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**AN ATTEMPT TO EVALUATE THE ECONOMY OF UPPER SILESIA
BY OWNERSHIP SECTORS BASED ON THE ECONOMETRIC MODEL
AND THE FINAL EQUATION**

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Abstract

The paper attempts to use the econometric model in the evaluation of trends in the economy of the Śląskie Voivodeship. The analysis is mainly based on the final equation of the model. The characteristic roots indicate that the economy of Upper Silesia does not have a cyclical nature (the lack of complex roots) and the private enterprise sector is characterised with higher efficiency than the public enterprise sector, since the curve for endogenous variable y_t is converging with the asymptote $y = 0$ more quickly in the case of public enterprises than in the case of private enterprise sector.

Keywords: econometric model, final equation, the roots of the final equation, public and private enterprise sector.

JEL classification: C51, C52.

Introduction

The econometric model of an economic enterprise is one of the tools used for the analysis and evaluation of economic processes. The study is an attempt to evaluate the construction, estimation and analysis of the dynamic properties of a mini-model of the economy of the Śląskie Voivodeship by ownership sectors. It aims to verify the hypothesis that the growth dynamics in the sector of public enterprises differs from growth dynamics in the sector of private enterprises. The dynamic properties of the model are analysed using the final equations of selected variables. The mechanism for developing the final equation of a variable is based on the elimination through consecutive substituting of all other unlagged and lagged endogenous variables in a given equation. As a result, we obtain an equation which features only a researched endogenous variable and its subsequent lags and unlagged and lagged exogenous variables. Formally speaking, due to y_{it} , the final equation is a difference equation. The analysis of the solution to this equation allows for drawing conclusions on the proper motions (an independent trend) and stability of the system. The system will be stable when the impact of lagged endogenous variables on variable y_{it} will be expiring in nature, which means that the trajectory of proper motion will strive to reach equilibrium.

1. The model of relationships between variables characterising the economy of the Śląskie Voivodeship

The estimation and analysis were based on annual data¹ regarding variables characterising the selected aspects of the economy of the Śląskie Voivodeship in the years 1999–2009. We specified and estimated two types of models in the enterprise sector due to ownership form:

- the model of relationships between variables characterising the economy of the Śląskie Voivodeship in the public enterprise sector,
- the model of relationships between variables characterising the economy of the Śląskie Voivodeship in the private enterprise sector.

In order to make a comparative analysis based on ownership form, we use the same categories of variables in the models. The econometric analysis uses the values published in the Statistical Bulletins of the Śląskie Voivodeship¹ and they were marked in the following way:

PZSPU_{*t*} – average paid employment in enterprise sector, public sector, in thousands people,

$PZSPR_t$ – average paid employment in enterprise sector, private sector, in thousands people,

$PWBPU_t$ – average gross wages and salaries in public enterprise sector, in PLN,

$PWBPR_t$ – average gross wages and salaries in private enterprise sector, in PLN,

$PSPPU_t$ – sold production of industry, public sector, in PLN million,

$PSPPR_t$ – sold production of industry, private sector in PLN million,

$PCDPU_t$ – revenues from total activity of enterprises, public sector, in PLN million,

$PCDPR_t$ – revenues from total activity of enterprises, private sector, in PLN million,

$KUPPU_t$ – costs of obtaining revenues from total activity, public sector, in PLN million,

$KUPPR_t$ – costs of obtaining revenues from total activity, private sector in PLN million,

$WFNPU_t$ – net financial result in public enterprise sector, in PLN million,

$WFNPR_t$ – net financial result in private enterprise sector, in PLN million,

$NISPU_t$ – investment outlays, public sector, in PLN million,

$NISPR_t$ – investment outlays, private sector, in PLN million.

