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THE JACKSON QUEUEING NETWORK MODEL BUILT USING POISSON MEASURES. APPLICATION TO A BANK MODEL

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Abstract

In this paper we will build a bank model using Poisson measures and Jackson queueing networks. We take into account the relationship between the Poisson and the exponential distributions, and we consider for each credit/deposit type a node where shocks are modeled as the compound Poisson processes. The transmissions of the shocks are modeled as moving between nodes in Jackson queueing networks, the external shocks are modeled as external arrivals, and the absorption of shocks as departures from the network.

Keywords: Jackson queueing networks, Poisson measures, banking.

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Introduction

First of all we will present some elements on the interest theory that are applied in financial operations, such as credits and deposits. The interest for a financial operation represents the sum added periodically to the initial sum S_0 during the operation time T until its end. The interest can be simple or compound[1].

Denoting by S_t the amount at the moment t from the initial moment to the maturity T, and by R the interest rate (the ratio of the initial sum added periodically), we obtain:

$$S_t = S_0 \left(1 + R \cdot t\right) \tag{1}$$

The above interest rate can be expressed either as a number between 0 and 1 (the ratio between the periodically added sum and the initial one), or as percentage: $100 \cdot R = p\%$. If we want to compute the initial capital in terms of the final capital, the duration of the operation and the interest rate, we obtain:

$$S_0 = \frac{S_T}{1 + R \cdot T} \tag{1'}$$

Next, we define the equivalence by interest of two financial operations. For this purpose let us suppose that the financial operations A and B are decomposed in m, respectively n simple financial operations, characterized by initial sums $S_0^{(i)}$, the interest rates R_i and the duration T_i, respectively by initial sums $S_0^{\prime(j)}$, the interest rates $R_j^{\prime}$ and the duration $T_j^{\prime}$.

Definition 1. *The financial operations A and B are equivalent by interest in the case of simple interest, and we write* $A \sim_{ü\ ;} B$ *if*:

$$\sum_{i=1}^{m} S_0^{(i)} \cdot R_i \cdot T_i = \sum_{j=1}^{n} S_0^{\prime(j)} \cdot R_j^{\prime} \cdot T_j^{\prime}.$$

By a multiple replacement operation we maintain two of the three elements that define the operation (the initial capitals, S_k, the interest rates, R_k, and the durations, T_k), and the third element becomes identical for all the n components, finally obtaining an equivalent operation. The initial expected replacing capital is:

$$S = \frac{\sum_{k=1}^{n} S_k \cdot R_k \cdot T_k}{\sum_{k=1}^{n} R_k \cdot T_k} \tag{2}$$

The annual expected replacing interest rate is:

$$R = \frac{\sum_{k=1}^{n} S_k \cdot R_k \cdot T_k}{\sum_{k=1}^{n} S_k \cdot T_k} \tag{3}$$

The expected replacing maturity is:

$$T = \frac{\sum_{k=1}^{n} S_k \cdot R_k \cdot T_k}{\sum_{k=1}^{n} S_k \cdot R_k} \tag{4}$$

By a unique replacement operation we fix first two of the three elements (the initial capital, the annual interest rate and the duration) for all the *n* components, and next we fix the third element so that we obtain an equivalent financial operation by simple interest[2]. The initial unique replacing capital is:

$$S = \frac{\sum_{k=1}^{n} S_k \cdot R_k \cdot T_k}{R \cdot T} \tag{5}$$

The annual unique replacing interest rate is:

$$R = \frac{\sum_{k=1}^{n} S_k \cdot R_k \cdot T_k}{S \cdot T} \tag{6}$$

The unique replacing duration (maturity) is:

$$T = \frac{\sum_{k=1}^{n} S_k \cdot R_k \cdot T_k}{S \cdot R} \tag{7}$$

The above equivalence can be defined in terms of the actual value, i.e. the sum of initial capitals if we know the final capitals, the interest rates and the maturities. More precisely, we have the following definition:

Definition 2. *Let A and B be financial operations that are decomposed in* n*, respectively* m *simple financial operations. Denote by* S_i*,* R_i *and* T_i*, respectively by* S'_j*,* R'_j *and* T'_j *the final capitals, the interest rates and the maturities of components of the two financial operations. The financial operations* A *and* B *are equivalent by actual value in simple interest regime if*

$$AV(A)=\sum_{i=1}^{n}\frac{S_i}{1+R_i\cdot T_i}=\sum_{j=1}^{m}\frac{S_j'}{1+R_j'\cdot T_j'}=AV(B).$$

Analogously to the case of equivalence by interest we define the financial operation with multiple substitutions and the financial operation with unique substitution, as follows. The expected final replacing capital is:

$$S=\frac{\sum_{i=1}^{n}\frac{S_i}{1+R_i\cdot T_i}}{\sum_{i=1}^{n}\frac{1}{1+R_i\cdot T_i}} \tag{8}$$

The expected replacing annual interest rate is computed by solving the nonlinear equation with the variable R:

$$\sum_{i=1}^{n}\frac{S_i}{1+R_i\cdot T_i}=\sum_{i=1}^{n}\frac{S_i}{1+R\cdot T_i} \tag{9}$$

and the expected replacing maturity is computed by solving the nonlinear equation with the variable T:

$$\sum_{i=1}^{n}\frac{S_i}{1+R_i\cdot T_i}=\sum_{i=1}^{n}\frac{S_i}{1+R_i\cdot T} \tag{10}$$

In the case of the unique replacing operations we have closed formulae for the three elements. The unique replacing final capital is:

$$S=(1+R\cdot T)\cdot\sum_{i=1}^{n}\frac{S_i}{1+R_i\cdot T_i} \tag{11}$$

the unique replacing interest rate is:

$$R=\frac{1}{T}\left(\frac{S}{\sum_{i=1}^{n}\frac{S_i}{1+R_i\cdot T_i}}-1\right) \tag{12}$$

and the unique replacing maturity is:

$$T = \frac{1}{R}\left(\frac{S}{\sum_{i=1}^{n} \frac{S_i}{1+R_i \cdot T_i}} - 1 \right) \tag{13}$$

In the case of compound interest the maturity is divided into n periods T_1, T_2, ..., T_n. The final capital is in this case[3]:

$$S_T = S_0 \cdot \prod_{i=1}^{n} \left(1 + R_i T_i\right) \tag{14}$$

where S_0 is the initial capital, and R_i is the interest rate on the period T_i.

Usually T is divided into equal time periods, the common length of these time periods becoming time unit ($T_i = 1$). If the interest rate is constant, R, over all the duration T, the formula (14) becomes:

$$S_T = S_0 \left(1 + R\right)^T \tag{14'}$$

Remark 1. *Sometimes the maturity T, is not supposed to be an integer, considering* $T = n + T_{n+1}$ *with* $T_{n+1} \in (0,1)$. *In this case the above value* S_T *is the trading solutionand the rational solution is* $S_T = S_0 \left(1+R\right)^n \cdot \left(1 + R \cdot T_{n+1}\right)$.

In the case of deposits at a given term, also the solution with lost interest is used: $S_T = S_0 \left(1+R\right)^n$.

The initial capital is[4] computed using (14) and (14'), as in the case of the simple interest. We obtain the general formula:

$$S_0 = \frac{S_T}{\prod_{i=1}^{n} \left(1 + R_i T_i\right)} \tag{15}$$

and in the particular case when $T_i = 1$ and $R_i = R$:

$$S_0 = \frac{S_T}{\left(1+R\right)^T} \tag{15'}$$

Obviously, we can derive analogue formulae for the rational solution and for the solution with the lost interest.

Analogously to the case of the simple interest we define the equivalence by interest and the equivalence by actual value. For this we denote by $D = S_T - S_0$ the interest of a financial operation, where S_T is the final capital computed using (14). Given the final capital, the actual value is the initial capital computed using (15).

