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# Metods ef Considering Risk in Programming Models Used in Agriculture

Metody uwzględniania ryzyka w modelach optymalizacyjnych stosowanych w rolnictwie

Методы учета риска в оптимализационных моделях, применяемых в сельском хозяйстве

Farm organization planning calls for taking into account many variants of possible solutions of the problem as well as for adjustment to many constraints imposed by natural and economic conditions. Due to this fact, linear programming is recognized as an efficient instrument of optimizing production and investment plans, although not free from defects. One of them is that conventional linear programming cannot adequately cope with fluctuations of crop yields, prices and of other parameters. Consequently, there was a long-prevailing opinion that linear programming could be used in very rare cases only. Over the last several years, however, significant progress has been made in the so-called stochastic programming, especially in its theory. Numerous methods also appeared which could be, and indeed were, applied in agriculture. It would be useful to present at least some of the most important formulations. For the interest in linear programming is relatively high while there is little information in Polish scientific literature on the methods of risk considering.

## I. FORMULATION OF THE PROBLEM

A standard version of the linear programming problem is the following:

maximize $\mathbf{m}^T\mathbf{x}$, such that:

$$\mathbf{Ax} \leqslant \mathbf{b} \quad \text{and} \quad \mathbf{x} \geqslant 0$$

where:

$\mathbf{m}^T$ — a column vector of objective function parameter mean values,

$\mathbf{x}$ — vector of activities,

$\mathbf{A}$ — an input-output coefficients matrix,

$\mathbf{b}$ — vector of available amounts of scarce resources.

We have to assume that the vector **m** and also the matrix A are subject to fluctuation because fluctuations of crop yields and prices cannot be excluded. In some cases the vector **b** has to be considered as well: the amount of available labour in respective periods of different years can differ due to changing weather conditions.

## II. RISK CONSIDERING IN OBJECTIVE FUNCTION PARAMETERS

If we assume that prices are the only source of fluctuations of agricultural planning parameters or that all crops grown on the farm are cash crops, it is sufficient to concentrate on objective function parameters only. This assumption, apparently artificial in farm conditions, is useful to the extent that it permits to see the approach to the problem of risk in objective function parameters. The extension of chance action upon other elements of the linear programming model, that is an input-output coefficient matrix and a right-hand side vector, does not in any way affect the approach to the introduction of risk into the objective function. In the two oldest and best-known methods of considering risk in objective function parameters, formulated by Markowitz (14) and Freund (10), the measure of fluctuation is the total variance of objective function:

$V_{\mathbf{m}^T\mathbf{x}} = \mathbf{x}^T\mathbf{D}\mathbf{x}$,[1] where:

$\mathbf{D}$ — variance-covariance matrix of objective function parameters,

$\mathbf{x}^T$, $\mathbf{x}$ — column and row vectors of activities, respectively,

$V_{\mathbf{m}^T\mathbf{x}}$ — total variation of objective function.

The Markowitz method (14) was originally meant for choosing

[1] Or another way: $V_{\mathbf{m}^T\mathbf{x}} = \sum x_i\sigma_i^2 + \sum_{j=1}^{n}\sum_{i=1}^{n} x_i x_j \sigma_{ij}$, where

$x_i$ — i-th activity,

$x_j$ — j-th activity,

$\sigma$ — variance of objective function parameters of the i-th activity,

$\sigma_{ij}$ — covariance between parameter values of the i-th and j-th activities.

a stocks combination, hence its name of "portfolio selection". It is founded on the assumption that the goal of financial activity is to maximize the profit, which, translated into formulas of linear programming, means a maximization of $\mathbf{m}^T\mathbf{x}$ objective function, with $\mathbf{Ax} \leqslant \mathbf{b}$ and $\mathbf{x} \geqslant 0$. The $\mathbf{Ax} \leqslant \mathbf{b}$ constraints are necessary because the amount of money that could be spent for stocks by any individual or company is limited just as is the amount of a single firm's stocks available on the market. It also follows from this assumption that the maximization should be such that the total variation of profit does not exceed a certain value, which could be accepted by a decision-maker. This means that an additional constraint has to be imposed on $\mathbf{m}^T\mathbf{x}$, that is $\mathbf{x}^T\mathbf{Dx} \leqslant \alpha$, where $\alpha$ the maximum admissible value of objective function variation. Since this is an entirely subjective value and it is difficult to assume any relation between $\alpha$ and $\mathbf{m}^T\mathbf{x}$ in advance, the most convenient way of solving this problem is to use parametric programming, with the problem formulated as follows:

maximize: $\mathbf{m}^T\mathbf{x}$, such that

$$\mathbf{Ax} \leqslant \mathbf{b}$$

$$\mathbf{x}^T\mathbf{Dx} \leqslant \alpha$$

$$\mathbf{x} \geqslant 0, \quad \text{where:}$$

$\mathbf{m}^T$, $\mathbf{x}$, $\mathbf{A}$, $\mathbf{b}$, $\mathbf{D}$ and $\alpha$ as above.

