σ)) but he tries to realize an idea that individual constants do not always name something. For this reason he enriches Σ_5 language by individual constants and he obtains system $M\Sigma_{5C}$ which

[6] Cf. D. Kalish, R. Montague, *On Tarski's Formalization of Predicate Logic with Identity*, Archiv für Math. Logik und Grundlagenforschung, 7/3–4(1963)81–101.

is characterized by all general closures of formulae described by (01)-(03), (Q1), (Q2), (ID1) and also:

(ID2*) $\tau=\tau' \rightarrow (A \rightarrow A(^{\tau}/_{\tau'}))$ for $A \in AT_{J\forall C}$

(AM*) $A(c) \rightarrow \neg\forall v \neg(v=c)$ for $A \in AT_{J\forall C}$

Again we take as a primitive rule in $M\Sigma_{5C}$ only (MP).

Introduced axioms are mentioned to realize Leibniz's intentions. (ID2*) expresses Leibnizian conception of identity (like in Σ_5, it is restricted to atomic formulae, but with possible instantiations for individual constants). (AM*) corresponds to Leibniz's principle according to which, what does not exist does not have any attributes (*nihil nullae propertates sunt*). We should note that Mates does not extend an application of schema (ID1), so he does not accept schema:

(ID1*) $\neg\forall v\neg (v=\tau)$

Actually he does not want to take as theses formulae of the shape: $\neg\forall v\neg (v=c)$. This solution together with the general interpretation of individual constants and actualistic meaning of quantifiers is intended to realize the crucial intention of Mates: not every individual constant names in σ-world w_i some individual concept from w_i domain (*nota bene*: even if all individual concepts have names, we could say that for every individual constant there is some possible world in which it does not name anything, but also every individual constant names something in some world). An original idea of linking actualistic quantification with global interpretation of individual constants and local interpretation of predicates (this effect is realized by (1γ) and ($i\vDash$)) brings difficulties.

At first let us note that every universal closure of the shape:

(OM) $\forall v\, A \rightarrow A$[7],

is a thesis of Σ_5 but it is not $M\Sigma_5$ thesis and it should be restricted to:

(OM*) $\forall v\, A \rightarrow A(^{v}/_{v'})$

because some formulae of the shape (OMC*) $\forall vA \rightarrow A\,(^{v}/_{c})$ are not logically valid[8].

[7] Cf. Ibid.

[8] This is noted also by Mates.

On the other hand, we can easily show that for formula $\neg$A where, $A \in AT_{J\forall C}$ the mentioned schema generates logically valid formulae so it seems that $M\Sigma_{5C}$ is not complete in F.

The reasons why attempts to "improve" Mates' proposals seem to be not attractive are given by Garson[9]: $M\Sigma_{5C}$ involves linking assumptions of a global interpretation of terms with a local interpretation of predicate letters and with the actual conception of quantifiers in a semantics with variable domains of possible worlds, but with classical rules for quantifiers. Garson points to at least two solutions to the encountered difficulties, from which each accepts the conception of variable domains, the global interpretation of terms and predicates (which, however, can have different extensions in different possible worlds) and the actual conception of quantifiers. The first solution eliminates the use of constants and *de re* modalities.[10] The other preserves individual constants in the language, and uses *de re* modalities, but also introduces the primitive predicate E (the extension of which in a given possible world w is the domain of the world w) and modifies rules for quantifiers, leading us to a modal version of free logic. In the present analysis, we choose the second option. To justify this decision, let us note that the enriching of our language by predicate E allows for the expression of the intention of Mates to take a local interpretation of predicates – we will obtain this effect in connection with a later defined interpretation function γ^*.

4.2 System S5MFLID

We provide our analysis in a minimal free logic with identity and S5 modalities proposed by Garson. System S5MFLID is axiomatized by tautologies of classical sentential logic ((01)-(03)) and all $J_{\forall C\Box E}$ formulae of the following forms:

[9] J. W. Garson, *Quantification in modal logic*, in: *Handbook of Philosophical Logic*, vol. 3, Kluwer Ac. Publ., Dordrecht 2001, 267–323.