Definition 3. *Consider the financial operations A and B that are divided in* m*, respectively n financial operations. Suppose also that their maturities are* T_i*,* $i = \overline{1,m}$ *and* $T_j^{'}$*,* $j = \overline{1,n}$*, which are decomposed in* m_i*, respectively* n_j *sub-periods with constant interest rates.*

Given the initial capitals, the financial operations A and B are called equivalent by compound interest if the sum of interests for the components of A is equal to the sum of interests for the components of B.

Given the final capitals, the financial operations A and B are called equivalent by actual value in compound interest regime if the sum of actual values for the components of A is equal to the sum of actual values for the components of B.

For replacing operations, we suppose first of all that each component of a financial operation has the interest rate constant during its period. Otherwise, in both cases[5] we solve the equation:

$$\left(1+R\right)^{T} = \prod_{i=1}^{n}\left(1+R_i \cdot T_i\right) \tag{16}$$

Therefore we can next assume that the financial operation A is decomposed in n components, having the interest rates R_{i} and the maturities T_{i}, where $i = \overline{1,n}$.

Next, we define the multiple replacing operations and the unique replacing operations in the case of equivalence by compound interest.

The initial expected replacing capital is:

$$S = \frac{\sum_{i=1}^{n} S_i\left(\left(1+R_i\right)^{T_i} - 1\right)}{\sum_{i=1}^{n}\left(1+R_i\right)^{T_i} - n} \tag{17}$$

where S_{i} are the initial capitals.

The expected replacing annual interest rate is computed by solving the nonlinear equation with the variable R:

$$\sum_{i=1}^{n} S_i\left(\left(1+R\right)^{T_i} - 1\right) = \sum_{i=1}^{n} S_i\left(\left(1+R_i\right)^{T_i} - 1\right) \tag{18}$$

and the expected replacing maturity is computed by solving the nonlinear equation with the variable T:

$$\sum_{i=1}^{n} S_i\left(\left(1+R_i\right)^{T}-1\right)=\sum_{i=1}^{n} S_i\left(\left(1+R_i\right)^{T_i}-1\right) \tag{19}$$

The initial unique replacing capital is:

$$S=\frac{\sum_{i=1}^{n} S_i\left(\left(1+R_i\right)^{T_i}-1\right)}{n\left(\left(1+R\right)^{T}-1\right)} \tag{20}$$

The unique replacing annual interest rate is:

$$R=\left(\frac{\sum_{i=1}^{n} S_i\left(\left(1+R_i\right)^{T_i}-1\right)}{nS}+1\right)^{\frac{1}{T}}-1 \tag{21}$$

and the unique replacing maturity is:

$$\frac{\ln\left(\frac{\Sigma S_i\left(\left(1+R_i\right)\quad -1\right)}{nS}\quad 1\right)}{\ln\left(1\qquad\right)} \tag{22}$$

The replacing operations in the case of actual value in compound interest regime are defined as follows. The final expected replacing capital is:

$$S=\frac{\sum_{i=1}^{n}\frac{S_i}{\left(1+R_i\right)^{T_i}}}{\sum_{i=1}^{n}\frac{1}{\left(1+R_i\right)^{T_i}}} \tag{23}$$

where S_i are the final capitals.

The expected replacing annual interest rate is computed by solving the nonlinear equation with the variable R:

$$\sum_{i=1}^{n}\frac{S_i}{\left(1+R\right)^{T_i}}=\sum_{i=1}^{n}\frac{S_i}{\left(1+R_i\right)^{T_i}} \tag{24}$$

and the expected replacing maturity is computed by solving the nonlinear equation with variable *T:*

$$\sum_{i=1}^{n} \frac{S_i}{\left(1+R_i\right)^T} = \sum_{i=1}^{n} \frac{S_i}{\left(1+R_i\right)^{T_i}} \tag{25}$$

