Roxanne Marie Kurtz

Quinean Ontological Commitment Derailed

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Abstract

What should we believe exists? The Quinean response is straightforward: We should believe in all and only those objects over which we must quantify in our best scientific theories. Let us call this view Ontological Commitment = Quantifier Commitment, or **OC=QC**. The author draws upon resources from Jody Azzouni and Stephen Yablo, who reject this criterion to disrupt a central argument for platonism in mathematics. The project has two parts. First, the negative project is to argue that **OC=QC** is misguided because we ought not read our ontological commitments off of our quantifier commitments. Second, the positive project is to suggest an alternative criterion to **OC=QC** that allows us to accept the idea that statements that quantify over mathematical objects that would be abstract if they existed are indispensable to our best scientific theories, but nevertheless reject the existence of numbers.

Keywords: ontological commitment, indispensability thesis, fictionalism, platonism, formalism, thick epistemic access, existence of numbers, metaphor

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1. The task at hand

This is a discussion on metaphor, language, and the existence of numbers. What should we believe exists?[1] The Quinean response is straightforward: We should believe in all and only those objects over which we must quantify in our best scientific theories. Let us call this view Ontological Commitment = Quantifier Commitment, or **OC=QC**.[2]

    If we accept **OC=QC**, then we get a straightforward argument that if we trust science, then we should believe in the existence of mathematical objects:[3]

> **OC=QC:** We should believe in the existence of all and only those objects over which we must quantify in our best scientific theories.

> **INDISPENSABILITY:** We must quantify over mathematical objects in our best scientific theories.

> **C1:** So, we should believe that mathematical objects exist.

> **ABSTRACTNESS:** If mathematical objects exist, then they are abstract objects.

> **PLATONISM:** Thus, we should believe that abstract objects (including numbers and other mathematical objects) exist.

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[1] Note that this is a different question from 'what exists?' Perhaps on the planetoid Pluto there are cute, purple-feathered flying orange fish. If so, I take it we have no good reason to believe in their existence.

[2] **OC=QC** blends two theses: (1) we should believe in the existence of those things that the theories we believe say exist and (2) we should believe our best scientific theories. Thanks to an anonymous reviewer who pressed me on this point, correctly assessing that my target is (1), and noted that I followed cases in the literature in which the two claims are stated as a combined thesis. I agree. I chose to use **OC=QC** rather than (1) to sidestep exegetical questions about Azzouni's work and to bypass questions that may arise when we pull them apart. For instance, we might ask what counts as a theory that is not also a scientific theory, and we might wonder why we "ought" to believe in the things of which our theories speak if they are not our best theories. Because these issues need not be settled for my discussion to proceed, I took the simpler path. But thanks again to the reviewer for the encouragement to take up this topic in a further paper.

[3] This is simply a version of an indispensability argument for the existence of mathematical objects that cashes out "indispensability" in terms of quantifier commitment. For a useful discussion of such arguments see Colyvan (2011).

I invite you to join me in accepting **Indispensability**[4] and **Abstractness** for the sake of argument**.** Our aim here is to seek good reasons to deem **OC=QC** untenable and to thereby disrupt the justification for **Platonism**. Our method is exploratory: We consider how work by Jody Azzouni on quantifier commitments and the powers of observation to yield thick epistemic access to concrete objects might complement Stephen Yablo's work on hermeneutic fictionalism/figuralism with respect to number talk in the sciences, and vice versa. We have a negative project: to argue that **OC=QC** is misguided because we ought not read our ontological commitments off of our quantifier commitments. We also have a positive project: to suggest an alternative criterion to **OC=QC** that allows us to accept **Indispensability** and **Abstractness**, yet to reject **C1** and so **Platonism**.

To give just a taste of context on these two authors, Azzouni is a nominalist who not-so-controversially holds that mathematical objects do not exist (nor subsist), yet he is also a formalist who quite controversially claims that statements that quantify over mathematical objects can nevertheless be literally true. For, he thinks, we can have a coherent notion of truth that appeals to formalization alone.[5] In contrast, I think we can fairly say that Yablo is neither nominalist nor platonist. He quite controversially holds instead that in at least some important cases, there is no fact of the matter as to whether abstract objects exist. He also (less controversially) holds, like many other fictionalists/figuralists, that statements that quantify over mathematical objects are not (or perhaps more weakly, are not meant to be taken as) literally true, but nevertheless that they remain useful as their important content is given by their metaphorical or figurative content.[6]

Despite the clear differences between the two philosophers, they both (largely) take **Indispensability** and **Abstractness** as given and locate the problem with the argument for **Platonism** with the untenability of **OC=QC**. They concur that issues with how and why we quantify over mathematical objects in our best scientific theories make it the case that we ought not

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[4] This is an empirical claim about science that we will take as given, though it faces some strong opposition. Hartry Field, for instance, offers strong arguments with compelling examples to show that we may well be able to eliminate talk of mathematical objects in science. See Colyvan (2012) for a discussion.

[5] See Azzouni (2005 and 2011).

[6] Yablo (2010) and Turner (2012).

read our ontological commitments off of the quantifier commitments of our theories. In this way, the views of Azzouni and Yablo are congenial with respect to what we should believe exists.

First, I briefly recount the highlights of Yablo's argument that our metaphorical use of language makes equating the quantifier commitments of a first order theory (with all nonessential metaphors paraphrased away) with its ontological commitments a dubious project. Next, I put forward Azzouni's picture of ontological commitment that distinguishes between those entities that theories quantify over because of the nature of the world, and those that they quantify over because of the nature of language. With these views on the table, I then look at how they provide mutual support against **OC=QC** and in favor of Azzouni's notion of thick epistemic access as a criterion of ontological commitment. Finally, I sketch some possible replies from this fortified position to the philosopher who bucks at the new version of ontological commitment and to the mathematician who insists that she does have thick epistemic access to mathematical objects.