Such problems could not be solved in the early fifties when the "portfolio selection" method was formulated. The converse problem, that of minimization of objective function variation, with the assumption that the mean value of the profit will not decrease below a certain value, could already be solved owing to the earlier work by Kuhn and Tucker (12). Its mathematical solution turned out to be identical with the original problem. The final version of the Markowitz method can thus be formulated as follows:

minimize: $\mathbf{x}^T\mathbf{Dx}$, such that:

$$\mathbf{Ax} \leqslant \mathbf{b},$$

$$\mathbf{m}^T\mathbf{x} \geqslant \beta,$$

$$\mathbf{x} \geqslant 0, \quad \text{where}$$

$\mathbf{m}$, $\mathbf{x}$, $\mathbf{A}$, $\mathbf{b}$, $\mathbf{D}$ as above

$\beta$ — parameter determining the minimum acceptable profit.

The solution to this problem are pairs of mean profit value and profit variations, and a corresponding set of values of each activity involved.

A choice is made according to individual preferences of profit height and its variance. In other words, solutions after the "portfolio selection" method provide information that with a given mean profit value, variation equal to $\mathbf{x}^T\mathbf{D}\mathbf{x}$ cannot be avoided, and that in this case all activities have to asume the values as in the optimum solution to the foregoing problem.

Freund founded his method on the "utility theory" formulated by von Neuman and Morgenstern (9). The central point of this theory is the assertion of a decrease in money value following its acquisition uncertainty. This means that of two farm enterprises with the same amount of profit, the one with a lower profit variation is "more useful". Moreover, two enterprises with different profits and with a different rate of profit variation have equal "utility" if the enterprise with a higher profit variation obtains this profit higher by a definite amount. This value varies with every individual farm operator. The relative measure of this value is referred to as a "risk aversion coefficient". The relation between profit height and its variation and profit utility is called utility function.[2] The one proposed by Freund for farmers has the following form:

$f(u) = 1 - e^{-ar}$, where

e — natural logarithm base,
a — risk aversion coefficient,
r — profit height.

The bigger a is, the less readily a farm operator will take up risk, and the higher profit has to be obtained to level higher variation in alternative activities. Assuming $\mathbf{r}$ to have a normal distribution, the expected utility value will be as follows:

$E(u) = \mu - a\sigma^2/2$, where:

$\mu$ — mean profit value,
a — risk aversion coefficient,
$\sigma^2$ — profit variation.

Translated into linear programming, this means:

maximize: $\mathbf{m}^T\mathbf{x} - \frac{a}{2}\mathbf{x}^T\mathbf{D}\mathbf{x}$ such that:

$$\mathbf{A}\mathbf{x} \leqslant \mathbf{b}$$

$$\mathbf{x} \geqslant 0$$

[2] Freund called this relation the "utility of money function". In other papers, the term "utility of function" can be encountered.

Solution of the above problems, where risk has been dealt with according to the two presented methods, requires quadratic programming. Available computers solve that problem easily, nevertheless, quadratic programming is far less convenient than linear programming, mainly because the size of problems is then much more limited. Hence there were attempts to modify and adapt the portfolio selection and Freund method to the simplex procedure, and to linearize the objective function.

The best-known linearization of the portfolio selection method is the so-called MOTAD proposed by Hazell (11). Its guiding idea is to replace variation by absolute deviation. Hazell assumes further that it is sufficient to take into account negative deviations only. The resulting formula is as follows:

minimize $\sum_{i=1}^{n} y_j^-$ such that:

$Ax \leqslant b$

$\mathbf{m}^T\mathbf{x} \geqslant \beta$, where

$y_j^-$ — negative absolute deviation of j-th activity from its mean profit value.

