Anna Łatuszyńska

Multiple-criteria decision analysis using TOPSIS method for interval data in research into the level of information society development

Folia Oeconomica Stetinensia 13(21)/2, 63-76

2013

Folia Oeconomica Stetinensia
DOI: 10.2478/foli-2013-0015

MULTIPLE-CRITERIA DECISION ANALYSIS USING TOPSIS METHOD FOR INTERVAL DATA IN RESEARCH INTO THE LEVEL OF INFORMATION SOCIETY DEVELOPMENT

Anna Łatuszyńska, Ph.D.

University of Szczecin
Faculty of Economics and Management
Institute of IT in Management
Mickiewicza 64, 71-101 Szczecin, Poland
e-mail: latuszynska@gmail.com

Received 20 September 2013, Accepted 17 January 2014

Abstract

When making choices and decisions, very seldom does the situation arise that a decision-maker very seldom bases their assessment of the options available on only one criterion. Frequently, many aspects of the available solutions are considered – both in terms of potential benefits and costs. In order to support decision makers, the Multiple-Criteria Decision Analysis (MCDA) is used for selecting the solution which is the best in several respects.

There are many methods of multi-criteria decision- analysis such as AHP, ELECTRE, PROMETHEE, VIKOR or TOPSIS. In the article a modification of the last of these methods is used. With the use of TOPSIS method with interval arithmetic the analysis of the level of information society (IS) development in the European Union countries between 2005 to 2010 is performed and its results are presented.

Keywords: information society development, composite indicator, multi-criteria decision analysis.

JEL classification: C00, D83.

Introduction

When making choices and decisions, the situation when a decision-maker bases their assessment of the options available on just one criterion. Frequently, many aspects of the available solutions are taken into consideration – both in terms of potential benefits and costs. In order to support decision makers, the Multiple-Criteria Decision Analysis (MCDA) is used for selecting the solution which is the best in several respects.

In the multi-criteria decision-making problems, a solution that optimizes all objective functions (criteria) is often unavailable. In such situations, the concept, called Pareto optimal solution or an effective solution is introduced. Usually, there are several options which are then considered in terms of their adoption as a final solution[1]. Therefore, the decision analysis allows to select just the poly-optimal variant (i.e. no worse than any other) of the given set of acceptable variants, but it does not lead to designating the best solution.

One of the most difficult stages in constructing the decision model is a selection of an appropriate set of criteria, which has a significant influence on the quality of a solution obtained. Two most desirable features of such criteria can be mentioned. On the one hand, there should be as few variables as possible to make the decision making process the easiest and shortest as possible, and to allow the decision maker to mentally grasp and understand the impact of all the criteria for the implementation of the objective function. On the other hand, there should be enough variables to guarantee every relevant information regarding the problem.

In the most general form, multi-criteria decision-analysis problems can be represented as a decision matrix with the corresponding weight vector:

	C_1	C_2	...	C_n
A_1	f_{11}	f_{12}	...	f_{1n}
A_2	f_{21}	f_{22}	...	f_{2n}
f_m	f_{m1}	f_{m2}	...	f_{mn}

$$W = \left[w_1, w_2, \ldots, w_n\right] \tag{1}$$

where A_1, A_2, ..., A_m are possible solutions (options), among which the choice is made; C_1, C_2, ..., C_n are the criteria which are used to assess individual solutions; f_{ij} is the value of the criterion function with respect to the j-th criterion of the i-th solution and w_j is the weight of the j-th criterion.

Subsequent stages of the multi-criteria decision-analysis are as follows[2]:

a) establishing criteria for the evaluation of the system that relate to the purposes system capabilities;
b) generating alternative options with the use of mathematical or physical models, or through experiments;
c) assessing alternatives in terms of the criteria (determining the values of the criterion function);
d) introduction of weights of the criteria;
e) adoption of a single alternative for the "optimal" (preferred) one;
f) if the final solution is not acceptable, gathering new information and proceeding to the next iteration of the multi-criteria optimization.