The relationships between variables are presented in this scheme:

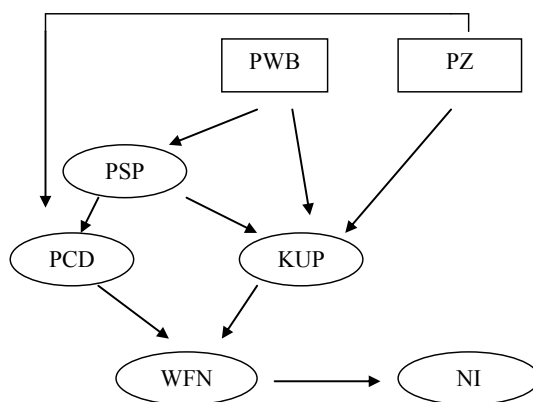


Fig. 1. The scheme of relationships between variables characterising the economy of the Śląskie Voivodeship

Source: own study.

In order to make a comparative analysis based on ownership form, we use the same categories of variables in the models.

The scheme of relationships between variables shows that this model is recursive and dynamic, as it includes lagged endogenous variables. The exogenous variables of the model are average paid employment in enterprises and average gross wages and salaries.

The results of the estimation of the model of relationships between variables characterizing the economy of the Śląskie Voivodeship in the public enterprise sector:

- The equation of sold production of industry in the public enterprise sector:

$$PSPPU_t = 0.3956 PSPPU_{t-1} + 89.9621 PZSPU_t + 3.8633 PWBPU_t - 14162.3745$$

$$R^2 = 0.75; D-W = 2.56,$$

- The equation of revenues from total activity:

$$PCDPU_t = -0.1831 PCDPU_{t-1} + 1.4305 PSPPU_t + 55.8736 PZSPU_t + 759.6728$$

$$R^2 = 0.923; D-W = 2.81,$$

- The equation of costs of obtaining revenues:

$$KUPPU_t = -0.0598 KUPPU_{t-1} + 0.5917 PSPPU_t + 197.6487 PZSPU_t +$$

$$+ 5.8877 PWBPU_t - 32400.940$$

$$R^2 = 0.93; D-W = 1.38,$$

- The equation of the total net financial result:

$$WFNPU_t = 0.0133 WFNPU_{t-1} + 0.8504 PCDPU_t - 0.8532 KUPPU_t - 95.3974$$

$$R^2 = 0.99; D-W = 1.38,$$

- The equation of total investment outlays:

$$NIPU_t = 1.0095 NIPU_{t-1} + 0.1047 WFNPU_t + 25.2009$$

$$R^2 = 0.80; D-W = 1.12.$$

The results of the estimation of the model of relationships between variables characterizing the economy of the Śląskie Voivodeship in the private enterprise sector:

- The equation of sold production of industry in the private enterprise sector:

$$PSPPR_t = 0.4044 PSPPR_{t-1} - 181.9157 PZSPR_t + 51.3089 PWBPR_t + 26537.9872$$

$$R^2 = 0.96; D-W = 2.56,$$

- The equation of revenues from total activity in the private enterprise sector:

$$PCDPR_t = 0.1139 PCDPR_{t-1} + 1.2180 PSPPR_t + 54.2245 PZSPR_t - 17692.9027$$

$$R^2 = 0.99; D-W = 1.87,$$

- The equation of costs of obtaining revenues:

$$\begin{aligned} KUPPR_t = & 0.1981 KUPPR_{t-1} + 0.9338 PSPPR_t + 36.4274 PZSPR_t + \\ & + 8.3156 PWBPR_t - 18293.6130 \\ R^2 = & 0.99; D-W = 1.77, \end{aligned}$$

- The equation of the total net financial result:

$$\begin{aligned} WFNPR_t = & -0.0658 WFNPR_{t-1} + 0.8715 PCDPR_t - 0.8642 KUPPR_t - 944.5454 \\ R^2 = & 0.99; D-W = 2.16 \end{aligned}$$

- The equation of total investment outlays:

$$\begin{aligned} NIPR_t = & 0.5166 NIPR_{t-1} + 0.3236 WFNPR_t + 1301.1485 \\ R^2 = & 0.90; D-W = 1.80. \end{aligned}$$

All the equations of particular types of models are characterised with high conformity, measured by the coefficient of determination, the lowest value of which is 0.8037 and applies to the equation on investment outlays in the public enterprise sector.