2. Yablo's make-believe

> *I was trying to find a way of being unserious about theoretical entities in math*
> *that didn't force me to take a similar view of theoretical entities in physics.*
> Stephen Yablo in a 2011 interview.[7]

Let us take two key points away from Yablo's "Apriority and Existence" and "A Paradox of Existence."[8] First, Yablo argues that we ought to take statements that quantify over platonic objects like numbers figuratively: We ought to understand them as existential *metaphors* rather than commitments, with their value to scientific theories reducing to metaphorical truth, utility or aptness. Second, because of the challenges of unscrambling metaphorical and literal quantifiers within scientific theories, Quine's criterion for ontological commitment will not work: We ought not read our ontological commitments off of a theory's quantified statements when some ought to be taken metaphorically.

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[7] Bateman (2011).

[8] Yablo (2000a; 2000b).

In broad strokes, Yablo defends his position by offering it as a solution to a dilemma. If we hold statements that quantify over platonic objects to be literally true, then existence proofs for platonic objects become immediate and trivial thanks to straightforward a priori bridge principles conjoined with uncontroversial a priori or a posteriori premises. The triviality of such proofs is an untenable result for the philosopher who either accepts Quine's position that a successful argument for the existence of platonic objects must be an a posteriori, holistic effort that shows why the kind of entity in question is required by the best version of the theory at hand, or believes that we can prove the existence of platonic objects only through sophisticated a priori arguments. On either horn, existence proofs for platonic objects ought not to be so easy. Let us step back a bit to see how he arrives at this position.

To Yablo, objects are platonic "relative to an area of discourse due to the combination of something positive—the discourse depends for its truth-value on how objects like that behave—with something negative—the discourse is not about objects of that type."[9] So, relative to the claim, 'there are as many black dogs as red dogs in my house,' numbers are platonic objects because: (a) the truth value of the claim hinges on whether the number that numbers the black dogs in my house also numbers the red dogs, and (b) the claim concerns my mutts, not some kind of mysterious entities called numbers. Now, here is the rub:

(1) There are as many black dogs as red dogs in my house.
(2) There are as many black dogs as red dogs in my house iff the number of black dogs in my house also numbers the red dogs.

(3) Thus, the number of black dogs in my house also numbers the red dogs, *and further, numbers exist*!

This argument is valid and apparently sound—we may easily establish the truth of (1)[10] and we seem to accept the truth of (2) on a priori grounds. But if things are as they appear to be, then the proof for the existence of numbers becomes unfortunately immediate. Yablo neatly avoids this outcome by denying the literal truth of bridge principles like (2). To make his position

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[9] Yablo (2000b: 199).
[10] Yablo (2000b: 202) also shows how this difficulty may arise when the first premise is an a priori truth. His example: an argument is valid iff it lacks counter models; there is an invalid argument; thus, counter models exist.

plausible, he suggests an alternative to (2) along with a sustained argument meant to assuage our reluctance to give up (2).

Yablo notes that one strategy we might use to deny that the proof for numbers is trivial is to require an a posteriori justification for (2). If so, then the truth of (2) would depend on the ontological status of the platonic objects in question. Yablo resists this move, however. He holds, and let us concur, that we do not wait for the answers to ontological questions before committing ourselves to (2). We simply do not agree that (2) is "'hanging by a thread' until the empirical situation sorts itself out."[11] Further, the strength of our beliefs about bridge principles like (2) does not vary with the strength of our beliefs that the relevant platonic objects will be shown to exist through empirical investigation. If our justification for (2) were a posteriori, then we could not remain so carefree about these matters.[12] Presumably, this message generalizes to other claims about platonic objects, such as those we target in this paper. In particular, we do not care whether or not objects like number exist when we endorse claims that quantify over numbers in our scientific theories.

Instead, Yablo invokes the idea of quantifiers that are not meant to be taken literally to block the easy proof of the existence of numbers—on his view we speak metaphorically or figuratively when we say that there is an x such that x is a platonic object. He argues that because bridge principles employ existential metaphors, "we were never committed in the first place to their truth."[13] Thus, we ought to see (2) as a metaphorical claim that literally says:

(2*) There are as many black dogs as red dogs in my house iff, *supposing that numbers exist*, the number of black dogs in my house also numbers the red dogs.

Arguably, (2*) has the virtue that the ontological status of numbers is irrelevant to its truth, yet it still does the theoretical work that we require from a bridge principle.

To support (2*) as an alternative to (2), Yablo responds to three objections from those who believe that the philosophical benefits of accepting

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[11] Ibid.: 200.

[12] Ibid.: 201–202.

[13] Ibid.: 202.

(2) as literally true force us to accept the ontological consequences. First, critics argue that if platonic objects offer significant power to clarify our discourse that it would otherwise lack, then we ought to embrace these objects and not shy away from the associated ontological commitments. Yablo replies that increased clarity in our discourse is owed to our *conception* of platonic objects in itself. The existence of platonic objects would add nothing to enhance these conceptions as tools of clarification.[14]

Second, a critic might claim that bridge principles like (2) are necessary to metalogical proofs that we cannot do without. As an example, Yablo considers a proof that includes a bridge principle that quantifies over models which yields the conclusion that if an argument is valid, then any argument with the same conclusion and premises will likewise be valid. Yablo counters that while it seems that such proofs depend on the truth of the bridge principle, instead this feature of valid arguments stems from our notion of validity in itself. An argument that employs a bridge principle modified à la (2*) can bring this out and, "[o]nce again, we gain as much purchase on the content of the concept by aligning it with a condition on assumed objects as would be gained by treating the objects as real."[15]

Third, and perhaps of most concern, is the claim that if we want our discourse to be seen as objective, then we must accept the platonic objects it references into our ontology. For, what chance do theories have of being objectively true if they refer to nonexistent things? To diffuse this objection, Yablo poses a dilemma to those who insist we ought to accept that platonic objects exist for the sake of objectivity. Either we have a complete conception of the nature of the platonic objects at issue, or we do not. Suppose we do: Our conception determines an objective answer as to the truth or falsity of a bridge principle like (2*), so we do not a need principle like (2). Suppose we do not: We will need to determine the truth or falsity of bridge principles like (2) and this will require empirical investigation of the platonic objects mentioned, but this takes us back to the untenability of a posteriori justifications for such bridge principles discussed above.[16]

Having offered significant arguments in defense of the claim that we basically pretend that platonic objects exist when we say things like (2) and

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[14] Ibid.: 203–205.