There are many methods of multi-criteria decision – analysis, such as: AHP (Analytic Hierarchy Process)[3], ELECTRE (fr. ELimination Et Choix Traduisant la REalité)[4], PROMETHEE (Preference Ranking Organization METHod for Enrichment of Evaluations)[5], VIKOR (sr. VIseKriterijumska Optimizacija I Kompromisno Resenje)[6] and TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution)[7] can be mentioned.

The purpose of this article is to use TOPSIS method and its modification applying the interval arithmetic to analyse the level of information society development in the European Union countries between 2005 and 2010.

1. TOPSIS method for real and interval data

TOPSIS method is based on the concept that the selected alternative should have the smallest geometric distance from the positive ideal solution (zenith) and the largest distance from the anti-ideal solution – nadir. Ideal solution consists of all best criteria values available, and anti-ideal solutions – the worst of all criteria values achievable.

The main advantage of the method is that it limits the subjectivity introduced by policy makers bringing it down mainly to the stage of determining the weights of the criteria[8]. Other benefits of the method include[9]:

– the logic representing rational human choice,
– a simple calculation procedure that is easy to program,

– the results obtained for all the solutions can be visualized with the use of a polyhedron, at least for any two dimensions.

The above advantages make TOPSIS the leading multi-criteria decision-analysis method. However, the method has its weaknesses, the main one being the methodology for deriving the weights and checking the consistency of decision makers[10].

Particular steps of the TOPSIS method are made in the following order[11]:

– the creation of a standardized decision matrix; normalized value of r_{ij} is calculated according to the formula:

$$r_{ij} = \frac{f_{ij}}{\sqrt{\sum_{i=1}^{m} f_{ij}^2}}, j = 1, ..., n; I = 1, ..., m \quad (2)$$

– the calculation of the weighted standardized decision matrix, the weighted normalized value defined as v_{ij} is calculated using the formula: $v_{ij} = w_j r_{ij}$, where w_j is the weight of the j-th attribute or criterion and $\sum_{j=1}^{n} w_j = 1$; weights can be equal, determined by means of linear regression or the centroid method[12] and by means of the AHP method[13];

– determination of the ideal and anti-ideal solutions according to the formulas:

$$A^* = \{v_1^*, \ldots, v_n^*\} = \left\{\left(\max_i v_{ij} \mid j \in I'\right), \left(\min_i v_{ij} \mid j \in I''\right)\right\} \quad (3)$$

$$A^- = \{v_1^-, \ldots, v_n^-\} = \left\{\left(\min_i v_{ij} \mid j \in I'\right), \left(\max_i v_{ij} \mid j \in I''\right)\right\} \quad (4)$$

where I' is related to the benefit, and I'' to the cost criteria.

– calculation of the distance measure using n-dimensional Euclidean distance (in the Rother variants of the Manhattan or Chebyshev method distances are also used[14]); the distance of each alternative from the ideal solution is expressed by the formula:

$$d_i^* = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^*\right)^2} \quad (5)$$

similarly, the distance from the anti-ideal solution is calculated on the basis of:

$$d_i^- = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^-\right)^2} \quad (6)$$

– calculation of the relative closeness to the ideal solution, the relative closeness of the alternative A_i in relation to A^* is defined as:

$$CC_i^* = \frac{d_i^-}{d_i^* + d_i^-} \tag{7}$$

Such a formula combines the two distances, allowing the selection of a solution that offers both the greatest possible profit and, at the same time, the smallest possible loss;

– ranking the order of preference.

An important element of the method is the appropriate choice of a standardization method. In its original form, the TOPSIS method uses a vector normalization[15], which, depending on what units criterion the function has (e.g., meters or kilometres), may influence the other values in a decision matrix and the choice of a solution. Therefore, in the subsequent modifications to the method, linear normalization is usually used[16].

In recent years, TOPSIS has been widely used, e.g. in human resources management[17], transport[18], design of products[19], production[20], water resources management[21], quality control[22] and location analysis[23].

