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DECIMAL POSITIONAL FRACTIONS*. THEIR USE FOR THE SURVEYING PURPOSES (FERRARA, 1442)

Some years ago, Giovanni Bianchini (ca.1400–ca.1470) was introduced into the history of mathematics as the author of the tables of decimal trigonometric functions, the first such tables in the West[1]. In these tables, however, the numerical values were not tabulated as fractions[2]. The question that I posed then to myself was about the presence of decimal fractions, in proper sense of the word, in other Bianchini's works till now unpublished or even uncatalogued. Lastly, while preparing the first edition of Bianchini's *De arithmetica* – so is entitled the treatise that opens his *Flores Almagesti* – I wondered whether it would be possible to attribute the invention of decimal positional fractions to Bianchini on the grounds of a fragment that runs as follows:

> "...omnis figura firmata in ordine numerorum denotat fractionem decenariam loci figurae immediate sequentem ad sinistram, ut puta 342. Dico quod 2 denotat duo decima unius decenae et 4 sunt 4 decima unius centenarii etc."[3]

Actually, as it results from the quotation, even if Bianchini uses the term "fractio decenaria", nevertheless he does not go as far as to consider tenth parts of a unit[4].

Fortunately, however, the *De arithmetica* appears not to be Bianchini's ultimate expression about decimal fractions, since his mathematical and astronomical legacy furnishes still another source valuable for the subject, namely the treatise *Compositio instrumenti*[5]. In this treatise, finally, the concept of decimal positional fractions, together with the rules for the operations on them, are clearly exposed.

The *Compositio instrumenti* is devoted to the construction and use of a surveying instrument. The calculations of the altitudes and distances of inaccessible objects are based either on the theorem of similar triangles or on the Pythagoras' theorem. All is referred to the *Elements.*

Bianchini describes an instrument that at a glance seems to be related to Ptolemy's *regulae* (*Almagest* V, 12). It is only a closer look at the principles of its construction and use that permits to discover the connection of the instrument with the observational quadrant (the *scala mathematica* at the revers of a quadrant) rather than with the *regulae*[6]. Although Bianchini does not give any specific name to the instrument, according to Domenico Fava such instruments were known under the name *biffa*[7].

The instrument deserves an exhaustive study for itself, as well as for its place in the Renaissance tradition of the surveying instruments. For the sake of this paper, however, I confine myself to the study of what is the essence of Bianchini's achievement, namely to the study of principles of both, construction and use of the instrument, the special attention being paied to its scales.

In fact, while to scale the instrument Bianchini introduces fractions of the tenth progress, and subsequently while operating on the data read off from the instrument and expressed in decimal positional fractions, namely while multiplying, dividing and extracting roots, he follows rules used in our days, and employs the decimal point to distinguish the whole number parts (integers or "zero integers") from fractions.

The *Compositio instrumenti,* dedicated to the Marquess Leonello d'Este of Ferrara, the patron and friend of Bianchini, had to be ready not later than in the course of 1450 – the year of the death of Leonello. The analysis of the manuscript text, however, permits to suppose that the treatise was offered to Leonello as early as 1442, some month only after he succeded his father Niccolo (died in 1441) to the rulership of Ferrara[8]. As for the copy of the treatise preserved at the Biblioteca Estense, Modena, Cod. Lat. 145 (α.T.6.19), till now the unique item known to historians, it seems to be done twenty five years later[9].

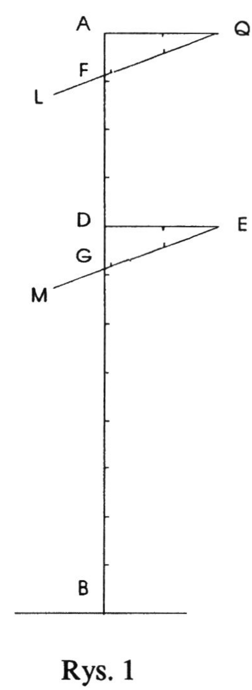
Furthermore, the Estense copy, although written on vellum and provided with the border decoration (for these reasons, probably, currently taken for the original offered once to Leonello), contains faults that alter the sense of Bianchini's exposition. It seems, therefore, that this copy was even not done directly from original but from one of the copies that possibly circulated in Italy in the second half of the fifteenth century. In certain places the law quality of the text is surely due to the copyist, ignorant of the subject.

1. THE PRINCIPLES OF THE CONSTRUCTION AND OF THE SCALE OF THE INSTRUMENT

In the letter of dedicace opening the treatise, Bianchini mentions Euclid and Ptolemy as the sources of inspiration, and displays the contents of the treatise disposed in five parts. While in parts one and five the construction of the instrument and its adaptation to various purposes is discussed, in parts two, three and four the use of the instrument is shown, illustrated with examples. Each example is provided with a diagram that interprets geometrically the situation resulting from the sighting. The calculations are done either by the rule of three or by the Pythagoras' theorem. The results of calculations given in numbers with decimal fractions, close the procedure.