[15] Ibid.: 205–207.

[16] Ibid.: 209–211.

that instead we literally mean (2*), and furthermore, that claims like (2*) will do the work philosophers need from a bridge principle, Yablo (playing with a crime metaphor) asks about means—how this subterfuge sneaks in; motive—what good the pretense does; and opportunity—why this make-believe goes unnoticed by philosophers.

As to means, Yablo argues that some figurative speech is so embedded in our language, and so ordinary, that we miss its figurative nature. It is easy to see the metaphor when we speak of Italy as a boot. It is less easy to catch figures of speech that are "too familiar, insufficiently picturesque, too boring," such as, "[t]hey put *a lot of hurdles* in your path, there's *a lot* that could be said about that, there's *no precedent* for that, *something* tells me you're right, *there are some things* better left unsaid…" Thus, it is reasonable to believe that we may similarly overlook the mundane existential metaphors that he holds occur with references to platonic objects.[17]

As to opportunity, first he demonstrates that attending too much to the nature of platonic objects leads us away from the subject matter—consider above: My subject was red and black dogs, not numbers. Thus, to avoid missing the point, we allow platonic objects to remain "unobtrusive."[18] Second, he argues that the content of some metaphors remains unsettled, and that one of the possible contents is the literal content in the case of what he calls a "maybe metaphor. For instance, maybe I speak literally maybe I speak figuratively when I say that the number of orange fish is two. In such cases, the difficulty of seeing the figurative nature our utterances arises because the possible literal content obscures the possible metaphorical content.[19] I will revisit this kind of metaphor again later.

As to motive, Yablo identifies three ways in which metaphors may benefit us: They may be representationally essential, cognitively advantageous or anticipatorily truthful.[20] A metaphor is representationally essential if the language of discourse does not allow the content to be expressed literally. His example—the literal content of "[t]he average star

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[17] Ibid.: 212–214. Also note that over time, Yablo increases his emphasis on his figuralism (rather than mere metaphor). See his collection *Things* 2010, and Turner's (2012) helpful discussion of the evolution of Yablo's thought in his review of *Things*.

[18] Ibid.: 218–219.

[19] Ibid.: 221.

[20] Ibid.: 214–218.

has 2.4 planets"—is inexpressible in English without the use of platonic objects (for instance, the average star or numbers) because the required infinite disjunction is an impermissible construction ("[t]here are 12 planets and 5 stars or 24 planets and 10 stars or…").[21]

A metaphor is cognitively advantageous to Yablo if it allows us to grasp the intended content more fully because its presentational force stimulates important psychological processes or promotes productive thought due to its suggestiveness (re: the average star, "then how many electrons does the average atom have?"), or helps us reason by making our thoughts logically tractable (the Davidsonian move of going from 'my red dog barked' to 'there was a barking done by my red dog' yields significant logical benefits).[22]

A metaphor that I have described as anticipatorily truthful is one "in which the speaker's sense of the potential metaphorical *truthfulness* of a form of words outruns her sense of the particular truth(s) being expressed," and may be pregnant, prophetic or patient. [23]

Yablo gives 'the state is an organism' as an example of a pregnant metaphor.[24] Notice how, upon continued reflection, the metaphor delivers new contents: the interdependency of the parts of a state; the dependence of a state's survival upon what happens in a larger world and so on.

A prophetic metaphor is one whose content is determinate but becomes evident only over time: Yablo again—someone tells Macbeth that "none of woman born" will harm him. But we, and Macbeth, see the particular truth expressed only at the end of the play.[25]

Yablo offers number talk as an example of a patient metaphor, that is, a metaphor whose initial content is open and remains open until and unless we find one interpretation to be superior. If we see (2) as a patient metaphor, then, depending on how the world turns out to be, the content of (2) might be that which it literally expresses or it might be that which (2*) literally expresses and (2) metaphorically expresses.[26]

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[21] Ibid.: 217.

[22] Ibid.: 217–218.

[23] Ibid.: 220.

[24] Ibid.: 217, footnote 37.

[25] Ibid.: 217, footnote 38.

[26] Ibid.: 220–221

This sketch illustrates how Yablo seeks to support the position that platonic objects may remain useful even if we take them to be objects of existential metaphor and to explain how so many philosophers have been snookered into taking references to platonic objects literally. Next, Yablo strengthens the intuitive plausibility of his position by bringing out several properties that some platonic objects have in common with some make-believe objects. These include paraphrasability (references to either kind of object may often be eliminated from the discourse without "felt loss of subject matter"), impatience (if someone questions whether the platonic or pretend object of which we speak exists, we become impatient), translucency (when making sense of claims that reference such an object, the object often fails to be part of the content), insubstantiality (any property ascribed to such an object generally follows from our conception of it), indeterminacy (the identity conditions of such objects may be indeterminate), silliness (questions about aspects of such an object that are not answered by what follows from our conception of it seem silly), expressiveness (reference to such objects may allow us to express things about other entities not otherwise expressible), irrelevance (such objects may be used to explain facts that would continue to hold in their absence), disconnectedness (usually, we attribute no causal power to such objects) and availability (our only access to such objects is through our conceptions of them, not through the world).[27] To foreshadow, I will suggest this list can help us flesh out Azzouni's criterion of ontological commitment.