In its initial version the TOPSIS method was created for the data presented in a form of real numbers. In some cases, however, accurate determination of the criteria is difficult and it is much easier to present such value as an interval[24]. This approach is used primarily in cases where the value of the criterion function is not precisely determined, but is known to be in a certain range of $f_{ij} \in \left[f_{ij}^L, f_{ij}^U\right]$[25].

In this situation, the individual steps of TOPSIS method are slightly modified and can be represented as follows:

– the creation of a standardized decision matrix; normalization of interval boundaries for each of the criterion function, according to the formulas:

$$\bar{r}_{ij}^L = \frac{f_{ij}^L}{\sqrt{\sum_{i=1}^{m}\left(f_{ij}^L\right)^2 + \left(f_{ij}^U\right)^2}}, \quad j = 1, \ldots, n; \quad i = 1, \ldots, m \tag{8}$$

$$\bar{r}_{ij}^U = \frac{f_{ij}^U}{\sqrt{\sum_{i=1}^{m}\left(f_{ij}^L\right)^2 + \left(f_{ij}^U\right)^2}}, \quad j = 1, \ldots, n; \quad i = 1, \ldots, m \tag{9}$$

– taking into account the relevance of each criterion by introducing weights, creating the weighted normalized interval decision matrix by means of the following formulas:

$$\overline{v}_{ij}^{L} = w_i \overline{r}_{ij}^{L}, \quad j = 1, \ldots, n; \quad i = 1, \ldots, m \tag{10}$$

$$\overline{v}_{ij}^{U} = w_i \overline{r}_{ij}^{U}, \quad j = 1, \ldots, n; \quad i = 1, \ldots, m \tag{11}$$

where w_j is the weight of the j-th criterion; $\sum_{j=1}^{n} w_j = 1$;

– determination of the ideal and anti-ideal solutions according to the formulas:

$$\overline{A}^{*} = \left\{ v_1^{*}, \ldots, v_n^{*} \right\} = \left\{ \left(\max_i \overline{v}_{ij}^{U} \mid j \in I' \right), \left(\min_i \overline{v}_{ij}^{L} \mid j \in I'' \right) \right\} \tag{12}$$

$$\overline{A}^{-} = \left\{ v_1^{-}, \ldots, v_n^{-} \right\} = \left\{ \left(\min_i \overline{v}_{ij}^{L} \mid j \in I' \right), \left(\max_i \overline{v}_{ij}^{U} \mid j \in I'' \right) \right\} \tag{13}$$

where I' is related to the benefit, and I'' to the cost criteria;

– the distance of each of the considered options to the ideal and anti-ideal solution is calculated on the basis of formulas:

$$\overline{d}_i^{*} = \sqrt{\sum_{j \in I'} \left(\overline{v}_{ij}^{L} - \overline{v}_j^{*} \right)^2 + \sum_{j \in I''} \left(\overline{v}_{ij}^{U} - \overline{v}_j^{*} \right)^2}, \quad i = 1, \ldots, m \tag{14}$$

$$\overline{d}_i^{-} = \sqrt{\sum_{j \in I'} \left(\overline{v}_{ij}^{U} - \overline{v}_j^{-} \right)^2 + \sum_{j \in I''} \left(\overline{v}_{ij}^{L} - \overline{v}_j^{-} \right)^2}, \quad i = 1, \ldots, m \tag{15}$$

– determination of the closeness coefficient, which is the foundation for the solution ranking:

$$\overline{CC}_i^{*} = \frac{\overline{d}_i^{-}}{\overline{d}_i^{*} + \overline{d}_i^{-}}, \quad i = 1, \ldots, m \tag{16}$$

the closer the value $\overline{CC}_i^{*}$ to one, the better the solution.

To fully demonstrate the approach described in the article, a calculation example will be presented. In the example, in accordance with the TOPSIS method for interval data, a ranking of members of the European Union EU-27 will be created. The classification will be made according to the level of information society development in the studied countries.