2. The construction of the final equations of the models

If endogenous variables have only one-period lags in a G equation econometric model, due to the selected endogenous variable the final equation of the model is as follows³:

$$y_t + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \dots + \alpha_G y_{t-G} = L(X_1, X_2, \dots, X_k) \quad (1)$$

where: $L(X_1, X_2, \dots, X_k)$ is a linear function of exogenous variables of the model.

The presentation of the dependence of y_t on t as an explicit formula of a certain function $y = F(t)$ is obtained as the sum of the general solution to the homogeneous equation and the particular solution to the nonhomogeneous equation. The homogeneous equation is expressed as:

$$y_t + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \dots + \alpha_G y_{t-G} = 0 \quad (2)$$

and its general solution is dependent on G initial conditions and on the roots of the characteristic equation:

$$\lambda^G + \alpha_1 \lambda^{G-1} + \dots + \alpha_G = 0 \quad (3)$$

Since the general solution to the homogeneous equation expresses the mechanism of proper motions of the system, the analysis of the homogeneous equation and the resultant characteristic equation informs on the nature of proper fluctuations, in particular an independent trend and possible periodic fluctuations.

In the case of real and single roots of the characteristic equation, the general solution to the homogeneous equation is expressed as⁴:

$$y_t = \sum_{j=1}^G A_j \lambda_j^t \quad (4)$$

where:

λ_j – is the j -th characteristic root,

A_j – is the j -th initial condition.

In this situation, the proper motion of the system does not include periodic fluctuations, whereas in the case when $|\lambda_j| < 1$ the curve of motion is an asymptotic downward curve towards the axis $y = 0$ (when $t \rightarrow \infty$).

Whereas, in turn, the solution to the characteristic equation is r real roots and $2s$ complex roots ($\lambda_j = a_j \pm i b_j$), the solution can be expressed as follows:

$$y_t = \sum_{j=1}^r A_j \lambda_j^t + \sum_{j=r+1}^{r+s} A_j \rho_j^t \cos \left(\frac{2\pi t}{\Theta_j} + \delta_j \right) \quad (5)$$

where:

$\Theta = \arctg \frac{b_j}{a_j}$ – defines the length of periodic fluctuations,

$\rho_j = \sqrt{a_j^2 + b_j^2}$ – defines the changes in the amplitudes of fluctuations compared to the initial amplitude,

δ_j – is determined based on the initial conditions defining for what value t the j -th periodic component has the maximum value.

Based on the results of the estimation of the structural form of the econometric model, the homogeneous equations and the corresponding characteristic equations can be developed.

In the public enterprise sector:

– The homogeneous equation for the variable $PSPPU_t$ is expressed as:

$$PSPPU_t - 0.3956 PSPPU_{t-1} = 0,$$

and the characteristic equation $\lambda - 0.3956 = 0$, so the characteristic root equals $\lambda = 0.3956$.

– The homogeneous equation for the variable $PCDPU_t$ is expressed as:

$$PCDPU_t - (0.3956 - 0.1831) PCDPU_{t-1} + [0.3956 (-0.1831)] PCDPU_{t-2} = 0,$$

The characteristic equation: $\lambda^2 - 0.2125 \lambda - 0.07243436 = 0$ has two characteristic roots, which are real numbers $\lambda_1 = -0.3662$ and $\lambda_2 = 0.3956$.

- The homogeneous equation for the variable $KUPPU_t$ is expressed as:

$$KUPPU_t - (0.3956 - 0.0598) KUPPU_{t-1} + [0.3956 (-0.0598)] KUPPU_{t-2} = 0.$$

The characteristic equation: $\lambda^2 - 0.3358 \lambda - 0.02365688 = 0$ has two characteristic roots, which are real numbers: $\lambda_1 = -0.0598$ and $\lambda_2 = 0.3956$.