We now have Yablo's view, presented, defended and supported. If he is right about our extensive use of existential metaphor, then, as he recognizes, **OC=QC** is in trouble. For, the philosopher who wants to use **OC=QC** would first need to strip theories of figurative speech before determining her commitments.[28] But, it looks as if this may turn out to be an unrealistic project, given that the figurative speech in a theory is often pretty much invisible, and even when identified it may be near impossible to eliminate because of its being representationally essential, cognitively advantageous or anticipatorily truthful. Yablo makes this point concisely: "Quine lost the debate, in my mind, because he trusted science to drive out the metaphors,

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[27] Ibid.: 224–226.
[28] Yablo (2000a).

leaving behind the literal truth. I didn't see why metaphorical entities couldn't earn a permanent place in our theories."[29]

So, we see that Yablo gives us reason to let go of (2). His view also offers an explanation as to why it might be reasonable to deny the existence of numbers even though our best scientific theories quantify over them: (i) our conceptions of numbers serve us equally as well as platonic numbers, and, (ii) our continued metaphorical use of numbers is fruitful. Whether or not this captures the value of math in science is debatable. Certainly, Azzouni and Yablo diverge here. But set that aside. The point I wish to make here is that even if we get on board with Yablo at this point, we are left without a positive criterion for ontological commitment. It is here that I think Azzouni's thick epistemic access as a criterion of ontological commitment meshes especially well with Yablo's hermeneutic fictionalism. If something is not pretend, after all, we should be able to interact with it in the material world (or at least understand why it is part of the material world even if we cannot do so.)

3. Azzouni's untouchable numbers

> *One of the little tragedies of my early childhood is that I never saw a hobbit;*
> *and because I did not, I regretfully had to conclude that there were no such things.*
> Jody Azzouni[30]

In "Thick Epistemic Access: Distinguishing the Mathematical from the Empirical,"[31] Azzouni argues against the Quinean view that a first order theory's quantifier commitments[32] are, in themselves, sufficient to establish its ontological commitments. Rather, he contends that quantifiers range over a theory's posits, some of which enter the theory because of facts about the

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[29] Bateman (2011).

[30] Azzouni (1997: 472).

[31] Azzouni (1997).

[32] In footnote 5, Azzouni (1997), explains his use of 'quantifier commitment': "Be forewarned: while Quinean doctrines are being denied, I shall not use the term 'existential commitments' to describe implications of a theory which are in the form of an existential quantifier followed by a formula with one free variable; rather, I shall adopt more neutral terminology: 'quantifier commitment.'"

world and some of which enter the theory because of facts about language. Ontological commitment ought to follow from quantifier commitment only in the case of some of the former kind of posits—those to which we have thick epistemic access.

Azzouni initially considers observation as he develops the notion of thick epistemic access (**TEA**) to demonstrate the difference between these two kinds of posits. Observation via our sensory experiences is a form of thick epistemic access in virtue of (at least) four epistemic properties. Fuller descriptions will follow, but for now I will give a very concise gloss of these. Our observations are robust—they do not hinge (too much) on our expectations. They are refinable—we can get a better look at things using appropriate strategies. We can use them to monitor and track an object's properties through time. And finally, observations can ground our explanations of why we can or cannot know things about what properties an object does or does not have.[33]

Now, it would be hard to deny a theory's ontological commitment to posits to which the theorist, via observation, has thick epistemic access (and I will ignore philosophers who would want to); but clearly the ontological commitments of many theories go beyond things that we can directly observe. This is not a problem for Azzouni, as he asserts that there are other forms of thick epistemic access: for one, access to objects through the use of instruments. Such instrumental access can possess all the virtues of observation enumerated above (although the robustness of instrumental access derives from the robustness of observation), thereby delivering the possibility of thick epistemic access to unobservable objects such as electrons.

Azzouni contrasts thick epistemic access to thin epistemic access. A posit to which we have thin epistemic access is any posit in a theory that has the Quinean virtues of "simplicity, familiarity, scope, fecundity, and success under testing." Quine's criterion of ontological commitment amounts to commitment to any posit to which we have thin epistemic access; that is, it amounts to quantifier commitment.[34] On this view, Azzouni points out, the ultimate epistemic ground for claiming that a posit exists is the same for

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[33] For three discussions on thick epistemic access, see especially Azzouni (1997: 474–476; 2004a: 129; 2004b: 383).

[34] Azzouni (1997: 479).

every posit in a theory; thus, to claim that some posits exist while others do not would be arbitrary.[35] To hold that to be is to be the value of a variable is to hold that the values of all variables are on equal footing in terms of ontological commitment.

Azzouni aims to use this distinction between thick and thin epistemic access to explain how we might reasonably be ontologically committed to electrons but not to mathematical objects. Few would argue against the idea that thick epistemic access to an object is sufficient for our being ontologically committed to its existence; and clearly, we have no such access to things like numbers, so his project looks promising.[36] However, to go further and claim that thick epistemic access is necessary for our being ontologically committed to an object's existence would be to make the mistake of the verificationist. Azzouni carefully avoids this error by introducing a defeasability condition: "a theoretical item can be one we think exists, even if we do not have appropriate instrumental access to it, provided that we can tell a decent story, in terms of its properties, about why we cannot have such instrumental access to it."[37] (Azzouni calls these defeasibility stories.) So, although we have no thick epistemic access to the Higgs boson,[38] it is not untenable to assert its existence because, first, we can explain our lack of thick epistemic access and, second, we have other good reasons to think such particles exist. We might now, as a first stab, say that the following is Azzouni's criterion for ontological commitment:

> **TEA**: Thick epistemic access is sufficient for ontological commitment. Further, we may maintain the existence of any posit that we believe has properties that let us tell a defeasability story about why we cannot gain thick epistemic access to it.