2. The procedure for the ranking creation on the example of the study on the level of the information society development

The analysis of the information society, especially the one determining the level of its development, that is conducted on the basis of individual indicators can only be partial. It does not give the possibility of a coherent description of the phenomenon which, because of its complexity, is described by several different variables. Therefore, it seems that the measurement and evaluation of the information society development should use a multi-criteria approach. In common practice, the rankings are formed on the basis of data from a single year. In such a case we have to deal with the values expressed by real numbers and the traditional TOPSIS method may be used. However, if the interest is the ranking which summarizes a few years you can use a modification of the method including the interval arithmetic. The data for each indicator of the period of time are presented in the form of an interval, the limits of which are, respectively – the smallest and largest value of the variable in the accepted time frame. The conditions and procedure for the selection of indicators to assess the degree of development of information society presented in this article are shown in the publication[26].

The data were selected on the basis of the experts survey, while the indicators were collected from the Eurostat. Although already at the stage of selecting the variables only these which have complete representation in the database were taken into account, the resulting data matrix was characterized by the presence of certain deficiencies. Despite these deficiencies, in order to continue the calculation process the author decided to eliminate the objects/variables and to supplement missing information The adoption of specific practices was primarily determined by the number of empty cells in the matrix for a given indicator or country. Since the study covered such a short period of time, from the very beginning the possibility of deleting a data set representing a specific year was excluded. First, the author eliminated the indicators and the countries that were characterized by too many deficiencies to supplement their data without losing credibility. Then, the remaining deficiencies in data were completed by means of linear regression.

First, the characteristics of the occurrence of the missing data was considered for each indicator separately. At that stage, the author separated indicators that showed a significant incompleteness of the data for all the EU-27 countries. They were identified on the basis of empirical rule of “three Sigmas”[27] that showed outliers (observations which have more deficiencies than the others). A following formula was used:

$$t_i = \left| \frac{x_i - \overline{x}}{s} . \right| > 3 \tag{17}$$

where x_i is the total number of deficiencies of the i-th indicator, $\overline{x}$ and s are, respectively, mean and standard deviations of the deficiencies number in the chosen set of indices.

Having eliminated the indicators leading in terms of the number of deficiencies in the data, the next step was to find the countries in which a set of indicators did not have a single value throughout the period considered. In the case of those countries, it was taken into account whether there was a total absence of data for one, or for a larger number of indicators. When there was only a single indicator, that index was eliminated; if there were more such indicators – the country was eliminated. This procedure is justified in terms of the final results of the calculations and the final ranking of the EU-27 where we should include as many countries as possible. The depletion of the set of indicators still leaves enough indices to calculate the ranking.

In the second stage of the procedure designed to eliminate the problem of missing data, linear regression was used to estimate the unknown values of the available data To get a bit more accuracy in the data preparation stage, values for 2004, which is not directly taken into account in the audited period, were also loaded. The calculations used the regression model in the form[28]:

$$y = a \cdot x + b \tag{18}$$

for which the parameters are determined by the formula[29]:

$$a = \frac{\sum_{i=1}^{n} \left[(x_i - \overline{x}) \cdot (y_i - \overline{y}) \right]}{\sum_{i=1}^{n} (x_i - \overline{x})^2} \tag{19}$$

$$b = \overline{y} - a \cdot \overline{x} \tag{20}$$

where a is the slope of the regression line with respect to the x-axis, and b is the point where the regression line intercepts the y axis. In our case, the x-axis included subsequent years for which data were collected. On the y-axis there were values of the corresponding indicators. In the process of calculation, vector data for each country and for each indicator was dealt with separately, but it was applied only to those cases for which in the years 2005–2010 data were missing. By using the formulas described by (18) – (20), all the data were supplemented.