In the private enterprise sector:

- The homogeneous equation for the variable $PSPPR_t$ is expressed as:

$$PSPPR_t - 0.4044 PSPPR_{t-1} = 0,$$

and the characteristic equation $\lambda - 0.4044 = 0$; accordingly, the characteristic root equals $\lambda = 0.4044$.

- The homogeneous equation for the variable $PCDPR_t$ is expressed as:

$$PCDPR_t - (0.4044 + 0.1139) PCDPR_{t-1} + 0.4044 \cdot 0.1139 PCDPR_{t-2} = 0,$$

The characteristic equation: $\lambda^2 - 0.5183 \lambda + 0.04606116 = 0$ has two characteristic roots, which are real numbers: $\lambda_1 = 0.1139$ and $\lambda_2 = 0.4044$.

- The homogeneous equation for the variable $KUPPR_t$ is expressed as:

$$KUPPR_t - (0.4044 + 0.1981) KUPPR_{t-1} + (0.4044 \cdot 0.1981) KUPPR_{t-2} = 0.$$

The characteristic equation: $\lambda^2 - 0.6025 \lambda + 0.08011164 = 0$ has two characteristic roots, which are real numbers: $\lambda_1 = 0.2031$ and $\lambda_2 = 0.4044$.

The characteristic roots that we achieved fulfil the condition $|\lambda| < 1$ in every system, which proves its stability. Moreover, in each equation of the same system we have the same value of the characteristic root⁵:

- for the public enterprise sector – $\lambda = 0.3956$,
- for the private enterprise sector – $\lambda = 0.4044$.

The analysis of the characteristic roots shows that the only characteristic roots are the roots that are real numbers. The lack of complex roots may prove the lack of cyclical nature of the economy of the Śląskie Voivodeship in the analysed period of time.

If we substitute the values of the respective characteristic roots in formula (4), we can see that in the case of the private sector the values of endogenous variables converge with the asymptote $y = 0$ more quickly, (with no external incentives) due to lower values of λ_j . The values of coefficients for the public and private enterprise sectors are presented in Figures 2 and 3.

Table 1. The comparison of the characteristic roots of the final equation of particular variables by ownership sectors

Endogenous variables	Ownership sector	
	Public	Private
Sold production of industry	0.3956	0.4044
Revenues from total activity	0.3956 -0.3662	0.4044 0.1139
Costs of obtaining revenues from total activity	0.3956 -0.0598	0.4044 0.2031

Source: own calculations.

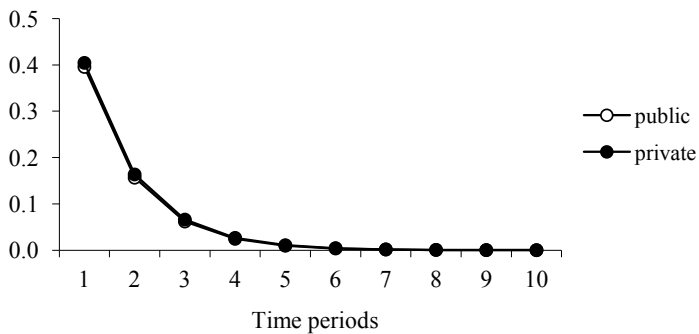


Fig. 2. The values of coefficients of sold production of industry for the public and private sectors (the values of coefficients λ^t in particular time periods)

Source: own study.