But this is too easy. As Azzouni himself recognizes, in terms of ontological commitment, **TEA** once again puts mathematical objects on equal footing in terms of ontological commitment with the Higgs boson,

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[35] Ibid.

[36] Ibid.

[37] Ibid.: 480.

[38] Let us pretend this is before CERN's recent announcement concerning the Higgs boson and we do not yet have instrumental access to it. We can always substitute any scientific posit to which we lack thick epistemic access but still hope to find it.

though not atoms—that is, with any posit to which we currently fail to have thick epistemic access. After all, we do have a defeasability story: mathematical objects exist outside of space and time, thus, of course we have no route to thick epistemic access of them.[39] Azzouni responds to this difficulty by arguing that we ought not to take such a defeasability story seriously because, "one already understands that, of course, they [mathematical objects] are not the sorts of things one would even want to get in instrumental or observational contact with."[40] Thus, we need to recast **TEA** as:

> **TEA***: Thick epistemic access is sufficient for ontological commitment. Further, we may maintain the existence of any posit that we believe has properties such that **we ought** to tell a defeasability story about why we cannot gain thick epistemic access to it.

For this response to be anything but ad hoc, Azzouni must provide a criterion for when we ought to tell a defeasability story and when we ought not. This he does in the final section of his paper by separating posits that enter our theories because of how the material world is from those that enter because of how our language works. He uses the oft-cited example of numbers to suggest how it could be the case that all mathematical objects are of the latter type (though even if we accept this example, he does not suggest that this serves as a proof for such a claim). For any upper limit on the number of Fs and Gs, first order logic can express that the number of Fs is the number of Gs without quantifying over numbers—suppose, at most, there are two Fs and Gs, then we may say:

$$\{\neg\exists xFx\wedge\neg\exists xGx\}\vee\{\exists x[Fx\wedge\forall z(Fz\rightarrow z=x)]\wedge\exists x[Gx\wedge\forall z(Gz\rightarrow z=x)]\}$$
$$\vee\{\exists x\exists y[Fx\wedge Fy\wedge x\neq y\wedge\forall z(Fz\rightarrow(z=x\vee z=y))]\wedge\exists x\exists y[Gx\wedge Gy\wedge x\neq y\wedge\forall z$$
$$(Gz\rightarrow(z=x\vee z=y))]\}$$

However, because of the inexpressibility of infinite disjunctions in first order logic, we cannot say that the number of Fs is the number of Gs without establishing an upper limit first. This linguistic limitation forces

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[39] Azzouni (1997: 480).

[40] Ibid.

our use of numbers in many first order theories.[41] Azzouni's point is that in such cases, "we are not implicitly using an inference to the best explanation, hypothetical-deductive methods, or any other traditional scientific method for establishing the existence of something."[42] Thus we get the basis for the normative claim that Azzouni needs: If something is a posit merely because the limitations of language force it into our theory, then we ought not try to tell a defeasability story about it.

4. Why team up to derail the Quinean train?

As discussed early on, Azzouni and Yablo sharply disagree on some key questions regarding mathematical objects, especially with respect to the truth conditions of mathematical claims when we take them literally and the facts about the existence of mathematical objects. Nevertheless, I hope my discussion thus far has suggested to the reader that we may still find reasons informed by and consistent across both positions to pursue our tasks of rejecting **OC=QC** and exploring a possible alternative criterion. Letting the theories inform each other seems particularly productive because (in some sense) they are two sides of the same coin. I think it is fair to say in terms of ontological carving work, Azzouni's approach focuses on the distinctiveness of concrete objects while Yablo's approach focuses on special features of platonic objects.

5. OC=QC: Off-track

One reason we can combine views to call **OC=QC** into question is the congeniality of Azzouni's view to Yablo's idea of mathematical objects as metaphorical objects essential to the expressive power of language. We see this quite clearly when Azzouni writes that we quantify over mathematical objects because "the language we use to describe things has to be formulated

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[41] Azzouni gives an interesting second example from geometry, which I will not rehearse here.

[42] Azzouni (1997: 482).

a certain way if we want greater facility to manipulate descriptions of objects and recognize similarities among them…"[43]

The important move that Yablo and Azzouni share is to separate quantifier commitment from ontological commitment because we ought not to treat all quantifiers alike. Azzouni argues that if some posits are indispensable because of how language is rather than how the world is, then we ought not to be ontologically committed to them, regardless of our use of quantifiers. Yablo explains more fully why we may permissibly fail to follow quantifiers blindly to our ontological commitments by bringing out that the quantifiers in claims that enter theories because of language alone are metaphorical and can be representationally essential. Further, he smoothes the way a bit for our acceptance of the failure of **OC=QC** with a discussion of how such posits enter language, how they remain in use regardless of our attempts to remove them from scientific theories and why we might not want to remove them in any case.

Further, if we read Azzouni in a way that allows some quantifiers to be metaphors, then Yablo's fuller account of the benefits of using existential metaphors in our theories adds depth to the notion of a posit entering theory for reasons of language alone. For instance, Yablonian considerations of the value of cognitively advantageous or anticipatorily truthful metaphors ought to strike scientists as very appealing when designing a fruitful theory (though it may leave some questions unanswered when patient metaphors are used). Permitting such existential metaphors into a theory may make the ideas within the theory more easily and fully communicated and their logical connections better understood, encourage ongoing creative thinking, allow work to progress until relevant facts become evident, allow work to progress without regard to the truth or falsity of claims irrelevant to the subject matter, and allow work to progress toward "something right" before all the details are known.