In the further development of a set of indicators, additional elimination of variables was carried out using the ratio of features significance[30]:

$$V_k = \frac{s_k}{\bar{x}_k} \tag{21}$$

where s_k is a standard deviation and $\bar{x}_k$ – mean the value of the k-th variable. It is assumed that the variables whose calculated ratio falls within the range of $\langle 0\,;0.1\rangle$ or is smaller than the predetermined value are quasi-fixed variables and should be eliminated from the set of the variables considered, since it does not have any significant information value (they change to a very small extent). In the presented case, the coefficient limit of 0.1 was adopted.

In the next stage of eliminating, the correlation coefficients between all the indicators which had not been eliminated in the previous steps were calculated. The correlation is calculated using the formula[31]:

$$r(X,Y) = \frac{\sum_{i=1}^{n}(x_i - \bar{x})\cdot(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \cdot (y_i - \bar{y})^2}} \tag{22}$$

With the use of the values calculated with the formula (22) the procedure for parametric method was performed[32]. That method allowed for a final reduction of features and for narrowing the set to those that have the highest informative value. The final set of indicators is shown in Table 1.

Table 1. The indicators used to create the ranking (by category)

	Indicator
ICT availability	Percentage of households with internet access at home
	Broadband coverage: percentage of population reached by broadband access
	Percentage of households with internet access at home via broadband
Citizens' access to and use of the Internet	Frequency of individual use of the Internet in the last 12 months (at least once a month; less than once a month)[33]
	Percentage of individuals using the Internet for e-banking
	Proportion of individuals who used the Internet in the last 12 months
Enterprises' access to and use of ICTs	Percentage of enterprises with broadband access
e-Public Services	Percentage of individuals using the internet for interacting with public authorities

Source: own elaboration.

Data prepared in this way were processed in accordance with further steps of the TOPSIS method for interval data, and the ranges have been developed on the basis of the minimum and maximum values of the period considered (for each country separately). All the characteristics were used as the benefit criteria. The result of the calculation allowed to obtain a ranking which is shown in Table 2.

Table 2. Results of the ranking obtained
with the use of the TOPSIS method for interval data

No.	Country	Result
1.	Great Britain	0.5632
2.	Finland	0.5605
3.	Sweden	0.5602
4.	Germany	0.5372
5.	Netherlands	0.5307
6.	Luxembourg	0.5306
7.	Denmark	0.5237
8.	France	0.4962
9.	Spain	0.4943
10.	Austria	0.4541
11.	Belgium	0.4526
12.	Estonia	0.4480
13.	Ireland	0.4141
14.	Slovenia	0.4027
15.	Malta	0.3862
16.	Slovakia	0.3577
17.	Latvia	0.3566
18.	Czech Republic	0.3533
19.	Italy	0.3482
20.	Poland	0.3395
21.	Cyprus	0.3349
22.	Lithuania	0.3317
23.	Portugal	0.3303
24.	Hungary	0.3106
25.	Bulgaria	0.1909
26.	Romania	0.1697

Source: own elaboration.

Depending on the values of the closeness coefficient which determines the position of the country in the ranking, the examined countries were divided into four classes, as shown in Figure 1 in the form of a map.

The analysis of the results obtained in calculations confirms the significant dominance of the Scandinavian countries, Germany, the UK and the Netherlands. In these countries, we can talk

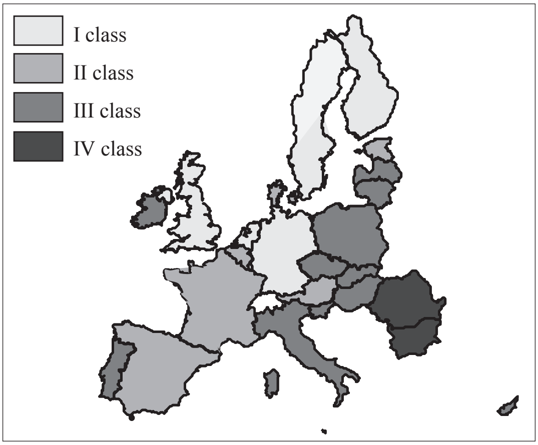

Fig. 1. The European Union countries EU-27 divided into classes according to values of the closeness coefficient

Source: own elaboration.

about the highest level of the information society development in 2005–2010. Much weaker is the position of the countries whose European Union membership is the shortest – especially Bulgaria and Romania. The position of Poland compared with countries which became the EU members in 2004 is low. With such a rapidly growing economy, the third ten is not a satisfactory result.