According to Bianchini's description here is a matter of a large device, in the style evoking the future Tychonian instruments. The vertical post AB (*cathecus*) is about four meters (duodecim pedes) high. The peculiarity of Bianchini's instrument lies in two p a i r s of bars: AQ, LQ and DE, EM. The respective use of them enabled the surveyor to take two sights of an object without changing the position of the instrument itself.

While the horizontal bars AQ, DE (*bases*) are fixed at right angles to the post AB, (in points A, D), the transversal bars LQ, EM (*hypothenuse*) are pivoted at the extremities (Q, E) of the horizontal bars. Each of the transversal bars is provided either with two pinnule (*due tabelle otoni, perforate*) or with a sighting tube (*canella*). During the sighting the lower extremity of a transversal bar slides freely along the scale of the post[10]. Actually, the transversal bars are the alidads, but since Bianchini avoids the names that are not linked with the classic tradition, he calles them not *alidade* but simply *rige* or *linee transversales.* (I mark in my sketch the scales that are missing in the manuscript, and I use, at variance with the copyist of the manuscript, the upper cast letters)[11].



Rys. 1

At the very beginning of the treatise Bianchini remarks that the way to scale the post and the bars is especially chosen to facilitate arithmetical operations. He speaks then of multiplication and of division but in the course of the treatise, when the Pythagoras' theorem is used, the question of the extraction of square roots emerges as well.

The unit of integers used in the scale of the instrument is foot (*pes*). Bianchini presents the idea of the division of foot in ten parts, and of the subsequent decimal subdivision of each of its parts etc., as a new device. He does not underline, however, his-own authorship of the procedure[12]. While applying names to the new units of length Bianchini uses "traditional" terms. He calls *untia* one tenth of *foot*, *minuta* one tenth of *untia*, *secunda* one tenth of *minuta*, *tertia* one tenth of *secunda* etc. In this way Bianchini presents the idea of decimal positional fractions using a melange of names originally attributed to the units of time, of angle and of length. The eclectic terminology, however, does not alter the clarity of Bianchini's exposition.

According to Bianchini's own words:

> "...every section [of the post and of each bar, horizontal and tranversal as well], having the lenght of one foot, has to be divided into ten equal parts. [...] These divisions will be called *uncie*. The *uncie* will be also divided [each] in ten parts, and the [units resulting from] this division will be called *minutes*. Then *minutes* will be divided [each] in ten parts, [the space] permitting, [...] and these divisions will be called *seconds*. And if minutes [for the lack of the space] can't be divided in ten parts, be divided in five [parts], so each division will be [equal to] two *seconds*. Remark – concludes Bianchini – that these divisions are always ended with ten, and this in order to facilitate multiplications and divisions. I will teach it below"[13].

2. THE SURVEYING

Bianchini describes the use of the instrument for such purposes as the determination of the height and of the distance of an inaccessible tower, the measurement of the depths of a terrain the instrument being situated in the upper part of it, finally the measurement of a distance between two inaccessible objects. Aa a rule the sightings have to be taken with each of alidads respectively. Exceptionally, however, the upper bar is taken off, being rather a hinder than a help in the work, so the sightings are accomplished with the lower alidad alone[14].

Now we will look closly at the first example of the use of the instrument, concerning the determination of the height and of the distance of an inaccessible tower. The other examples will be referred to only if they supply more information concerning operations on decimal fractions.

While determining the height of the tower the measurements are taken twice, by means of the pinnule situated at the upper (pivoted) and lower (sliding) ends of each of two alidads. The results of sightings, expressed in feet and their decimal subdivisions etc., are read off from the scale of the post. These results, in the example presented by Bianchini, are respectively:

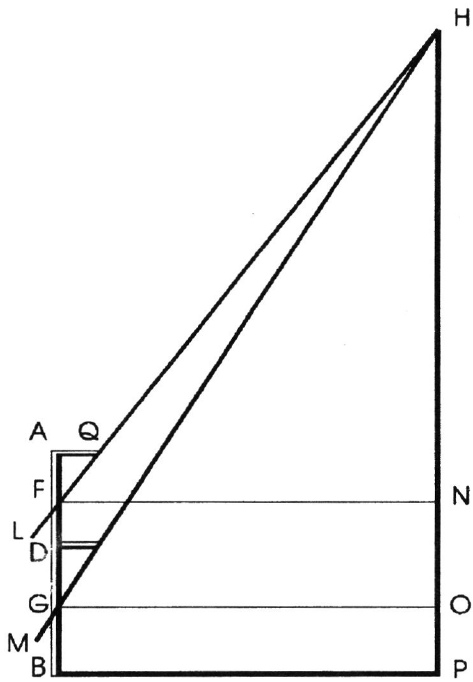
pedes .0. untie .7. minuta .4. et secunda .6.