Recognizing the diverse benefits of existential metaphors brings out how useful they may be in trying to make a theory simple, familiar, of useful scope, fecund and successful under testing (to borrow from Quine). And this fact can help someone who likes Azzouni's approach (even rejecting his formalism) explain why we fail to have thick epistemic access to far more

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[43] Ibid.: 482–483.

posits mentioned in important bits of critical theories than those that can be explained away by reference to purely representational concerns alone.

Then, Azzouni delivers a final blow by providing a suitable alternative to **OC=QC** that does not make us look at a theory's claims and divide them into two piles—one of literal claims and one of figurative claims—and then reformulate the theory without using claims in the second pile before deciding what there is. This is especially important given Yablo's position that we might simply never be able to eliminate figurative claims from our best scientific theories. And why should we strive to, given their utility and fruitfulness, once we have other means of arriving at our ontological commitments?

A Quinean defender might want to argue that Azzouni's criterion could serve as a way to distinguish metaphorical from non-metaphorical claims and that Quine could revise his criterion to limit ontological commitment to posits mentioned in our best theory's non-metaphorical claims. But this seems to distort Quine's intentions too much. Using Azzouni's criterion in such a way would mean that the revised Quinean criterion would require us to look at the world with one eye and study our theory with the other, to say what there is. Further, even if it were an acceptable distortion, it would amount to Azzouni's criterion with a Quinean brand.

6. Contrasting Silly Access to Platonic Objects Thick Epistemic Access

Some philosophers who otherwise might be moved by Yablo's arguments may resist coming fully to his side because it would seem to leave them without a viable theory of ontological commitment—given the stubborn difficulties we face in isolating literal claims from metaphorical claims. Not only does Azzouni's criterion fill this void, it cleanly avoids the problems for Yablo's view that Quine's view creates. In separating quantifier commitment from ontological commitment, metaphorical existential claims become ontologically unproblematic. On Azzouni's view we look not to the quantifiers, but to our access to things in the world, to decide our ontological commitments. And this seems fair enough.

Indeed, it appears probable that philosophers have misread the evidence when, to use Yablo's metaphor, they believe that there is a crime

to be investigated—the smuggling of metaphorical claims into our theories. Scientists, or normal people anyway, may never have wanted to believe that all things we need to quantify over exist. To repeat an earlier point, **TEA*** goes a long way toward making this claim tenable because it offers a principled way to identify the kind of posits that Yablo calls platonic objects. And it does so without a reliance on our intuitions about the roles of various bits of language or on our creativity (or lack thereof) to find solutions that dispense with them.

Moreover, if we carry our Yablonian intuitions over to our musings on **TEA***, we might uncover more features of thick epistemic access or better understand some of those already put on the table by Azzouni. I try to capture some of the possible back-and-forth in the table below. The table offers a fuller description of Azzouni's four features of observation, in virtue of which it counts as a form of thick epistemic access, along with some connections to Yablo. Beyond the four features already discussed, I have expanded the list. Yablo's discussion of the similarities between platonic objects (which I take to be the same thing as the posits of a theory to which we fail to have thick epistemic access) and make-believe objects suggests properties of observation that look to be additional properties in virtue of which observation is a form of thick epistemic access because of opposites (though I do not mean to suggest that either Azzouni or Yablo would go along with all of these indicators of **TEA***).

To motivate the idea that Revealingness is an important property of observation in virtue of which we gain thick epistemic access to the things observed, contrast our observation of the activities of some members of a species that reveals new things about the species with our study of numbers from which "[a]ll the really important facts … follow from (2nd order) Peano's Axioms."[44] In the latter case, anything we learn about numbers follows from our concepts of them. The point here is that if a theory mentions a posit for reasons of language alone, then discoveries about that posit should not extend beyond those implied by our concept of what the posit would be like. With respect to Contingency on Existence, this merely highlights that our observations of non-platonic objects stop if they cease to exist, in contrast to platonic objects.

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[44] Yablo (2000b: 225).

Indicator of Thick Epistemic Access	Azzouni	Yablo
Robustness: although some of our observations are mistaken, affected by training or understood only if given some theory, in the main, our observations are robust across most circumstances and "largely independent (epistemically speaking) of what the recipient(s) expect" to observe (Azzouni 2004a: 129).	Original component of Azzouni's thick epistemic access. For three formulations, see especially Azzouni (1997: 474–476; 2004a: 129; 2004b: 383).	Yablo helps us contrast the robustness of observation with our beliefs about things like numbers which are only *conceptually available* to us. Our observations of mathematical objects are robust only in the trivial sense that they remain steady in their absence.
Refinability: we can refine and adjust our observations—for instance, by moving closer to an object—to help us distinguish between a thing's true properties and properties it may merely seem to have because of perceptual error.	Original component of Azzouni's thick epistemic access. For three formulations, see especially Azzouni (1997: 474–476; 2004a: 129; 2004b: 383).	Perhaps it is because of the *translucency* and *insubstantiality* of platonic objects that we have no chance to refine our observations of them.
Monitoring: our observations over time allow us to learn about various properties of an object and to track changes in an object's properties over time.	Original component of Azzouni's thick epistemic access. For three formulations, see especially Azzouni (1997: 474–476; 2004a: 129; 2004b: 383).	Here the problems of monitoring platonic objects are evident, but below I suggest some Yablo-inspired indicators of **TEA*** that are either part of Azzouni's monitoring or related to it.
Grounding: we can use observed properties of a thing to explain how we can know (or fail to know) about those or other of its properties—we can see books because they are solid, opaque objects.	Original component of Azzouni's thick epistemic access. For three formulations, see especially Azzouni (1997: 474–476; 2004a: 129; 2004b: 383).	Notice that this indicator helps the significance of Yablo's point that platonic objects are like pretend objects in that they are only *conceptually available* to us. As such, we could not use observed properties of numbers, for instance, to explain our knowledge of their properties.
Revealingness: observation reveals theoretically significant properties of the object studied over and above those that our conception of them already entails.	In correspondence, Azzouni shared with me that he intends his tracking indicator to capture. Fair enough. I have tried to capture this in "Monitoring" above. This also may be part of "Robustness." Still, I keep Revealingness separate here to show how Yablo's discussion emphasizes the importance of this aspect of the indicator.	Yablo's discussion of both the *insubstantiality* of platonic objects and the *silliness* of questions that go beyond those answered by our conceptions of them highlight the importance of this indicator.