[10] Such an approach was proposed by S. Kripke in *Semantical Considerations on Modal Logic*, Acta Philosophica Fennica 16(1963), 83–94.

(T) $\Box A \rightarrow A$
(K) $\Box(A\rightarrow B) \rightarrow(\Box A\rightarrow\Box B)$
(5) $\neg\Box\neg A \rightarrow \Box\neg\Box\neg A$
(Q1) $\forall v(A\rightarrow B) \rightarrow (\forall v\, A \rightarrow \forall v\, B)$
(Q2) $A \rightarrow \forall vA$, where $v \notin FV(A)$
(Q3) $\forall vA \wedge E(\tau) \rightarrow A(^{v}/_{\tau})$
(ID0) $\tau=\tau$
(ID2*) $\tau=\tau' \rightarrow (A \rightarrow A(^{\tau}/_{\tau'}))$, for $A \in AT_{J\forall C\Box E}$
(ID3) $\tau=\tau' \rightarrow \Box\ (\tau=\tau')$

Primitive rules: (MP), (RG) $A \vdash \Box A$, and (FUG) $A \rightarrow (E(\tau) \rightarrow B(\tau)) \vdash A \rightarrow \forall v\, B(v)$, τ does not occur in: $A \rightarrow \forall v\, B(v)$ as a constant or a free variable

Let us note that S5MFLID theses are formulae mentioned by Mates as schemata of necessary sentences: S5MFLID does not change the meaning of classical connections, possibility and the existential quantifier may be defined in a standard way:

($\Diamond/\Box$) $\Diamond A\leftrightarrow \neg\Box\neg A$
($\exists/\forall$) $\exists vA \leftrightarrow \neg\forall v\neg A$

S5MFLID theses are also: Kripke's axioms of system M[11], formulae of the following shapes: $\Diamond(c=c)$; $c=c' \rightarrow \Box(A\rightarrow A(^{c}//_{c'}))$ for $A \in AT_{J\forall C\Box E}$ and (AM*/E) $E(c) \rightarrow \neg\forall v\neg(v=c)$ (the substitution of (AM*)).

Garson describes semantics in which S5MFLID is complete.

He considers models of the shape $<W, R, D, Q, a>$, where: W is a set of possible worlds, $R \subseteq W \times W$ is the accessibility relation, which is in our case (because of S5) universal: $\forall_{wi,\, wj}\ w_iRw_j$, D is a non empty set of possible objects, Q: $W \rightarrow 2^D$ and a is an interpretation function such that:

(1a) a: $NM \rightarrow D$
(2a) a: $(PRED \rightarrow(W \rightarrow 2^{D}))$
(2aE) $a(E)(w) = Q(w)$
(3a) $a(=) = \{<d,d>: d \in D\}$

[11] Ibid.

The function a is extended on the set of formulae $FOR_{J\forall C\Box E}$ with values in {1,0} and for formulae with ∀ we have:

(♦) $a(\forall vA)(w) = 1$ iff for every $d \in Q(w)$: $a(^{v}/_{d})(A)(w) = 1$

Schemata of Barcan formulae and converse of them generate invalid formulae:

(FB) $\forall v\Box A \rightarrow \Box\forall vA$, (CFB) $\Box\forall vA \rightarrow \forall v\Box A$

The same applies to schema:

(OMC*) $\forall vA \rightarrow A\ (^{v}/_{c})$ for $A \in AT_{J\forall C\Box}$

Mates also considers FB, CFB and OMC* as schemata of invalid formulae.