Chen and Backer (7) proposed a linearization of objective function $E(u) = \mathbf{m}^T\mathbf{x} - \frac{a}{2}\mathbf{x}^T\mathbf{D}\mathbf{x}$, founded on the assumption that no activity can be activated beyond the point where its marginal utility assumes a zero value. This means that that the value of any activity can be increased as long as its increase adds anything to the sum of total utility. If this limit is exceeded, total utility decreases. This marginal utility equals:

$$\frac{\delta E(u)}{\delta x_i} = m_i - \frac{a}{2}\sum_{j=1}^{n} s_{ij}x_j, \text{ where:}$$

$s_{ij}$ — covariance between the i-th and j-th activities [3].

[3] The value of convariance between the objective function parameters of the i-th and i-th activity is the variance of objective function parameter of the i-th activity, with the marginal utility thus being as follows:

$$\frac{\sigma E(u)}{\sigma x_i} = m_i = \alpha S_i^2 x_i + 2\alpha \sum_{j=1}^{n} S_{ij}x_j, \text{ where } S_2^i \text{ is of course the variance}$$

of objective function parameter of the i-th activity.

Consequently, the problem to be solved is as follows:

maximize: $\mathbf{m}^T\mathbf{x}$, such that

$$\mathbf{Ax} \leqslant \mathbf{b}$$

$$m_i - \frac{a}{2}\sum_{j=1}^{n} s_{ij}x_j$$

where $i = 1 \dots n$, $j = 1 \dots n$

n x the number of activities

Or another way:

maximize: $\mathbf{m}^T\mathbf{x}$, such that:

$$\mathbf{Ax} \leqslant \mathbf{b}$$

$$\mathbf{Dx} \leqslant \frac{2}{a}\mathbf{m}$$

$$\mathbf{x} \geqslant 0$$

Unfortunately, so simply formulated a problem can be solved only if $\mathbf{x}$ assumes a positive values. If any activity of which the $\mathbf{x}$ vector consists, say $x_k$, assumes a zero value, it could turn out that the constraint:

$$m_k - \frac{a}{2}\sum_{j=1}^{n} s_{kj}x_j \geqslant 0$$

is restrictive to other activities although $x_k$ should not have any influence on the optimum solution, because it is an idle activity. Chen and Backer developed a multi-stage algorithm for this purpose, which gradually removes all idle activities and their corresponding constraints which ensure the assumption of its non-negative marginal utility. The algorithm is the following:

1. Find an optimum solution of a parametric L.P. problem:

maximize: $\mathbf{m}^T\mathbf{x}$, subject to:

$$\mathbf{Ax} \leqslant \mathbf{b}$$

$$\mathbf{Dx} \leqslant \frac{2}{\alpha}\mathbf{m}$$

$\mathbf{x} \geqslant 0$ where:

$\alpha$ — a parameter assuming values from $+\infty$ to 0.

2. Record the whole set of solutions and their objective function values, if none of the dual solutions associated with the constraints which are to preserve non-negative $x_j$ utility, assumes a positive value.

3. Remove from the **x** vector all the activities which are not in the basis and all the corresponding constraints ensuring non-negative $x_j$ utility.

4. Find a new set of solutions. Come back to step 2 and record only solutions with a lower $\mathbf{m}^T\mathbf{x}$ value than previously obtained.

Another way of the linearization of objective function in the Markowitz method is "Separable Programming" (27).

It consists in the division of the $\mathbf{x}^T\mathbf{D}\mathbf{x}$ function into a sum of single-argument functions, which permits their spatial linearization.

The third approach to the problem of objective function fluctuations is founded on the theory of games. In the Polish economic literature this approach has been described in detail by T. Marszałkowicz (15). It appears, however, that a further discussion will be more lucid if the ısic tenets of the theory are explained at this point.

In the farm organization planning or other decision making, the so-called games with nature are selected out of a number of games covered by this theory. These games have such a property that the opponent in the game — nature — although ruthless, is not spiteful. It is therefore assumed that a player — in this case a decision maker — faces m possibilities, each of them having n realizations of the value under consideration. The problem is to select one out of m possibilities, the choice in no way affecting the opponent's action. The selection need not be limited to the choice of one possibility, which is called "pure strategy". This can also be any combination of possibilities, which is then called "mixed strategy". Mean values or variances as a criterion of choice cannot be applied as they cannot be calculated because nothing is known about the probability of any m realizations.[4] The only information we have is the set of values which every m possibility can assume. To deal with this really difficult situation, the minimax rule is adopted if the realizations of m are costs, and the maximin rule if the realiations of m are incomes.