And:

pes .0. untie .8. minuta .3. et secunda .4.[15].

After the sighting a diagram is drawn, expressing geometrically the situation.

3. THE *DEMONSTRATIO IN QUANTITATE CONTINUA* OR THE GEOMETRICAL BASE FOR CALCULATIONS



Rys. 2

While the aim of the sighting was to determine the sides of the small triangles given in the instrument (FAQ and GDE), the aim of the geometrical considerations is to prove the similarity of these triangles to the large ones (FNH and GOH respectively), given in the field.

(1) Δ FAQ is equiangle with Δ FNH, and Δ GDE with Δ GOH[16].

< FAQ =< FNH, < AQF=< NFH, < AFQ=< NFH

< GDE=< GOH < DEG=HGO < DGE=< GHO

since according to VI,4: In equiangular triagles the sides about the equal angles are proportionals,

(2) AQ:FN=AF:H N and DE:GO=DG:HO,

but since AQ=DE and FN=GO, Bianchini applies to (2) *Elements* VII, 9, that in his own wording run as follows; Si fuerint quator numeri quorum primus secundi tota pars fuerit quota tertius quarti, erunt permutatim primus tertii tota pars seu partes, quota secundus quarti,[17]
and obtains the following proportions:

(3) AF:AQ = HN:FN and DG:DE = HO:GO.

The further transformations lead eventually to the formula:

(4) HO = FG · DG : (DG - AF)[18]

The Regula brevis, given by Bianchini in the next chapter, explains the use of the instrument as follows:

[1] Take the first sighting with the LQ.
[2] Take the second sighting with ME.
[3] Subtract the result of the first sighting from the result of the second one.
[4] Take from the post AB the value corresponding to FG.
[5] Multiply it by the value of the second sighting, DG.
[6] Divide the product by DG - AF, and you obtain HO = FG · DG : (DG-AF).

The height of the tower is equal to HO + OP, the quantity OP = BG being taken directly from the instrument.

(By the similar procedure Bianchini finds the distance of the tower (FN = GO). Then, two sides of the right-angled triangle known, he finds the distances FH and GH by Pythagoras' theorem).

Once proportions established and the formula found, Bianchini proceeds, according to his own expression, from "continuous quantities" to "discrete numbers"[19]. In a word: Bianchini replaces lines by numbers (decimal fractions) and passes to calculations.

4. THE OPERATIONS ON DECIMAL FRACTIONS

Bianchini uses two ways in the writting down the decimal fractions. The first way occurs while the readings are taken off from the decimal scale of the post:

pedes .0. *untie* .7. *minuta* .4. *et secunda* .6.,

The same number, however, while used in the arithmetical operations, assumes a purely decimal form and is expressed as .746. seconds uderstood as 0.746 foot:

"To facilitate operations reduce *pedem* .0. *untias* .7. *minuta* .4. *secunda* .6. to the last fraction, the smallest one namely to *seconds*. This is just for this purpose that I did decimal divisions, for you can put the invented similar parts [the smallest units] continuously, namely: .746. Thus there will be .746. seconds. And in the same way reduce *untias* .8. *minuta* .3. *et secunda* .4. to the same kind, and will be .834. seconds"[20].

Bianchini explains the operations on decimal fractions beginning with additon and subtraction. It happens only in the examples of these operation that the successive numerals of a decimal fraction are accompagned by the names of units they represent. The contrary occurs when Bianchini explains multiplication, division and extraction of roots. Then he abandons the metrological names for decimal places as such, and a decimal fraction is written as a "continuous number" (*numerus continuus*).

$$AF = .746. \quad DG = .834. \quad FD = .834. - .746. = .88.$$

$$NO = FG = FD + DG = 4 + (.834. - .746.) = 4.088.$$

$$HO = \frac{.834 \cdot 4.088.}{.88.} = 38.7.4.3\frac{1}{11}$$ [21]

or: *pedes* .38. *untia* .7. *minuta* .4. *secunda* . $3\frac{1}{11}$

Bianchini rounds it to 38.7.4.3., adds the value corresponding to OP taken off from the instrument, and receives finally the height of the tower.

As a rule, in the *Compositio* only the final results of the operations are given. That is why I had to recur to other Bianchini's treatise, namely *De arithmetica*, while reconstructing the calculations. Besides, the "rules" of calculations one finds in the *De arithmetica* are very similar to the "rules" currently explaned in the fifteenth-century arithmetical treatises. With the one exception, however, namely the rule of signs. It was Bianchini the first to formulate it and to "prove" it geometrically. I signal here the rule of signs, (even if it has nothing to do with the use of the *biffa*), because of its special place in Bianchini's contribution to the Renaissance mathematics[22].

Example of addition:

pes .0. *untie* .8. *minuta* .3. *et secunda* .4.

plus

pedes .3. *untie* .2. *minuta* .5. *secunda* .4.

equal to:

pedes .4. *untie* .0. *minuta* .8. *secunda* .8.