Let us now consider the following model:

(i) W is the set $D_{/\backsim}$ generated by the relation $\backsim$:

Def ($\backsim$). $d_i \backsim d_j$ iff $[\sigma(d_i)] = [\sigma(d_j)]$

Elements of $D_{/\backsim}$ are *possible worlds of individuals* (*i-worlds*): w_i^*, w_j^*, ... such that:

(w^*/w) $d_j \in w_i^*$ iff $\sigma(d_j) \in w_i$, for every $i, j \in I$

(ii) D is the set of individuals D

(iii) Q fulfills the condition: $Q(w^*) = w^*$

(iv) a is the interpretation function μ such that:

(1μ) μ: NM $\rightarrow$ D

(2μ) μ: (PRED $\rightarrow$($D_{/\backsim} \rightarrow 2^D$))

(2μE) $\mu(E)(w^*) = Q(w^*)$

(3μ) $\mu(=) = \{\langle d,d\rangle : d \in D\}$

We extend μ:

($i\mu$) $\mu(P(\tau),w_i^*) = 1$ iff $\mu(\tau) \in \mu(P,w_i^*)$

($ii\mu$) $\mu(\tau = \tau', w_i^*) = 1$ iff $\mu(\tau) = \mu(\tau')$

conditions ($iii\mu$), ($iv\mu$) for $\neg A$ and $A \rightarrow B$ are standard

(We use the notation $V'(\mu, c)$ for the set of all c-variants of function μ , i.e. for the set of all interpretation with the same value as μ with the possible exception for c.)

We continue:

$(\mathrm{v}\mu)\quad \mu(\forall \mathrm{v}A, w_i^*) = 1 \quad \text{iff} \quad \mu'(A(^{\mathrm{v}}/_{c}), w_i^*)=1, \quad \text{where } c \notin NM(A),$
$\text{for every } \mu' \in V'(\mu, c), \text{ where } \mu'(c) \in w_i^*$

Condition $(\mathrm{v}\mu)$ is a counterpart of condition ($\blacklozenge$).

For formulae with $\Box$ we have:

$(\mathrm{vi}\mu)\quad \mu(\Box A, w_i^*) = 1 \quad \text{iff} \quad \mu(A, w_j^*) = 1 \text{ for every } w_j^* \in D_{/\infty}$

5. FROM WORLDS OF INDIVIDUAL CONCEPTS TO WORLDS OF INDIVIDUALS

Let us now to compare the S5MFLID model <W, D, Q, μ> with Mates' conception from par. 3. As we recall, Mates considers a constant interpretation function, which fulfills conditions described for any function γ. We modify these conditions in a way to keep the crucial assumptions of Mates as well to interpret S5MFLID. We keep the concept of global interpretation of individual constants and we extend it to individual variables; predicate letters are interpreted locally (this effect is obtained in a different way than in Mates' approach); we repeat the idea of converse predication. We also take an actualistic interpretation of quantifiers. Identity is interpreted globally.

We associate with every function g the function γ^* in the following way:

$(1\gamma^*)\quad$ for every constant c: $\gamma^*(c) = \gamma(c)$
$(1'\gamma^*)\quad$ for every variable v: $\gamma^*(v) \in CON$
$(2\gamma^*)\quad \gamma^*: PRED \times CON_{/\Gamma} \rightarrow 2^D$
$(2'\gamma^*)\quad \gamma^*(E, w_i) = \{d_j: \sigma(d_j) \in w_i\}$ for every $w_i \in CON_\Gamma$
$(3\gamma^*)\quad \gamma^*(=) = \gamma(=)$

We extend g * on the set of $J_{C\forall\Box E}$ formulae and we define truth conditions in the same way as μ. For atomic formulae we have:

$(\mathrm{i}\gamma^*)\quad \gamma^*(P(\tau), w_i) = 1 \quad \text{iff} \quad \gamma^*(P, w_i) \in \gamma^*(\tau)$

We note that in case of predicate E from condition $(\mathrm{i}\gamma^*)$ we get:

$(\mathrm{i}\gamma^*E)\quad \gamma^*(E(\tau), w_i) = 1 \quad \text{iff} \quad \{d_j: \sigma(d_j) \in w_i\} \in \gamma^*(\tau)$

Conditions $(\mathrm{ii}\gamma^*)$ - $(\mathrm{vi}\gamma^*)$ are the same as $(\mathrm{ii}\mu)$ - $(\mathrm{vi}\mu)$.