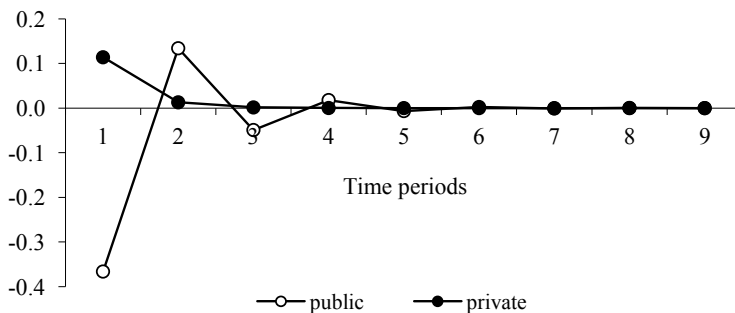


Fig. 3. The values of coefficients of revenues from total activity by ownership sectors (the values of coefficients λ^t in particular time periods)

Source: own study.

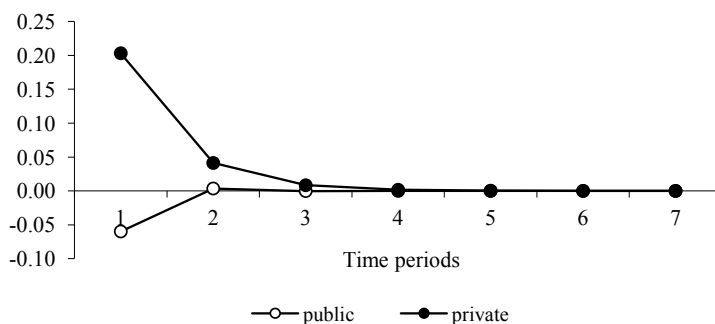


Fig. 4. The values of coefficients of costs of obtaining revenues for the public and private sectors (the values of coefficients λ^t in particular time periods)

Source: own study.

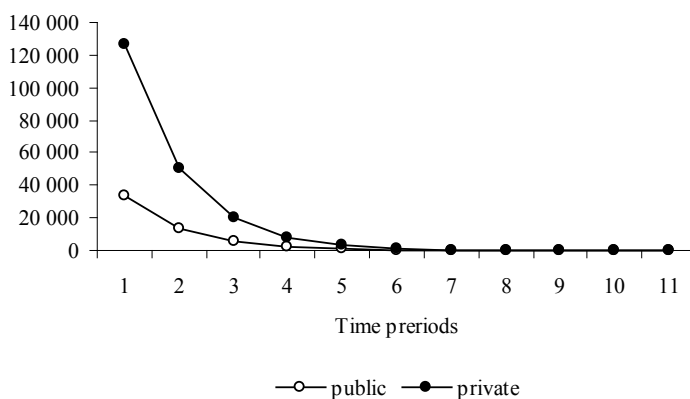


Fig. 5. Independent trends in sold production of industry by ownership sectors $PSP_t = PSP_0 \lambda^t$

Source: own study.

For the variables “Revenues from total activity” and “Costs of obtaining revenues from total activity” in the public sector, the oscillation occurs from both sides due to the negative values of the latter characteristic root.

Conclusions

The results of the estimation of the equations characterising the economy of the Śląskie Voivodeship both in the public and private sectors show high conformity, the higher one for the private enterprise sector.

The final equations, determined based on estimated equations, and the characteristic equations corresponding with particular variables and ownership sectors have the characteristic roots that are real numbers the value of which is $|\lambda| < 1$. This leads to the conclusion that the economy of the Śląskie Voivodeship did not show the tendency towards cyclical fluctuations in the analysed period of time, the system is stable, and in the case of the lack of external incentives over a longer period of time the economic system will be striving towards 0 both in the public and private sectors, with the public sector converging towards zero more quickly.

Notes

- ¹ Due to short time series, analyses presented in Biolik (2007), pp. 27–43, Biolik (2008a), pp. 45–55, Biolik (2008b), pp. 11–30, Biolik (2009), pp. 49–56, Biolik (2010), pp. 31–41 were based on quarterly data, which showed certain deficiencies.
- ² See Statistical Bulletins of the Śląskie Voivodeship.
- ³ See Pawłowski (1981).
- ⁴ See *ibidem*, p. 42.
- ⁵ This is the property of the characteristic roots of the final equation, because particular endogenous variables are affected by the same developmental mechanisms.

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