Contingency on existence: some of our observations change upon destruction of the object.	Plausibly, Azzouni would already count this as part of his tracking indicator.	Yablo's discussion of the *irrelevance* of platonic objects to science motivates this indicator — nothing in our observations will change if the number nine self-destructs. His point on the *impatience* we experience when asked about the existence of platonic objects is likewise suggestive for the same reason. Finally, his concern about the *disconnectedness* of platonic objects supports this as an indicator.
Language resistance: the value of information gleaned through observation is language resistant. It remains relevant to a theory's success no matter how one rephrases or paraphrases its hypotheses, so long as the subject matter does not change. Moreover, the information would remain relevant even if the expressive powers of our language were to change significantly.	This suits Azzouni's view that we ought not to take quantifiers that enter our theories for reasons of language alone seriously, though in correspondence he notes that such a suggestion is quite controversial.	Inspired by Yablo on the *paraphrasability* of apparent references to platonic objects without a "felt loss of subject matter," along with the *translucency* of platonic objects and, finally, the bonus in terms of *expressiveness* delivered by platonic objects.
Identity Establishing: usually, we can use observation to distinguish one thing from another thing, one token from another token. Further, when we cannot, it is not silly to ask why not.	Again, this indicator is congenial to Azzouni's view; perhaps we might understand it as part of the explanation of epistemic access.	Inspired by Yablo on the *indeterminacy* of platonic objects and the *silliness* of questions about them.

Yablo also tells us that we can paraphrase away talk of platonic objects in our scientific theories without feeling as though we have lost information. In contrast, we cannot likewise "disappear" the importance of observational data to science. This is what motivated Language Resistance. To see how the fact that the importance of observation reports is language resistant plays a role in having thick epistemic access to observed objects, let us again compare two cases. First, consider the relevance to the theory of zoology of any observed behavior of creatures that we call Donzos and classify as belonging to a certain genus, class, etc. The relevance of such observations to zoology remains constant, no matter how we try to paraphrase or rephrase claims about Donzos, or even if we try to eliminate them (perhaps because

they were false in virtue of the fact that we were wrong about the genus of Donzos). But now consider the relevance of proving certain mathematical claims to zoology (I assume some mathematical theory enters zoology). If we succeed in rephrasing, paraphrasing, or eliminating any mathematical claim in zoological theory, then that claim and whatever proof the claim depended upon become irrelevant to zoology. Observation allows us contact with the things that our theory is about. This explains why our observation reports remain pertinent to our theories no matter how we use language to spell them out.

Also in contrast with the indeterminate identity conditions of platonic objects and the perceived silliness of questions about platonic objects that are not answered by what follows from our conceptions, Yablo's view suggests that when we instead have thick epistemic access to an object, our observations should help us answer non-silly questions about identity: they should be Identity Establishing. Again, to see the import of this property of observation to the idea of thick epistemic access, let us see what difference it makes. Notice that we can establish that one frog is not another frog through observation, or, if observation fails to enlighten us, we can, at a minimum, explain what barriers to establishing identity would need to be removed. However, our access to numbers is not at all like this. Observation will never establish whether or not the Fregean number 7 is identical with the Zermelo number 7, or even if the number two that numbers my dogs is the same number two that numbers my feet.

I expect that Yablo's discussion could suggest more properties of observation than I have managed to think of that Azzouni might want to consider. But let us now turn to seeing if our fortified list of properties in virtue of which observation is a form of thick epistemic access helps either philosopher respond to critics of his position.

7. Replying to naysayers

Let us now consider how we may better address critics of both Yablo and Azzouni if we are willing to intertwine some aspects of their views. At the end of his article, Azzouni cautions us against taking his argument as one that purports to show that there are no mathematical objects; his argument has the more modest goal of demonstrating that quantifier commitment, in

itself, does not entail ontological commitment. In footnote 17, his claim becomes even more tentative:

> I am writing as if I have given an argument for this position, but perhaps I am only shifting burdens. What response is there to someone who argues, "You have to say, 'there is the same number of cats as dogs', and that is enough of a reason by itself to think that item (the number) exists. Who cares about whether this posit got into the theory 'the wrong way'; the point is that we cannot get it *out* again." What can anyone say to this?[45]

However, the neo-Yabounnian may well have a response to the footnote naysayer. What can anyone say? We can explain why we cannot get it out again by appeals to the indispensability of metaphor to our scientific theories as motivated by Yablo.

The OC=QC Loyalist

If the challenge is that if one accepts a theory, then he ought to believe in what it says there is, then it sounds as if the critic is holding tenaciously to **OC=QC**. However, as we saw above, Azzouni's champion who embraces Yablo's view is well positioned to reject this argument. Once the two views are combined, she has a reasonable alternative to **OC=QC** to offer the critic that better answers questions concerning what roles claims about pretend posits do in our best theories—we may use them in virtue of their representational, cognitive or truth-conveying advantages. Since we may obtain these advantages through existential metaphor without postulating the existence of these objects, then it makes sense to change our criterion of ontological commitment to that of Azzouni to avoid either sacrificing the utility of our theories or making unnecessary assumptions. This strikes me as a fairly significant something to say.