Examples of subtraction:

	.834.
minus	.746.
equal to :	.88.

pedes .4.

minus

pedes .0. *untie* .7. *minuta* .4. *secunda* .6.

equal to:

pedes .3. *untie* .2. *minuta* .5. *secunda* .4.[23]

Examples of multiplication and of the raising to the second power:

Multiplica ergo
per

```
     .4088.
      .834.
     16352
    12264
  32704
  3409392
```
[24]

According to Bianchini's notation the final result runs as follows: 34.0.9.3.9.2. The decimal point plays here a double role: it separates the integral part of a number from the fractional one, and it marks the decimal places of the remaining fractional units.

The raising to the second power of a number:

feet .92. *untias* .9. *minuta* .0. *secunda* .9.

gives to Bianchini an opportunity to explain the use of the decimal point in multiplication. First he presents the above value as *figure continue:* .92909. Their second power is equal to .8.6.3.2.0.8.2.2.8.1. The determination of the integral part of the number is based on the formula $10^m.10^n = 10^{m+n}$:

"Count then how many numerals are there in the multiplied besides integers. There is a fraction "909" which [is composed] of three numerals. The same is in the multiplicand because it is equal [to multiplied]. Both taken together count six numerals. Cut off six ultime numerals from the product and remain four, namely: .8632. These are feet. The .0.8.2.2.8.1. is the rest, namely *untie* .0. *minuta* .8. *secunda* .2. *tertia* .2. *quarta* .8. *quinta* .1.[25]

Example of division:

38743

3409392 : 88

264

769

704

653

616

379

352

272

264

8

The result is equal to the quotient .38.7.4.3. with a remainder of 8. At this point Bianchini could apply to the division the "rule of zeros", and to continue it decimally; instead of this he divides 8 by 88 in the usual way and receives "one eleventh of a second [!] The 1/11 is added to the last unit of the decimal fraction, and the final result expressed as .38.7.4.3 1/11[26]. (In the same way the products are recorded as .862.099173 $\frac{67}{121}$ and .1501.027093 $\frac{23}{3121}$)[27].

The procedure looks like a betrayal of the decimal idea. Is this plausible that Bianchini, while dividing decimal fractions, became "blinded by an equivocation between the decimal idea and the ordinary process of division" ?[28]

Although Bianchini's examples of division are not accompanied by the rule of a decimal point, the determination of the integral part of the quotient implies the use of the formula $10^m : 10^n = 10^{m-n}$

*

* *

While an outline of Bianchini's doctrine on decimal positional fractions is given in the *De arithmetica,* the complete exposition of this doctrine is found in the *Compositio instrumenti.* The establishing which of the two treatises was written earlier requires further studies. From the systematical point of view the *De arithmetica* seems to be antecedant to the *Compositio.* In fact, while the *De arithmetica* signals the principle of the extention of the decimal positional number system from integrals to fractions, the *Compositio,* at its turn, furnishes the explanation of the decimal doctrine as applied to the problems of metrology. The metrological framework, however, once being overpast, the universal meaning of the decimal fractions is eventually shown.

What seems to persist ambiguous in Bianchini's doctrine, is the place attributed in it to common fractions. While appended at the end of decimal ones, they generate together with them quite a strange hybrid. Actually, in Bianchini's treatise the common fractions do appear just in situations in which one would expect Bianchini to point to a possibility of an infinite expression of a decimal fraction (parallely to what he had written in the *De arithmetica* with regard to integrals).

One has not to be mistaken by Bianchini's use of words: *numerus continuus, figure continue.* What they mean is only a decimal fraction as considered in terms of its smallest units. For instance .0.7.4.6. foot is expressed "continuously" as .746. seconds. Thus, in the *Compositio* the "continuity" of a number seems to appear only at the level of notation, being an effect of the omission of metrological names, (or points that signale such names). In the *Compositio instrumenti* the "continuity" is not a quality inherent in the certain decimal entities (or forms), and meaning the possibility of their infinite development (or meaning an infinite set of fractions).

In fact, Bianchini abandoned "continuous quantities" for "discrete numbers" while he passed from geometrical proofs to calculations. Bianchini's *numeri continui* are in fact discrete numbers. In this point Bianchini's decimal idea seems to fail (even if Bianchini goes as far as to calculete values of the *quarta, quinta, sexta etc*).

Otherwise, the Bianchini's realizing of the concept of decimal fractions, (he uses even the term *fractio decenaria*), is surely superior to the Stevin's one. At variance with the Simon Stevin's "somewhat unwieldy" notation, the Bianchini's notation is almost as clear as the notation used in our days. This notation, however, once introduced into mathematics by means of a treatise offered to a Renaissance ruler in homage, eventually passed into oblivion. The regular use of the decimal point appears as late as the eighteenth century.