The final unique replacing capital is:

$$S = \left(1+R\right)^T \cdot \sum_{i=1}^{n} \frac{S_i}{\left(1+R_i\right)^{T_i}} \tag{26}$$

the unique replacing annual interest rate is:

$$R = \left(\frac{S}{\sum_{i=1}^{n} \frac{S_i}{\left(1+R_i\right)^{T_i}}} \right)^{\frac{1}{T}} - 1 \tag{27}$$

and the unique replacing maturity is:

$$T = \frac{\ln S - \ln\left(\sum_{i=1}^{n} \frac{S_i}{\left(1+R_i\right)^{T_i}} \right)}{\ln\left(1+R\right)} \tag{28}$$

Next, we present some notions on queueing systems and queueing networks. These models can be applied in a bank model because they use the exponential models for times/ Poisson models for the number of customers. The Poisson models are also used in finances for modeling shocks. Kleinrock[6] and Garcia et al.[7] presented the service systems with bulk arrivals and with bulk services. Technically, these service systems are built starting from the *M/M/1* system, for which the interarrival time is $\exp\left(\lambda\right)$, the service time is $\exp\left(\mu\right)$, one server and infinite queue. The difference is that instead of a single customer we have a group of *k* customers that arrive and are served one by one by the channel of the system.

It is proved that these service systems are equivalent to the service system with $\exp\left(\lambda\right)$ interarrival time and $E_k\left(\mu\right)$ service time, respectively with $E_k\left(\lambda\right)$ interarrival time and $\exp\left(\mu\right)$ service time. This is called the method of phases in the above mention books.

According to Zbăganu[8], a light tail distribution is the distribution of a random variable *X* such that the moments' generating function:

$$E\left(e^{-\xi \cdot X}\right) \tag{29}$$

is finite for any ξ complexes number in the neighborhood of zero. Otherwise, we have a heavy tail distribution.

A heavy tail distribution is the Pareto distribution[9], for which the cdf is:

$$F(t)=\begin{cases}1-\left(1-\dfrac{a(t-c)}{b}\right)^{\frac{1}{a}}, \text{if } a\neq 0\\ 1-\exp\left(-\dfrac{x-c}{b}\right), \text{if } a=0\end{cases} \tag{30}$$

From the presented method of moments, it results that the *r*-th moment exists if and only if $a>-\frac{1}{r}$. We can derive from here that the Pareto distribution is light tail for $a\geq 0$, and heavy tail for $a<0$.

Another heavy tail distribution is presented in Drăgan and Simionescu[10], namely the inverse Weibull distribution, with the cdf:

$$F(t)=\exp\left(-\frac{1}{\theta\cdot t^k}\right) \tag{31}$$

where $\theta, k>0$.

In the case of Pareto distribution, the method of moments $\left(a>-\frac{1}{3}\right)$ leads to a nonlinear equation in a, involving the skewness, *G*. The other two parameters are computed in terms of *a*, and the other two moments[11]:

$$\begin{cases}\bar{X}=c+\dfrac{b}{1+a}\\ S^2=\dfrac{b^2}{(1+a)^2(1+2\cdot a)}\\ G=\dfrac{2(1+a)\sqrt{1+2\cdot a}}{1+3\cdot a}\end{cases} \tag{32}$$

If we use the maximum likelihood method we obtain $c=\min(X_i)$, and the other two parameters are estimated from the nonlinear system:

$$\begin{cases} \sum_{i=1}^{n} \frac{X_i - c}{b - a\left(X_i - c\right)} = \frac{n}{1-a} \\ \sum_{i=1}^{n} \ln\left(1 - \frac{a\left(X_i - c\right)}{b}\right) = -n \cdot a \end{cases} \tag{32'}$$

For the inverse Weibull distribution, used by Drăgan and Simionescu[12] to model complex technical systems, the maximum likelihood method is used. For fixed $k > 0$ we have to solve the equation:

$$\frac{\partial \ln L}{\partial \theta} = -\frac{n}{\theta} + \frac{1}{\theta^2} \sum_{i=1}^{n} \frac{1}{X_i^k} = 0 \tag{33}$$

and from here:

$$\theta = \frac{1}{n} \sum_{i=1}^{n} \frac{1}{X_i^k} \tag{33'}$$

2. Modeling using the compound Poisson processes

Next we define the compound Poisson process[13] that will be useful in our model.