The minimax rule consists in the choice of such a pure or mixed strategy that has the lowest maximum cost value of all mixed and pure strategies. By analogy, the maximin rule selects such a mixed or pure

[4] The mean value as a criterion of selection is called the Laplace criterion. It is based on the assumption that if the probability of no m realizations can be determined, it is necessary to assume that the probability of each realization is the same. This approach has been criticized in paper (24).

strategy that has the highest minimum value of income of all mixed or pure strategies.

An example will serve as a better illustration. In Table 1 are shown the yields of four oats varieties in the course of five years. The results are given in pounds per acre (the example was drawn from Heady, Pesek and Walker (29).

According to the maximin rule, the B variety is the best because its lowest yield obtained in the first year of the experiment is higher than the lowest yield of any other variety under consideration.

Choosing a mixed strategy is much more complex. For it is impossible to make a set of all combinations since the number of proportions of each variety in such a combination is infinite, whereas the mixed stategy is supposed to have such proportions of each variety that a combination with a higher minimum yield could not be found. It is therefore necessary to solve the following L.P. problem:

maximize: $x_5$, such that:

$$
\begin{array}{rcrcrcrcrl}
1472x_1 & + & 1568x_2 & + & 1440x_3 & + & 1552x_4 & - & x_5 & \geqslant 0 \\
2112x_1 & + & 1984x_2 & + & 2368x_3 & + & 2688x_4 & - & x_5 & \geqslant 0 \\
1920x_1 & + & 1824x_2 & + & 2496x_3 & + & 2784x_4 & - & x_5 & \geqslant 0 \\
3620x_1 & + & 3104x_2 & + & 3552x_3 & + & 0x_4 & - & x_5 & \geqslant 0 \\
3072x_1 & + & 3328x_2 & + & 2848x_3 & + & 3200x_4 & - & x_5 & \geqslant 0 \\
x_1 & + & x_2 & + & x_3 & + & x_4 & & & = 1 \\
 & & & & & & x_1 & & & \geqslant 0 \\
 & & & & & & x_2 & & & \geqslant 0 \\
 & & & & & & x_3 & & & \geqslant 0 \\
 & & & & & & x_4 & & & \geqslant 0 \\
 & & & & & & x_5 & & & \geqslant 0
\end{array}
$$

The solution to the above problem is a mixed strategy consisting of 56% of B variety and 44% of C variety.

The whole problem of determining a mixed strategy can be generalized as follows:

$x_p$ maximization, such that

$$\mathbf{m}_1\mathbf{x} - x_p \geqslant 0$$

$$\mathbf{m}_2\mathbf{x} - x_p \geqslant 0$$

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$\mathbf{m}_n\mathbf{x} - \mathbf{x}_p \geqslant 0$

$$\sum_{i=1}^{n} x_i \leqslant d$$

$\mathbf{x} \geqslant 0$, where:

$x_p$ — value of a game,
$\mathbf{m}_1 \dots \mathbf{m}_n$ — vectors of m realizations,
$\mathbf{x}$ — activity vector,
d — value which the sum total of activity values cannot exceed (most often 1 or 100%),
$x_i$ — constituent activities of the $\mathbf{x}$ vector.

After this theoretical discussion, it is necessary to return to the application of the method in the construction of an LP matrix which is to determine the optimum program of production and possible investments. The set of constraints constituting the mixed strategy contains an $\sum_{i=1}^{n} x_i \leqslant d$ element. In the matrix constructed for the described task, the $\sum_{i=1}^{n} x_i \leqslant d$ is replaced by the whole input-output coefficients matrix. The problem can thus be formulated as follows:

maximize: $x_p$, such that:

$$\mathbf{Ax} \leqslant \mathbf{b}$$
$$\mathbf{m}_1\mathbf{x} - \mathbf{x}_p \geqslant \mathbf{0}$$
$$\dots\dots\dots$$
$$\dots\dots\dots$$
$$\dots\dots\dots$$
$$\mathbf{m}_n\mathbf{x} - \mathbf{x}_p \geqslant 0$$
$$\mathbf{x} \geqslant 0$$

The criterion of choice used in the foregoing example is not the only one, although the most popular. A detailed analysis of all criteria can be found in Adamus (1).