The concept of decimal fractions in positional notation have revived in Europe only about a century after Bianchini's death (ca 1470). The authors of the modern expression of the decimal idea, however, acted without reference to the Bianchini's achievement.

Notes

* It was E. J. D i j k s t e r h u i s, who postuleted the use of the expression *decimal positional fractions* for fractions written in the decimal system of positional notation (in contrast with fractions written in the decimal system of numereation). E. J. D i j k s t e r h u i s: *Simon Stevin. Science in Netherlands around 1600*. R. H o o y k a s, M.G.J. M i n n a e r t (eds). The Hague: Martin Nijhoff 1970 p. 17.

1 G. R o s i ń s k a: *Tables trigonométriques de Giovanni Bianchini*. "Historia Mathematica". Vol. 8 1981 pp. 46–55. E a d e m: Tables of Decimal Trigonometric Functions from ca.1450 to ca.1550. In: *From Deferent to Equant. Volume of Studies in theudies in the History of Science in the Ancient and Medieval Near East in Honor of E.S. Kennedy*. D. K i n g, G. S a l i b a (eds). "Annals of the New York Academy of Sciences" 1987 Vol. 500 pp.419–421.

2 In the tables of tangent and of cosecant Bianchini avoided fractions thanks to calculations with the radius R equal to a number of units of length expressed in the form 10^n. Since in these tables, (dressed certainly before 1463, or even before 1450, accordingly to the last discoveries), Bianchini assumed for *n* respectively 3 and 4, thus all quantities were tabulated as integers with a certain accuracy considered sufficient for astronomical calculations; the determination, however, of the real value of the tabulated quantities supposed the knowledge of *n*.

3 "...each numeral, fixed in the order of numbers, denotes decimal fraction [of the number that] follows immediatly at the left of the place [occupied] by this numeral, as 342. I say that 2 denotes two tenth of ten, and 4 are 4 tenth of hundred *etc*". I refere myself to the critical edition of Bianchini's *De arithmetica*, forthcomming in the Studia Copernicana series.

4 Bianchini seems to hold here the position that some hunderd forty years later will be held by Simon Stevin in the *De Thiende*. In fact, according to Stevin's *Preface* to this work the Thiendetalen ("tenth numbers") are integers. Simon S t e v i n: *La disme. Enseignant facilement expedier par nombres entiers sans rompuz, tous comptes se rencontrans aux affaires des Hommes. Premièrement descripte en Flameng, et maintenant en François, par Simon Stevin de Bruges*. In: *L'Arithmétique*. Leyde Christophe Plantin 1585 pp.132–148, particularily pp.139–140; George S a r t o n: *The first explanation of decimal fractions and measures (1585). Together with a history of the decimal idea and a facsimile of Stevin's Disme*. "Isis" No 65. Vol.XXIII(1) 1935 pp.153–244. (Facsimile on pp.230–238). It is generally admitted that Stevin "introduced decimal fractions for general purposes, and showed that operations could be performed as easily with such fractions as with integers", M.G.J. M i n n a e r t: *Simon Stevin*. In: *Dictionnary of Scientific Biography*. Vol.13, 1973 p.48; E.J. D i j k s t e r h u i s, op. cit. p.19, underlines the incompleteness of the Stevin's achievement: "...we may ask the question as to how far Stevin can be considered to have

introduced the decimal positional fractions by means of these tenth numbers. Strictly speaking, he obviously did not introduce fractions at all; his tenth numbers are integers, and he even considered this feature to be their principal advantage".

[5] Bianchini's treatise was mentionned by Domenico F a v a: *La Biblioteca Estense nel suo sviluppo storico. Con il Catalogo della Mostra Permanente e 10 Tavole.* (Modena: Libreria Editrice G.T. Vincenzi e Nipoti, 1925) p.36: "Al circolo di Leonello apparteneva pure il matematico ed astronomo Giovanni Bianchini, che con altissime cariche presso la Corte estense e del principe fu per qualche tempo consigliere. Di lui sono note le *Tabulae astronomicae* [...] inedita e quasi sconosciuta è'un altra sua opera latina dedicata a Leonello, che tratta dello strumento per misurare, chiamato solitamente "biffa", tramandatoci dal codice membranaceo lat. 145 (=α. T. 6. 19), che dev'essere stato esemplare di dedica, avendo per titolo le parole: *Illu. et Ex. Principi* [...]. The transcription of the *Compositio instrumenti* was recently published by Paolo G a r u t i, with a historical Introduction by Gino A r r i g h i: *Giovanni Bianchini. Compositio instrumenti* (Cod. Lat. 145=alpha.T.6.19) *della Biblioteca Estense di Modena.* A cura di Paolo Garuti con introduzione di Gino Arrighi. In: "Rendiconti Classe di Lettere e Scienze Morali e Storiche. Vol. 125(1), 1991 pp.95–127 (Istituto Lombardo Accademia di Scienze e Lettere, Milano 1992). In Gino Arrighi's Introduction (pp. 95–107), the contribution to the development of the surveying instruments due to Gerbert, Leonardo Pisano, Cristofano Gherardo di Dino, and Francesco di Giorgio Martini is presented. On p.103, note 10 G. Arrighi reports that from 16 March to 11 April 1987 the manuscript was exposed in the Estense Library, Modena, at the exibition "Materiali per la storia delle matematiche nelle raccolte delle Biblioteche Estense e Universitaria di Modena". Unusually enough, the publication of the *Compositio instrumenti* is devoided of a commentary on dating of the Estense manuscript as well as on the contents of Bianchini's treatise. The publication was meant by the Authors to make available the *Compositio* to historians of science. As for the P. Garuti's publication of Bianchini's *Compositio instrumenti,* I share Professor Arrighi's opinion on it, expressed at the end of the Introduction: "...gli storici della scienza e della technica hanno da essere grati al P. Paolo Garuti O.P. che ha compiuto la trascrizione che qui per la prima volta vien pubblicata." In my research I served myself of the Paolo Garuti's transcription, but above all I used a photocopy of the Estense manuscript kindly supplied to me by the Istituto e Museo di Storia della Scienza, Florence. While quoting the Latin text I refer to Paolo Garuti's edition, with the exception of cases in which my reading differs from Garuti's one; then I try to give both readings.