Definition 4. *A compound Poisson process with the intensity* λ *and the jump size* f *is a stochastic process*:

$$X_t = \sum_{i=1}^{N_t} Y_i,$$

where N_t *is a Poisson process with the intensity* λ*, and the random variables* Y_i, $i \geq 1$ *are independent and they have the same distribution, f.*

If the distribution *f* is such that $Y_i = 1$ with the probability 1, we have $X_t = N_t$. Therefore the Poisson process is a particular case of the compound Poisson process. We have the following property of the compound Poisson processes.

Proposition 1. *The stochastic process* $\left(X_t\right)_{t\geq 0}$ *is a compound Poisson process if and only if it is a Lévy process and its sample paths are piecewise constant functions.*

Next we define the Lévy process[14].

Definition 5. *A Lévy process is a stochastic process* X_t *such that:*

(a) *Independent increments: for any sequence of time moments* $t_1 < t_2 < \ldots < t_n$, *the random variables* $X_{t1}, X_{t2} - X_{t1}, \ldots, X_{tn} - X_{tn-1}$, *are independent.*

(b) *Stationary increments: for any t, h > 0 the distribution law of $X_{t+h} - X_t$ depends only on h, and it does not depend on t.*

(c) *Stochastic continuity: for any ε > 0 we have* $\lim_{h \to 0} P\left(\left|X_{t+h} - X_t\right| > \varepsilon\right) = 0$.

The examples of the Lévy processes are the Brownian motion, where the distribution law of $X_{t+h} - X_t$ is normal, and the above mentioned compound Poisson processes. Only the Brownian motion has continuous paths. It means that the point (c) from Definition 5 does not imply that sample paths are continuous. In fact, any Lévy process has the Lévy-Itô decomposition, as follows[15]. First of all, we define the Lévy measure.

Definition 6. *Let X_t be a Lévy process. The Lévy measure is defined on the Borelian sets in R (or in R^d if the dimension of the process is d > 1) such that*:

$$\nu(A) = E\left(\#\left\{t \in [0,1] \middle| \Delta X_t \neq 0, \Delta X_t \in A\right\}\right),$$

i.e. the measure of the number of jumps that have the value in A.

Theorem 2. *Let X_t be a Lévy process. There exist γ > 0, the Brownian motion B_t, the compound Poisson process X_t^1 and the family of compound Poisson processes $\left(\tilde{X}_t^\varepsilon\right)_{\varepsilon > 0}$ such that*:

$$\mathrm{X_t} = \mathrm{t} \cdot \gamma + \mathrm{B_t} + \mathrm{X_t^1} + \lim_{\varepsilon \searrow 0} \mathrm{X_t^\varepsilon},$$

where X_t^1 has Lévy measure greater than one, and X_t^ε has the Lévy measure between ε and one.

If in the definition of the compound Poisson process Y_i we change the unit of time and same for unit of Y_i (which is money unit in our case), we can consider the shocks on credits and on the deposits modeled as Poisson processes. The same we can say about absorbing the shocks. Now, using the well-known exponential distribution of times between events if the number of events per time unit is Poisson (same parameter λ), we can represent each type of credit/deposit as a node in a queueing network with shocks being the money the bank should pay (for credits or for interests at deposits), and services being the money the bank should receive (the interests paid by the customers, and the money received for deposits).

First we have to define the Jackson queueing network[16].

Definition 7. *The Jackson queueing network is an open queueing network with* n *nodes, exponential external arrivals* $\exp(\lambda_i),\ i = \overline{1,n}$, *exponential services* $\exp(\mu_i),\ i = \overline{1,n}$, *and, when they finish their service at the node* i, *a customer goes to the node* j *with the probability* p_{ij} p_{ij}, *or leaves the network with the probability* p_{i0}.