A similar approach to objective function fluctuations as in the theory of games can be found in the "safety-first" method. The idea of "safety first" was worked out by Roy (26) and Tesler (28). It was further developed and applied to LP by Maruyama (16) and by Petit and

Boussard (21). According to this method, a farm should be operated in such a way that the profit every year could be high enough for the farm to maintain its existence. This means that the farm's income has to ensure at least a social minimum for the farmer and his family, and to pay for all the charges (debet installment payments, interests, taxes etc.) every year irrespective of weather conditions and price fluctuations. It is not enough to have a high mean income because it can be spent if "bad harvest" is not expected. Moreover, previous incomes do not necessarily imply that they will be similar in the future. At best, it only follows that such and such incomes, profits, or yields will be obtained in the future. It is impossible to know how often this will happen for the sample is too small to infer anything from, the more so that the observations from the previous years cannot possibly be recognized as drawn out by lot.

Therefore, the LP matrix should be such as to prevent a situation where the mean profit or income is high, but its stability is not sufficient, which leads to a farm failure. In Maruyma's already-cited work, this problem is solved by:

$\mathbf{mx}$ maximization, such that:

$\mathbf{Ax} \leqslant \mathbf{b}$

$\mathbf{m_1x} \leqslant \mathbf{d}$

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$\mathbf{m_nx} \leqslant d$

$\mathbf{x} \geqslant 0,$ where:

$\mathbf{m}$ — mean objective function parameters vector,
$\mathbf{m_1} \dots \mathbf{m_n}$ — objective function parameters in each of n years,
$\mathbf{A}$ — input-output coefficient matrix,
$\mathbf{d}$ — the level below which income (profit) cannot drop in any year.

## III. INTRODUCTION OF RISK INTO INPUT-OUTPUT COEFFICIENTS MATRIX

Fluctuations of planning parameters are caused either by price fluctuation or yield change. Price fluctuations, in terms of LP, affect objective function parameters only. On the other hand, yield fluctuations affect also input-output coefficients. If farm planning is to be consistent with reality, this problem must be taken into account as well.

One of the methods of considering fluctuations of input-output coefficients is the so-called "Chance-constrained Programming" (5). The assumptions of this method are the following: if in some constraints there are parameters subject to random fluctuations, these constraints cannot be met with a 100% probability. To put it in another way, we can assume that the risk-affected constraint should be met with a probability of no less than for instance 0.90, 0.95 or 0.99. Using the latter approach as the starting point, it is necessary to add the 90%, 95% or 99% confidence interval to the sum of the products of parameters mean values by the value of their corresponding activities. Thus, if the deterministic formulation of the problem is the following:

$\mathbf{a}_k\mathbf{x} \geqslant \mathbf{b}_k$, where:

$\mathbf{a}_k$ — vector of input-output coefficients vector,

$\mathbf{x}$ — activity vector,

$\mathbf{b}_k$ — the minimum value of $a_k x$ ensuring the coherence of the program,

then it is necessary to replace $a_k x$ by:

$$a_k x - \frac{t}{\alpha}\sqrt{V\mathbf{a}_k\mathbf{x}}, \quad \text{where}$$

$t$ — standardized confidence interval,

$V\mathbf{a}_k\mathbf{x}$ — $\mathbf{a}_k\mathbf{x}$ variation;

if the constraint is to be met with the required probability. Further:

$V\mathbf{a}_k\mathbf{x} = \mathbf{x}^T\mathbf{G}_k\mathbf{x}$, where:

$\mathbf{G}_k$ — variance-covariance matrix of the $\mathbf{a}_k$ vector.

The whole equation can thus be presented as follows:

$$\mathbf{a}_k\mathbf{x} - \frac{t}{\alpha}\sqrt{\mathbf{x}\mathbf{G}_k\mathbf{x}}\ ,$$

and it has to be more than or equal to $b_k$. Then the whole problem is as follows:

maximize: $\mathbf{m}^T\mathbf{x}$ [5], such that:

$$\mathbf{A}\mathbf{x} \leqslant \mathbf{b}$$
$$\mathbf{a}_k\mathbf{x} - t\ \mathbf{x}^T\mathbf{G}_k\mathbf{x} \geqslant \mathbf{b}$$
$$\mathbf{x} \geqslant 0.$$

[5] The objective function has been formulated in a deterministic way to simplify the notation. There is no obstacle to formulating it in any other way.