Conclusions

The use of the multi-criteria decision-analysis TOPSIS method for the interval data to measure the level of the information society development made it possible to create the ranking which provides summary of a longer period of the phenomenon's development. In contrast to the analysis based on data from one year, and using real values of variables describing the investigated objects, a generalized picture of the period 2005–2010 has been obtained. It is not then possible to extract information needed for accurate classification in each of the years, but thanks to this approach further aggregation of the results is not necessary when the subject of the research is to assess changes in time.

The proposed approach is not limited to the issue of measuring the information society development. It can be successfully used in the analysis of other complex phenomena as an alternative approach to using composite indicators in creating the rankings and classifications of objects.

Notes

[1] Hwang, Yoon (1981).
[2] Opricovic, Tzeng (2004).
[3] Saaty (1980).
[4] Roy (1968).
[5] Brans (1982).
[6] Opricovic (1998).
[7] Hwang, Yoon (1981).
[8] Olson (2004).
[9] Shih, Shyur, Lee (2007).
[10] Ibidem.
[11] Chen, Hwang (1992).
[12] Olson (2004).
[13] Tsaur, Chang, Yen (2002).
[14] Olson (2004).
[15] Opricovic, Tzeng (2004).
[16] Lai, Liu, Hwang (1994).
[17] Chen, Tzeng (2004).
[18] Janic (2003).
[19] Kwong, Tam (2002).
[20] Milani, Shanian, Madoliat (2005).
[21] Srdjevic, Medeiros, Faria (2004).
[22] Yang, Chou (2005).
[23] Yoon, Hwang (1985).
[24] Tikhonenko (2011).
[25] Jahanshahloo, Hosseinzadeh, Izadikhah (2006).
[26] Łatuszyńska. (2012).
[27] Aczel (2000).
[28] Zeliaś (2000).
[29] Griffiths (2012).
[30] Kukuła (2000).
[31] Zeliaś (2000).
[32] the method is described in detail in Nowak (1990).
[33] The index was divided into four separate indicators defining appropriate proportion of the population that uses the internet every day, at least once a week, at least once a month and less than once a month.

References

Aczel, A.D. (2000). *Statystyka w zarządzaniu*. Warszawa: Wydawnictwo Naukowe PWN.

Brans, J.P. (1982). *L'ingénierie de la décision:* élaboration *d'instruments d'aide à la décision. La méthode PROMETHEE*. Laval: Presses de l'Université Laval.

Chen, M.F. & Tzeng, G.H. (2004). Combining gray relation and TOPSIS concepts for selecting an expatriate host country. *Mathematical and Computer Modelling*, 40, 1473–1490. DOI: 10.1016/j.mcm.2005.01.006.

Chen, S.J. & Hwang, C.L. (1992). *Fuzzy Multiple Attribute Decision Making: Methods and Applications*. Berlin: Springer-Verlag.

Griffiths, D. (2012). *Head First. Statystyka*. Gliwice: Helion.

Hwang, C.L. & Yoon, K. (1981). *Multiple Attribute Decision Making: Methods and Applications*. New York: Springer-Verlag.

Jahanshahloo, G.R., Hosseinzadeh, L.F. & Izadikhah, M. (2006). An algorithmic method to extend TOPSIS for decision-making problems with interval data. *Applied Mathematics and Computation*, 175, 1375–1384. DOI: 10.1016/j.amc.2005.08.048.

Jahanshahloo, G.R., Hosseinzadeh, L.F. & Izadikhah, M. (2006). Extension of the TOPSIS method for decision-making problems with fuzzy data. *Applied Mathematics and Computation*, 181, 1544–1551. DOI: 10.1016/j.amc.2006.02.057.