A manuscript copy of a fragment of a treatise on an instrument devoted to measurements of inacessible objects, attributed to Bianchini (preserved in the Bodleian Library, Oxford, ms. Canon. Misc. 501), was signaled by Lynn T h o r n d i k e: *Giovanni Bianchini's astronomical instrument.* "Scripta Mathematica" Vol. XXI 1955 p.136: "In it the discussion of the measurement of inaccessible bodies is followed on fol. 2r by *Canones,* "Ad verificandum stellas fixas...". L. Thorndike, *ibidem,* signals the treatise *De mensuratione rerum [in]accessibilium,* of which only a fragment is known, preserved at the Bibliothèque Nationale, Paris, ms. lat. 7271, f.1v. The desinit of this fragment seems to indicate that in the treatise the adaptation of an astronomical (observational) instrument for the surveying purposes is discussed: "...ut sicut res qui [*sic* GR] in celo videntur cum ipso mensurantur, ita res terrene visibiles etiam si inaccessibiles essent metiri possint." see L. T h o r n d i k e, op. cit. p.136. We are not interested here in Bianchini's treatises on instruments called "aequatoria".

[6] See, for instance, *Cartography, survey, and navigation to 1400.* In: *A History of Technology.* Charles S i n g e r, E.J. H o l m y a r d, A.R. H a l l, Trevor I. W i l l i a m s (eds.) Vol.III Oxford Clarendon Press 1957 pp.527–529.

[7] See above, note 5.

[8] Leonello d'Este (21. Sept. 1407 – 1. Oct. 1450), in 1442 iniciated, together with Guarino Guarini, the reform of the University of Ferrara.

[9] In fact, the manuscript reads on f.1v (I give here my-own reading of the text): "Imperauit autem Antoninus Adriani gener et filius adoptiuus post incarnati uerbi natiuitatem annis CXLII [Garuti reads "XLII"]. Id est annis iam M.CCC.XXV ["XXV" clearly cancelled in the manuscript], ex quo tempore ipsius Almagesti fides et ueritas comprobatur". Thus, taking account of the expression: "annis i a m" [elapsis] and of the cancellation of "XXV" in the date "M.CCC.XXV", we obtain 1300+142 = 1442. P. G a r u t i [ed.], p.108 omits "C" that exists in the manuscript, and so he reads "XLII" instead of "CXLII" (despite historical evidence concerning the dates of the ruling of Antoninus Pius). It is possible that Garuti was induced in this reading by some (vague) points about "C", that he took for a sign of suppression of the cipfer "C". As for the XXV written down and then cancelled, P. G a r u t i, *ib.*,annotation 1, comments it: "Il XXV pare cancellato". It seems possible that the years that result from the considering the "XXV" as if it was not cancelled are the date of the Estense copy. We have then 1325+142 = 1467. Certainly, however, the year 1467 can not be considered as the date of the treatise itself.