Two difficulties are connected with this problem. First, in order to be useful, it has to be resolvable and there must be an algorithm of the solution. Although this algorithm is available (32), it has a number of defects. Not least is its small effectiveness and very high restrictions on the size of the problem. The other difficulty with the chance-constrained programming is the assumption of a normal distribution of fluctuations of input-output coefficients, which is not always tenable. This inconvenience can be avoided by using the Tshebyshev inequality (24), in this case a considerable increase of the t $\alpha$ parameter has to be taken into account.[6]

To avoid all these inconveniences, attempts were made to simplify this method. Merill (17) and Chen (6) developed methods consisting in the interchange of the objective function and the constraint affected by input-output parameter fluctuations, when only one constraint is subject to them. Rahman and Bender (24) formulated a method applicable in a situation where covariance between input-output parameters does not exist or can be ignored. A more general and simplified method was worked out by Wicks and Guise (30). It permits the use of LP because it is founded on absolute deviation rather than standard deviation as a measure of fluctuation.

Madansky's method (13) has an entirely different background as it is derived from the theory of games. The method assumes that, if there is any parameter affected by fluctuations in the constraint, the constraint has to be met in each situation. In terms of the LP used in farm organization planning, this means that the constraints under consideration have to be met each year which is an information source. Thus, if the constraint has a deterministic formula:

$$\mathbf{a}_i\mathbf{x} \leqslant \mathbf{b}_i,$$

then in the case of the $\mathbf{a}_i$ vector fluctuations and using the Madansky assumption, this notation should be presented as follows:

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[6] The Tshebyshev inequality is: $[(Px_o - \bar{x}) \leqslant t\sigma^2] > 1 - \frac{1}{t^2}$ which means that the probability that the n-th realization will not not deviate from mean by no more than t times of $\delta$ is higher than $1 - \frac{1}{t^2}$ .Thus, if the constraint is to be met with the probability of not less than $1-\alpha$, then $1-\alpha = \frac{1}{t^2}$ hence $t = \sqrt{\frac{1}{\alpha}}$. For the probability equal 0.95, t $\approx$4.4, which is more than twice of $t_{0.05}$. The smaller the $\alpha$ parameter, the bigger that disproportion is.

$\mathbf{a}_{i1}\mathbf{x} \leqslant b_i$

$\mathbf{a}_{i2}\mathbf{x} \leqslant b_i$

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$\mathbf{a}_{in}\mathbf{x} \leqslant b_i$, where:

$\mathbf{a}_{ij}$ — realization of the $\mathbf{a}_i$ vector in each source-of-information year. If $b_i$ were also subject to fluctuations, the above notation could be modified as:

$\mathbf{a}_{i1}\mathbf{x} \leqslant \mathbf{b}_{i1}$

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. . . . . .

. . . . . .

$\mathbf{a}_{in}\mathbf{x} \leqslant \mathbf{b}_{in}$, where:

$b_{ij}$ — realization of the $b_i$ parameter.

Introduction of what has been previously achieved into the LP model is already obvious:

maximize: $\mathbf{m}^T\mathbf{x}$ [7], such that:

$\mathbf{A}\mathbf{x} \leqslant \mathbf{b}$

$\mathbf{a}_{11}\mathbf{x} \leqslant \mathbf{b}_{11}$

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$\mathbf{a}_{1n}\mathbf{x} \leqslant \mathbf{b}_{1n}$

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. . . . . . .

$\mathbf{a}_{kn}\mathbf{x} \leqslant \mathbf{b}_{kn}$

[7] As in note 5.

## IV. SUMMARY: EVALUATION OF METHOD USEFULNESS

Most of the presented methods have their practical application. The methods developed by Freund and by Markowitz are most frequently employed (2, 3, 4, 8, 25, 31), but those founded on the theory of games are also applied. However, there are no studies whatsoever that would compare all the methods in question. More often, we can encounter criticism of a particular method, with its weak points and defects emphasized.

A frequent object of criticism is the Freund method. According to Petit and Boussard (21), the fundamental objection to the method is that it requires an assumption of the normal distribution of yields and prices in order to obtain the objective function. This has not been proved so far whereas only absolute certainty would justify this assumption. Furthermore, there are reports that the distribution of crop yields and prices of farm products is not normal or even not symmetric. Petit and Boussard after Day (9). Another objection concerns the risk aversion coefficient, which is different for every decision-making farmer and has to be determined before optimization procedures. This must be determined by experiment, which is criticized by Moscardi and de Janvry (18) because the coefficient value so defined will be affected by the farmer's attitude towards gambling.