We can prove[17] that the total arrivals (external or from another node) are $\exp(\Lambda_i),\ i = \overline{1,n}$, where Λ_i is the solution of the system:

$$\sum_{j=1}^{n} P_{ji}\Lambda_j + \lambda_i = \Lambda_i \tag{34}$$

The Jackson queueing network is stable if $\Lambda_i < \mu_i$.

The Jackson queueing network representing a bank is in Figure 1, that follows.

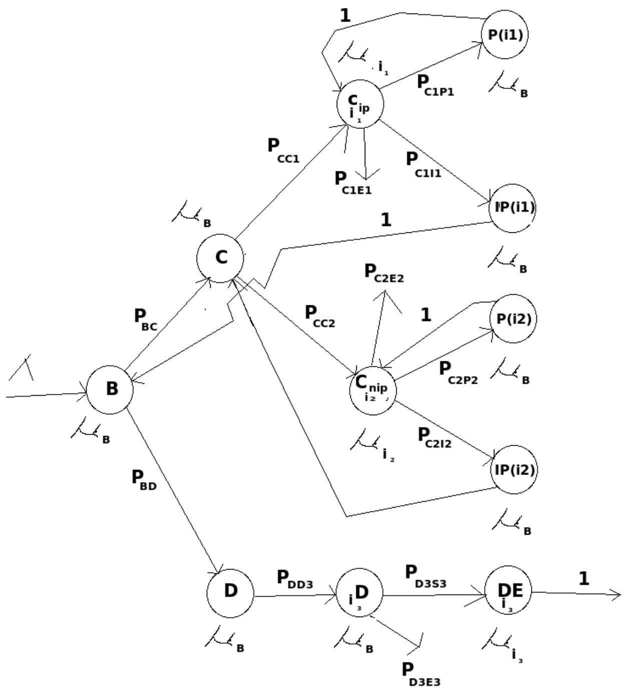

Fig. 1. The model for a bank using the Jackson queueing network

Source: own research.

In the above graphics, we have denoted the average borrowed sum per month by λ, and the average deposited sum per month by μ. The node B is the bank, the node C represents all the credits types contracted by the bank, and D is a similar node, but it represents the deposits. $C_{nip;i_1}$ while $C_{nip;i_2}$ represent the mortgage credit type i_1, respectively non-mortgage credit type i_2. Correspondingly, D_{i_3} is the deposit type i_3.

The only external arrivals are exp (λ) in B. We take the services exp (μ) in B, C, D, and other nodes with the parameter of this service such that it is large enough such that the corresponding nodes are stable (the interarrival Λ_j from system (29) are less than μ_B). For instance $\mu_B = \lambda + \mu$.

The other services are the expected sums that the bank must receive in one month from the corresponding type of credit/deposit. It means the expected sum of payments in the case of credits if they are actually paid (no historical debts, or defaults), and the average total sum deposited in deposit type i_3 for deposits.

The probabilities from C and D are proportional to the sums borrowed for the corresponding credit type/deposited for the corresponding deposit type. For credits we have two corresponding nodes: $P(i_j)$ and $IP(i_j)$, i.e. the historical debts, respectively the default. They have probabilities proportional to the sums that have not been yet paid by the customers who have only historical debts, respectively they are in the default situation. The probability to leave the network is proportional to the sum paid at (on?) time. P_{D3S3} is equal to $\frac{\lambda_{i_3}}{\mu_{i_3}}$, where μ_{i_3} is the expected sum deposited in the deposit type i_3, and λ_{i_3} is the average sum paid by the bank per month as interest. Note that the sum of probabilities for the exit from each node is one. If from a node we can move only to another given node, the probability is one, as we have represented in the graphics.

When the Jackson queueing network is not stable, the bank can increase μ (the expected total sum deposited). But it means that the bank must pay interests, hence it involves a long term loss.