[10] [Ms. Modena, B.E. Lat, 145, f.3r. P. G a r u t i [ed.], pp.109–110: Primo fiat una riga seu pertica ferrea aut otoni, vel alterius metalli, [...] Sitque longa pedes duodecim et paulo plus. [...] In cuius pertice medietate ab una eius extremitate ad alteram figetur linea recta longitudinis punctualiter pedum duodecim. Que linea sit .a.b. [...] In parte vero superiore fiat plaga per transversum in qua ponatur alia riga longitudinis pedum duorum et parum plus. In cuius medio etiam signetur linea secans lineam perpendicularem et ab ea intersecetur in puncto .a. ad angulum rectum; que sit linea .a.q. longitudinis punctualiter pedum duorum. Item supra lineam perpendicularem .a.b. descendendo in finem pedum quattuor, punctualiter signetur terminus .d.; que sit linea .a.d. in cuius extremitate fiat etiam plaga per transversum in qua ponatur alia riga longitudinis etiam pedum .2. et parum plus. In cuius medio signetur linea secans lineam perpendicularem .a.b. in puncto .d. ad angulum rectum. Que linea sit longitudinis punctualiter pedum .2. Et erit linea .d.e. Item linea .a.d. que est pedes quattuor, linea vero .d.b., que est pedes .8., et linea .a.q., que est pedes .2., et linea .d.e., que est etiam pedes .2., terminentur per lineas transversales ut dignoscatur pes quilibet per se.

[11] The scheme of the instrument is on f.2v; its reproduction in P. G a r u t i, op.cit. p.106.

[12] In the Introduction to the *Compositio* Bianchini presents the treatise as a result of the "intellectual peregrinations" of its author. These "peregrinations" were accomplished

in the realm of the Euclid's *Elements* and Ptolemy's *Almagest*. See Ms. Modena B.E. Lat 145, f.1–1v, P. G a r u t i [ed.], pp.107–108. Bianchini was aware of the value of the doctrine elucidated by him on only few folios: "[...] opusculum [...] corpore quidem pusillum, verum cognitione ac viribus amplum". Ms. Modena B.E. Lat. 145, f.2, P. G a r u t i [ed.], p.108.

13 [Ms. Modena B.E. Lat. 145, f.3. P. G a r u t i [ed.], p.110: "Ac etiam linea cuiuslibet pedis dividatur in partes decem equales terminantes per lineas minoris longitudinis quam sint linee terminantes pedes. Que divisiones untie vocitentur. Que untie etiam in partes decem dividantur et signentur per lineas etiam minores vel per punctos. Que divisiones minuta vocen[tur] [...] Et nota quod iste divisiones terminantur semper de decem in decem, ut multiplicationes et divisiones per eas faciende, per doctrinam quam inferius docebo, facilius operantur". Follows description of the pinnule.

14 [Ms.Modena B.E. Lat. 145, f.12v. P. G a r u t i [ed.], pp.125–126. In this chapter Bianchini considers the measurement of the distance between two inaccessible objects. For this purpose the reduction of the length of the bars to only one foot is advised. Moreover, Bianchini recommends the construction of a device that permits to avoid the transportation of the whole instrument. Actually, it is a matter of a model of the foot made in metal – "mensura unius pedis de metallo". The model, suplied with the decimal scale, permits to construct easilly an instrument in the field using all sort of objects accessible there such as "lanceas, perticas et similia", provided they remain perpendicular to the soil.

15 Ms. Modena, B.E. Lat 145, f.4v–5. P. G a r u t i [ed.], p.112.

16 *Elements* I,29; I,30

17 *Elements* VII,9. Cf. Th. L. H e a t h: *The thirteen books of Euclid's Elements*. Vol. II pp. 309–310

18 Ms. Modena, B.E. Lat.145, ff.5r-5v. P. G a r u t i [ed.], pp.113–114.

19 Ms. Modena, B.E. Lat 145, f.5v. P. G a r u t i [ed]. p.114: "Nunc vero prefactas conclusiones, que in quantitate continua demonstrate sunt, ad quantitatem discretam reducam, ut ad numerum propositum in numeris fiat conclusio". (I added the punctuation to the text).

20 *Ibidem.*

21 I give here my-own reading of the *Compositio*, Ms. Modena, B.E. Lat. 145, ff.6–6v: Iterumque catecus .d.g. est pes .0. untie .8. minuta .3. et secunda .4., cui addito catecus .f.d. erique totus catecus .f.g. pedes .4. untie .0. minuta .8. secunda .8., qui erunt .88. partes habito respectu ad .834. partes altitudinis turris a puncto .o. Quia concluditur quod sicut se habet .88. ad .834. et ita pedes [6ʋ] .4. uncie .0. minuta .8. et secunda .8. ad totam turrim.

22 G. R o s i ń s k a: *A chapter in the history of the Renaissance mathematics: negative numbers and the formulation of the law of signs (Ferrara, Italy ca.1450)*. In: "Kwartalnik Historii Nauki i Techniki" ("Quarterly Journal of the History of Science and Technology"). Vol. 40 1995 (1) pp.3–20.

23 Ms. Modena, B.E. Lat. 145, f.6, P. G a r u t i [ed.], p.116.

24 *Ibidem.*

25 I give here my own lecture of the *Compositio*, f.7r–7v: Numera ergo quot figure sunt in numerum multiplicantem ultra pedes, id est .909., que sunt fractiones et tres numero. Et totidem sunt in numero multiplicando, quia idem sunt. In utraque igitur erunt sex figure.