Janic, M. (2003). Multicriteria evaluation of high-speed rail, transrapid maglev, and air passenger transport in Europe. *Transportation Planning and Technology*, 26 (6), 491–512. DOI: 10.1080/0308106032000167373.

Kukuła, K. (2000). *Metoda unitaryzacji zerowanej*. Warszawa: Wydawnictwo Naukowe PWN.

Kwong, C.K. & Tam, S.M. (2002). Case-based reasoning approach to concurrent design of low power transformers. *Journal of Materials Processing Technology*, 128, 136–141. DOI:10.1016/S0924-0136(02)00440-5.

Lai, Y.J., Liu, T.Y. & Hwang C.L. (1994). TOPSIS for MODM. *European Journal of Operational Research*, 76, 486–500. DOI: 10.1016/0377-2217(94)90282-8.

Łatuszyńska, A. (2012). Key Indicators in Evaluation of the Level of Information Society Development. *Studies & Proceedings of Polish Association for Knowledge Management*, 60, 108–121.

Milani, A.S., Shanian, A. & Madoliat, R. (2005). The effect of normalization norms in multiple attribute decision making models: A case study in gear material selection. *Structural Multidisciplinary Optimization*, 29, 312–318. DOI: 10.1007/s00158-004-0473-1.

Nowak, E. (1990). *Metody taksonomiczne w klasyfikacji obiektów społeczno-gospodarczych*. Warszawa: Państwowe Wydawnictwo Ekonomiczne.

Olson, D. L. (2004). Comparison of weights in TOPSIS models. *Mathematical and Computer Modelling*, 40 (7–8), 721–727. DOI: 10.1016/j.mcm.2004.10.003.

Opricovic, S. (1998). *Multicriteria Optimization of Civil Engineering Systems*. Belgrade: Faculty of Civil Engineering.

Opricovic, S. & Tzeng, G. (2004). Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS. *European Journal of Operational Research*, 156, 445–455. DOI: 10.1016/S0377-2217(03)00020-1.

Roy, B. (1968). Classement et choix en présence de points de vue multiples (la méthode ELECTRE). *La Revue d'Informatique et de Recherche Opérationelle (RIRO)*, 8, 57–75.

Saaty, T.L. (1980). *The Analytic Hierarchy Process*. New York: McGraw Hill.

Shih, H.S., Shyur, H.J. & Lee E.S. (2007). An extension of TOPSIS for group decision making. *Mathematical and Computer Modelling*, 45 (7–8), 801–813. DOI: 10.1016/j.mcm.2006.03.023.

Srdjevic, B., Medeiros, Y.D.P. & Faria, A.S. (2004). An objective multi-criteria evaluation of water management scenarios. *Water Resources Management*, 18, 35–54. DOI: 10.1023/B:WARM.0000015348.88832.52.

Tikhonenko, A. (2011). Przedziałowe rozszerzenie metody TOPSIS, XII Studencka Konferencja Informatyczna, 15 June 2011. Częstochowa: Politechnika Częstochowska.

Tsaur, S.H., Chang, T.Y. & Yen C.H. (2002). The evaluation of airline service quality by fuzzy MCDM. *Tourism Management*, 23, 107–115. DOI: 10.1016/S0261-5177(01)00050-4.

Yang, T. & Chou, P. (2005). Solving a multiresponse simulation–optimization problem with discrete variables using a multi-attribute decision-making method. *Mathematics and Computers in Simulation*, 68, 9–21. DOI: 10.1016/j.matcom.2004.09.004.

Yoon, K. & Hwang C.L. (1985). Manufacturing plant location analysis by multiple attribute decision making: Part I—single-plant strategy. *International Journal of Production Research*, 23, 345–359. DOI: 10.1080/00207548508904712.

Zeliaś, A. (2000). *Metody statystyczne*. Warszawa: Polskie Wydawnictwo Ekonomiczne.