Another method is to decrease the probabilities of historical debts, and, more importantly, the probabilities of default. Both can be done by the stress test imposed by The National Bank of Romania. It means that, before receiving a credit, a customer must complete a questionnaire in order to inform the bank about their financial situation (and attaching some documents that confirm the answers are true). According to the answers (checked), the bank decides to grant the credit (considering that the customer will never be in the situation of historical debts, or even in the situation of default), or not. We denote by α the first degree error, i.e. the probability that the bank decides that the customer will not be in the corresponding situation, while in fact they will (consequently, the bank makes the error to give a credit to a customer that will not pay their debt in time). The second degree error β is the probability that the bank decides that the customer will be in the corresponding situation, while in fact they will not (consequently the bank refuses the customer, and the customer receives the same credit from another bank, and they will pay their debt in time). In both cases the bank makes the wrong decision: either to give the credit to a bad customer (α), or to refuse the credit to a good customer (β). These errors depend on the situation that is taken into consideration (historical debts, respectively default).

Next we consider the worst case, i.e. the default, and we act in the same way as in the case of historical debts. The corresponding interarrival time parameter from (29) is denoted by Λ, and the service parameter (the expected sum that the bank must receive in a month for the given credit type) by μ. The initial loss of the bank due to defaults is:

$$pierd = \mu \cdot P_{C_j I_j},\ j \in \{1,2\} \tag{35}$$

After applying the stress test, Λ decreases to Λ', because the probability of default has decreased by a multiplication by α. Consequently the new loss of the bank is:

$$pierd' = \mu' \cdot P'_{C_j I_j} + (\mu - \mu') \cdot P'_{C_j E_j} \tag{35'}$$

where we have marked the new values after the test as "prime". They are:

$$\begin{cases} P'_{C_j I_j} = \frac{P_{C_j I_j} \cdot \alpha}{P_{C_j I_j} \cdot \alpha + P_{C_j E_j} \cdot (1-\beta)} \\ P'_{C_j E_j} = 1 - P'_{C_j I_j} - P_{C_j P_j} \\ \mu' = \mu \cdot (1-\beta). \end{cases} \tag{35''}$$

Conclusions

In the Lévy process, the Brownian motion from the Lévy-Itô decomposition, as we can see in Geman, Madan and Yor[18,] captures the stable part of the process. The shocks, i.e. the problem with which this paper deals with, are captured by the Poisson part. It remains an open problem how we can model shocks using the third term from the Lévy-Itô decomposition[19].

When we compute the new loss of a bank we must take into account that due to the second degree error the bank loses customers, hence μ decreases as in (30"). An open problem is to check how the stress test and its errors influence the deposits. Therefore the loss of the bank decreases due to the falling probability of default/historical debts. But it must be added the term arisen from loss of customers, due to the second order error.

In the case of non-mortgage credits the bank can reschedule the credit and in this way decrease the "number of customers", i.e. the sum the bank must receive the next month (μ is the sum that the bank must receive next month in the case of lack of historical debts and of the defaults).

Note that the node to which the probability is one is the bank (B) in the case of mortgage credit, respectively the credits node, C if the credit is non-mortgage. This is because the bank sells the customer's property in the first case, but the customer takes a credit for buying it.

When at a node that represents a credit the current payment come together with historical debts, we can consider bulk arrivals at that node. Therefore, according Kleinrock[20] and

Garcia et al.[21], we can consider Erlang services. But an Erlang distribution is also a light tail one, because it is the convolution of *k* exponential services. A problem open for further investigation isthe queueing network from Figure 1 in this case, or in the case when services become heavy tail ones, as in Singh and Guo[22] and in Drăgan and Simionescu[23].

The same problems can be studied in the case of arrivals, that model the case of lack of confidence in banks, when banks have money, but they have only few credit customers.

Notes

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3 Ibidem.
4 Ibidem.
5 Ibidem.
6 Kleinrock (1975).
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