Although free from the foregoing objections, the Markowitz method has also its own defects, the most serious being that a dual solutions is not possible (20).

The above disadvantages of the two methods can be further strengthened by the fact that they require quadratic programming, which is more restrictive as to the size of the problem, while the information on which the methods are based, that is mean values and variance, is rarely credible. In order to obtain such figures, the data covering far more than ten years should be used. These data are not always available; moreover, the picture can be distorted by yield changes over a longer period due to new developments in technology, unless we have the data obtained from experiments. Prices can also be affected by such systematic changes.

All these defects of the two methods also hold for their modifications, except for that resulting from the use of quadratic programming.

Methods founded on the theory of games have their own defects as well. For example, Wickas and Guise (30) raise an objection that application of the theory of games increases the matrix size. This is an essential objection since the LP matrices employed in optimizing farm production and investment plans already have considerable sizes. Another objection Wicks and Guise discuss is that while applying the theory of games to risk consideration, we implicitly assume that the farmer's attitude towards risk can be described by that theory. There is no evidence to support this assumption. Still one more objection can be added that information drawn from the past, especially like that used in the theory of games, contributes very little to planning. Moreover, with the selection of data from previous years, an unconscious assumption is made that only those years and none other are representative and their number is sufficient as the information basis.

None of the discussed methods seems to be free from defects. Such a method is difficult to imagine, especially until the harmfulness of risk is defined. An attempt to deal with the problem in that way is part of the safety-first method but it is difficult to apply it in the case of fluctuations of input-output coeffients.

Tab. 1. Crop yields of four oats varieties in lbs. per acre in 1953—1957

| Year<br>Variety | 1953 | 1954 | 1955 | 1956 | 1957 |
|---|---|---|---|---|---|
| A | 1472 | 2112 | 1920 | 3520 | 3072 |
| B | 1568 | 1984 | 1824 | 3104 | 3328 |
| C | 1440 | 2368 | 2496 | 3552 | 2848 |
| D | 1952 | 2688 | 2784 | 0[1] | 3200 |

1. The D crop was destroyed by hail in 1956. This is the slowest-growing of the four varieties tested. Hail, which normally occurs after harvest, affected this variety in 1956 due to a prolonged vegetation period.

Source: O. L. Walker et al., Application of Game Theoretic Model to Decision Making, Agronomy Journal, no 2, 1964.

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## STRESZCZENIE

Celem przedstawionej tu pracy jest opis i porównanie metod uwzględniania ryzyka w modelach optymalizacyjnych stosowanych w rolnictwie. Przedmiotem opisu były przede wszystkim zagadnienia teoretyczne, a więc zarówno strona formalno-matematyczna prezentowanych metod, jak i formujący je zestaw założeń ekonomicznych.

Opisywane w niniejszej pracy metody uwzględniania wahań losowych parametrów funkcji celu oparte są na teorii użyteczności bądź na teorii gier, a służące do uwzględniania wahań parametrów techniczno-ekonomicznych również mają uzasadnienie teoretyczne w teorii gier oraz na tak zwanych ograniczeniach losowych (chance constraints). Próba oceny wykazała, iż większe nadzieje należy wiązać z grupą metod opartych na teorii gier. Do czasu ustalenia na czym polega szkodliwość ekonomiczna ryzyka trudno jest jednak wydawać jednoznaczne oceny.

## РЕЗЮМЕ

Цель настоящей работы — описать и сопоставить методы учета риска в оптимализационных моделях, применяемых в сельском хозяйстве. Предметом описания были прежде всего теоретические вопросы, в том чусле как формально-математическая сторона представляемых методов, так и формирующий их комплекс экономических предпосылок.

Описанные в настоящем исследовании методы учета случайных колебаний параметров функции цели опираются на теорию полезности или на теорию игр, а методы, служащие для учета колебаний технико-экономических параметров, теоретически обоснованы также теорией игр и, кроме того, так называемыми случайными ограничениями (chance constraints). Попытка оценки обнаружила, что больше надежд подае ттруппа методов, опирающихся на сеорию игр. Однако до установления, в чем состоит экономическая вредность риска, формировать однозначные оценки представляется затруднительным.