Et ideo resseca sex ultimas figuras de productis. Que relinquentur erunt quatuor, videlicet untie .0.minuta .8. secunda .2. tertia .2. quarta .8. quinta .1. (G a r u t i, p.116 reads ".1.909" instead of "id est .909.".

[26] G a r u t i op. cit. p. 115

[27] I d e m, p.116.

[28] These words are of George S a r t o n, op. cit. p.172. They were intended by their author as a possible justification of the procedures adopted by Pietro Borghi (1484), Francesco Pellizzati (1492) and Christoph Rudolff (1525).

Grażyna Rosińska

DZIESIĘTNE UŁAMKI POZYCYJNE. ICH ZASTOSOWANIE W MIERNICTWIE. (FERRARA, 1442)

Artykuł przedstawia wyniki kontynuacji badań prowadzonych przez autorkę w latach osiemdziesiątych i dotyczących pierwszych na Zachodzie tablic dziesiętnych funkcji trygonometrycznych (przypis 1). W tablicach tych jednak, wyliczonych przez Giovanniego Bianchiniego około połowy XV wieku, wartości liczbowe nie były ujęte jako ułamki (przypisy 2, 4).

W obecnej publikacji, autorka posługując się materiałem rękopiśmiennym z XV wieku oraz wydanymi ale nie opracowanymi dotąd źródłami, ukazuje pełną koncepcję ułamków dziesiętnych, opracowaną przez Bianchiniego w celu uproszczenia obliczeń wynikających z pomiarów dokonywanych przy zastosowaniu wynalezionego i opisanego przez Bianchiniego przyrządu mierniczego (przypisy 5, 6). Poza opisem konstrukcji i skalowania instrumentu, zachowanym w Biblioteca Estense w Modenie (sygn. Lat. 145) i opublikowanym, bez komentarza, w 1992 roku (przypis 5), autorka odwołuje się także do przygotowanej przez siebie do druku edycji krytycznej *De arithmetica* Bianchiniego, traktatu w którym wyjaśniona jest idea ułamków dziesiętnych. Bianchini używa nawet przy tej okazji terminu *fractio decenaria* (w przeciwieństwie do Stevina, który tego terminu nie używał), nie posuwa się jednak w koncepcji ułamków dziesiętnych dalej, niż to uczynił Stevin w *La disme*, dziele opublikowanym w 1585 roku (przypisy 3, 4).

W traktacie Bianchiniego o instrumencie mierniczym podane są, podobnie jak to zostało dokonane przez Stevina blisko 150 lat później, reguły działań na ułamkach dziesiętnych. Bianchini idzie dalej niż Stevin w tym sensie, że wprowadza oznaczenie (punkt) odzielający część całkowitą liczby od jej części ułamkowej. Na systematyczne wprowadzenie podobnego oznaczenia trzeba było czekać aż do połowy XVIII wieku.

Traktat Giovanniego Bianchiniego *Compositio instrumenti* powstał w roku 1442 (datacja ta została ustalona przez autorkę na podstawie analizy źródła oraz krytyki wewnętrznej tekstu) i został ofiarowany, w tymże roku, Leonello d'Este z Ferrary jako *homagium*.

Wyłożona w nim doktryna ułamków dziesiętnych najprawdopodobniej nie miała w przyszłości wpływu na prace Stevina i jego następców. Miałoby zatem miejsce „ponowne odkrycie" ułamków dziesiętnych ogłoszone wraz z *La disme*.

Seria publikacji na temat dokonań Bianchiniego w dziedzinie astronomii i matematyki (w tym publikacje w KHNiT), ukazuje w nowym świetle matematykę włoską XV wieku, zwłaszcza matematykę związaną ze środowiskami uniwersyteckimi. W przeciwieństwie bowiem do uniwersytetów, środowiska matematyczne kupieckich *scuole d'abbaco* zostały już w dużej mierze opracowane.

*

W kręgu wpływów Bianchiniego kształcił się w Ferrarze związany z Kopernikiem Domenico Maria Novara. Zanim jednak mógł Kopernik zetknąć się z dziełem Bianchiniego we Włoszech, wykorzystał on, we wpisach do swego studenckiego notatnika uczynionych w czasie krakowskich studiów, Bianchiniego tablice astronomiczne ruchu planet, znane od około połowy XV wieku w Uniwersytecie Krakowskim. (KHNiT 1984, R. 29, pp.637–644). Znał także wyliczone przez Bianchiniego dziesiętne tablice funkcji trygonometrycznych. (KHNiT 1981 R. 26 s. 567–577).

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Miejscem, w którym powstawało to studium, była Biblioteka przy Istituto e Museo di Storia della Scienza we Florencji oraz Biblioteka PAN, Biblioteka Instytutu Historii Nauki PAN i Biblioteka Instytutu Matematyki PAN w Warszawie. Kompetencja i życzliwość Bibliotekarzy była mi nieocenioną pomocą.