Further, our new properties of observation that we have identified as important to thick epistemic access may also be invoked to bring out aspects of the purported objects that the critic might not have previously considered. He will see that he must be willing to admit to the existence of objects such that: all he will ever know about them follows from what we know about our concepts of them; his affirmation of their existence will quickly change to

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[45] Azzouni (1997: 184).

denial if some way is found to eliminate mention of them; and the identity of the objects will remain, despite our best efforts, uncomfortably vague. At this point it seems that for his view to remain plausible, the critic needs to do more than reiterate the indispensability argument.

Mr. Serious

Alternatively, the challenge might be to rebut Mr. Serious who claims that given the high assertibility of claims about pretend posits in the middle of our best theories, they must be true.[46] It still seems as though scientists might still be going around saying things they do not mean all time. Mr. Serious demands to know how they could ever be justified in doing this.

Here, Azzouni's defender could appeal to Yablo's idea of existential metaphor together with the idea of apt metaphors. Yablo writes that a make-believe game:

> is apt relative to such and such a subject matter to the extent that it lends itself to the expression of truths about that subject matter. A particular metaphorical **utterance** is apt to the extent that (a) it is a move in an apt game, and (b) it makes impressive use of the resources that game provides. [Yablo: class notes, lecture on "Mathematics as Gameskeeping," p. 17]

So, for instance, the game of mathematics is particularly apt relative to physics. On this view, we say that scientists are playing make-believe when they make claims that refer to pretend posits, but they justifiably engage in this activity because all the non-metaphorical implicatures of apt metaphors are literally true. It is in virtue of this fact that the claims involving existential metaphors become assertible, and, indeed, why they seem so darn true to us.

Consider the sentence, 'Blue combined with yellow makes green.' Here, blue, yellow and green appear to be entities ('Flour combined with water makes paste'), rather than properties. Assuming properties do not exist, we can see this as a case of existential metaphor. The literally true implicatures that follow from this metaphor are sentences like, 'Blue paint combined with yellow paint makes green paint,' or, 'When one looks through two lenses, one yellow and one blue, things will look green.' These implicatures are

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[46] Yablo at one point suggested something like this, as well as the question about the mathematical platonist, to me.

literally true, as are a whole bundle of implicatures we implicitly understand follow from the metaphor, and this makes us very inclined to ascribe truth to the metaphor because it leads us to these truths.

So, the quick answer to Mr. Serious is that scientists may justifiably assert existential metaphors when their non-metaphorical implicatures turn out to be literally true.

Mathsense of a Philosopher-Mathematician

Finally, let us consider a problem facing Yablo that is also better addressed by the combined view. A philosopher-mathematician might accept Azzouni's alternative criterion for ontological commitment, but argue that she has thick epistemic access to numbers via some form of mathematical perception or intuition—call it mathsense. Such a person could remain committed to the literal truth of mathematical statements, even in light of Yablo's arguments, because Yablo undermines only the a priori and Quinean arguments for the existence of numbers.

Now, on the one hand, that we could conclusively prove that such a person lacks mathsense seems as unlikely as the possibility that we could prove that a Pope cannot hear God. On the other hand, our extended list of properties of observation that make it a form of thick epistemic access may cause some believers in mathsense to question themselves.

Our philosopher-mathematician already says she has thick epistemic access to numbers. To be consistent, then, she would have to argue that mathematical observation via mathsense counts as a form of thick epistemic access. But it seems she could do this using our indicators of thick epistemic access. Without blaringly obvious difficulties, she could argue that mathematical observation via her mathsense is *robust* across most circumstances and *refinable* through mathematical training. Further, she could claim that she could use her it to *monitor* the properties of mathematical objects over time (though, given the eternal nature of numbers, changes would likely be limited to relations to physical objects), and use it to *ground* her explanation of why we cannot observe mathematical objects through the other modes of perception—just like we can only taste flavors, hear sound and see light, we can only mathsense mathematical objects.

However, I think that she will find it more difficult to hold that mathematical observation could have the properties of thick epistemic access

that we appended. Through regular observation, we can discover properties of things that we could never have deduced that they would possess. However, the philosopher-mathematician with mathsense does not discover any properties of numbers that the less fortunate mathsense-blind individual could not have discovered through deduction. She will also be hard-pressed to explain why her mathematical observations that were essential to the development of a theory remain relevant to that theory even if paraphrased away later. Finally, she would need to tell us why her mathsense does not allow her to see which numbers are identical. Thus, our mathematician will face more troubles trying to endow mathematical observation with the expanded list of thick epistemic access indicators.

In light of our enriched notion of the nature of thick epistemic access, our philosopher-mathematician might finally concede that her access to numbers is lacking—perhaps even that it amounts to unusual familiarity and facility with mathematical concepts, and that it is her skill that makes it clear to her what properties objects standing behind mathematical concepts would have to have, if they existed.

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I end here without a neat conclusion to bring these bits and pieces of discussion on the fit of Yablo's story about our metaphorical talk of numbers (and such) and Azzouni's tale of what we should believe exists if we believe science. I can only reiterate the point I have made over and over: a joined view deserves serious consideration.[47]

It remains to be asked: *How* shall we join the views to yield the most defensible metaphysics? We may not merely sew them together as they include inconsistent metaphysical commitments. Should we import parts of Azzouni's machinery into Yablo's position when it is possible to do so without distorting Yablo's view, or vice versa? Ought we to draw from both

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[47] Thanks to Stephen Yablo and Jody Azzouni for helpful comments on a previous draft of this paper.

positions to arrive at a third alternative not yet considered? I will be happy to have shown these questions are worth asking.[48]

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[48] Thanks to an anonymous reviewer for asking for clarification on this point and the encouragement to pursue such questions in future work.

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