Now we can say that for any function γ:

$$(F,w_i) \models^{\gamma} P(c) \text{ iff } \gamma^*(E(c)\wedge P(c),w_i) = 1$$

and

$$(F,w_i) \models^{\gamma} (c=c') \text{ iff } \gamma^*(E(c)\wedge(c=c'),w_i) = 1$$

and if A does not contain any constants or free variables we have:

$$(F,w_i) \models^{\gamma} A \text{ iff } \gamma^*(A,w_i) = 1$$

The crucial connection between m and γ^* is:

(μ/γ^*) For any function μ there is γ^* such that: $\mu(A, w_i^*) = 1$ iff $\gamma^*(A, w_i) = 1$

and conversely:

(γ^*/μ) For any function γ^* there is μ such that: $\gamma^*(A, w_i) = 1$ iff $\mu(A, w_i^*) = 1$

Proofs (μ/γ^*) i (γ^*/μ) are inductive. We take any μ and we define γ^* in the same way as μ with the exception that: $\gamma^*(c) \in \gamma^*(P,w_i)$ iff $\mu(P, w_i^*) \in \mu(c)$ and this is valid because of (w^*/w): $w_i^* = \{d_j : \sigma(d_j) \in w_i\}$ and because for every set $X \subseteq D$: $d_i \in X$ iff $X \in \sigma(d_i)$ (cf. Def (σ).). In proving (γ^*/μ) we proceed conversely.

Let us note that the proofs of (μ/γ^*) and (γ^*/μ) are based on Mates' two basic assumptions from which the first is restricting the domain of considered concepts to s-objects – such that each of them always identifies exactly one individual d, and is maximal – all supersets of singleton {d} belong to each of them (cf. Def (σ)). The other key assumption is that Γ determines a partition in CON (domains of σ-worlds are always non-empty and non-overlapping), which makes it easily possible to construct a structure of i-worlds isomorphic to the structure of worlds (cf. (w^*/w)).

The equivalences (μ/γ^*) and (γ^*/μ) allow for it to be noted that the S5MFLID logic does not differentiate between models in which possible worlds are sets of individuals and models in which possible worlds are sets of individual concepts embodied in the individuals. Thus, S5MFLID neither establishes the conception of possible worlds as a collection of individual concepts, nor does it exclude, and hence, it can be a formal basis for a specific theory describing such an idea.

REFERENCES

Garson J. W., *Quantification in modal logic*, in: *Handbook of Philosophical Logic*, vol. 3, Kluwer Ac. Publ., Dordrecht 2001, 267–323.

Kalish D., Montague R., *On Tarski's Formalization of Predicate Logic with Identity*, Archiv für Math. Logik und Grundlagenforschung 7/3–4(1965), 81–101.

Kripke S., *Semantical Considerations on Modal Logic*, Acta Philosophica Fennica, 16(1963), 83–94.

Mates B., *Leibniz on possible worlds*, in: *Logic, Methodology and Philosophy of Science*, III, Studies in Logic and the Foundations of Mathematics, Logic, ed. B. van Rootselaar, J. F. Staal, Amsterdam 1968, 507–529.

Mates B., *The philosophy of Leibniz. Metaphysics and Language*, Oxford UP, New York – Oxford 1986.

Paśniczek J., *Predykacja. Elementy ontologii formalnej przedmiotów, własności i sytuacji* [*Predication. Elements of formal ontology of objects, attributes and situations*], Copernicus Center Press, Kraków (in print).

Przełęcki M., *On Possibility and Possible Worlds*, Poznań Studies in the Philosophy of the Sciences and the Humanities 4(2010)1–4, 27–36.

Świętorzecka K., *O możliwych światach pojęc jednostkowych. Formalna rekonstrukcja koncepcji B. Matesa.* [On possible worlds of individual concepts. A formal reconstruction of the idea of B. Mates], in: *Nauka i język (seria druga). Księga pamiątkowa Marianowi Przełęckiemu w darze na dziewięćdziesięciolecie urodzin [Science and Language (series IInd). Commemorative book for Marian Przełęcki as a gift to the ninetith anniversary of the birth]*, ed. A. Brożek, J. Jadacki, Norbertinum, Lublin, 125-137 (in print).