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# RATES OF RETURN DISTRIBUTIONS VARIATION – IMPLICATIONS FOR PORTFOLIO ANALYSIS

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K e y w o r d s: portfolio analysis, semi-variance, time series stationarity.

A b s t r a c t

The paper presents the properties of the rates of return distributions for Markowitz models and models with minimum semivariance. The special focus was placed on investigating the variation over time of the rates of return distributions for the studied portfolios. Non-parametric Kolmogorov--Smirnov tests and augmented Dickey-Fuller test were used for analysis of distributions over time.

The studies showed that the distributions of rates of return for portfolios developed, particularly for high assumed rates of return were characterized by high variation. Considering selected distribution parameters SEM portfolios were more favorable than Markowitz portfolios although they showed a higher variation of distributions over time.

## ZMIENNOŚĆ ROZKŁADÓW STÓP ZWROTU – IMPLIKACJE DLA ANALIZY PORTFELOWEJ

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S ł o w a k l u c z o w e: analiza portfelowa, semiwariancja, stacjonarność szeregów czasowych.

A b s t r a k t

W artykule zaprezentowano porównanie właściwości rozkładów stóp zwrotu modeli Markowitza i modeli o minimalnej semiwariancji. Szczególnie badano zmienność w czasie rozkładów stóp zwrotu badanych portfeli. Do analizy rozkładów w czasie wykorzystano nieparametryczne testy Kołmogorowa-Smirnowa i test stacjonarności szeregu Dickey'a-Fullera z augmentacją.

Badania wykazały, że rozkłady stóp zwrotu zbudowanych portfeli, zwłaszcza dla wysokich założonych stóp zwrotu, charakteryzowały się dużą zmiennością. Biorąc pod uwagę wybrane parametry rozkładu, portfele SEM były korzystniejsze od portfeli Markowitza, natomiast wykazywały one większą zmienność rozkładów w czasie.

## Introduction

The theory of modern finance is based on the assumption that the investor operates under the risk conditions, i.e. that the distributions of the rates of return for individual shares are known. In reality we very often deal with the conditions of uncertainty because we do not know the probability of occurrence of a specific rate of return. That probability is estimated on the basis of historical data (KRZEMIENOWSKI, OGRYCZAK 2002). If the distribution is not constant over time then the estimation of probability, and as a consequence the characteristic of the distribution can change in an unpredictable way. The fundamental problem then is the variation of the distributions of the rates of return.

The aim of this paper is to compare the properties of the distributions of rates of return for Markowitz models and models with minimum semivariance. The special focus was placed on investigating the variation over time of the rates of return distributions for the studied portfolios. The studies carried out according to the dynamic approach showed that the discussed models do not always work in practice (RUTKOWSKA-ZIARKO 2004; WDOWIŃSKI, WRZESIŃSKI 2003). That phenomenon can be caused by the variation of the rates of return (RUTKOWSKA-ZIARKO, MARKOWSKI 2005, RUTKOWSKA-ZIARKO, MARKOWSKI 2006).

## Models applied

The paper reviews two alternative models of stocks portfolio choice. The first of them is the classical Markowitz model (MARKOWITZ 1952), in which the risk is measured by the variance of portfolio rates of return. The other one is the SEM model (MARKOWITZ 1959), in which the semivariance from the assumed rate of return ($\gamma$– semivariance) is the measure of risk. In the classical Markowitz model (MARKOWITZ 1952), the risk is measured by the variance of rates of return. The weakness of variance as a measure of risk is the identical treatment of negative and positive deviations from the expected rate of return. In reality the negative deviations are undesirable while the positive ones create opportunities for a higher income. Aiming at measurement of negative deviations only Markowitz defined semivariance (MARKOWITZ 1959). The most important characteristic of semivariance is that it measures only the deviations below a certain specified level. The supporters of applying semivariance as the measure of risk stress that it is better in describing the actual preferences of investors than the variance (HOGAN, WARREN 1974, OGRYCZAK, RUSZCZYŃSKI 2001, SORTINO, SATCHELL 2001).

The problem of determining the shares of stocks in Markowitz model comes down to solution of the following optimization problem:
to minimize the variance of portfolio rates of return:

$$s_p^2 = \sum_{i=1}^{k} \sum_{j=1}^{k} x_i \, x_j k_{ij} \tag{1}$$

with the expectations:

$$\sum_{i=1}^{k} x_i = 1 \tag{2}$$

$$\sum_{i=1}^{k} x_i \, \bar{z}_i \geq \gamma \tag{3}$$

$$x_i \geq 0 \quad i = 1, ..., n \tag{4}$$

where:
$s_p^2$ – variance of stocks portfolio rate of return; $\gamma$ – rate of return defined in advance for the entire portfolio assuming that $\gamma \leq \max \bar{z}_i$; $\bar{z}_i$ – average rate of return of $i$ stock; $x_i$ – value share of $i$ stock in the portfolio; $k_{ij}$ – covariance of the rate of return for stocks $i$ and $j$.

SEM portfolio choice model is similar to the classical Markowitz model. The difference, however, is that another measure of risk $\gamma$ – semivariance of stocks portfolio rate of return ($ds_p^2(\gamma)$) is minimized. In Markowitz model a deviation both below the average rate of return and above the average rate of return is considered risky. In SEM model the risk is linked only to appearance of rates of return lower than the rate of return ($\gamma$) assumed by the investor. The problem of determining the shares of stocks in the SEM model comes down to solving the following optimization problem:
to minimize the $\gamma$-semivariance:

$$ds_p^2 \, (\gamma) = \sum_{i=1}^{k} \sum_{j=1}^{k} x_i x_j \, d_{ij}(\gamma) \tag{5}$$

with limitations (2-4),

where:

$$d_{ij}(\gamma) = \frac{1}{m-1} \sum_{t=1}^{m} d_{ijt}(\gamma) \tag{6}$$

$$d_{ijt}\ (\gamma)= \begin{cases} 0 & \text{for } z_{pt} \geq \gamma \\ (z_{it} - \gamma)(z_{jt} - \gamma) & \text{for } z_{pt} < \gamma \end{cases} \tag{7}$$

$d_{ij}(\gamma)$ – semicovariance for the assumed rate of return; $m$ – number of time units, in which stocks rates of return are registered $z_{it}$, $t = 1,2, ...., m$; $z_{pt}$ – portfolio rate of return at moment $t$. The issue of SEM portfolio design was described in detail in the papers (RUTKOWSKA-ZIARKO, OLESINKIEWICZ 2002, RUTKOWSKA-ZIARKO 2005).

## Rates of return distributions variation testing

Absence of variation in the distributions of the rates of return is the condition of persistency over time of the parameters of those models. Those parameters are estimated on the basis of the time runs of rates of return where it is assumed that the phenomenon develops similarly over the entire period studied and that it will not change during the future period, i.e. between portfolio purchase and sale. Presence of distribution variation causes that the portfolio effective during one period does not have to be effective during the next period.

The time run of rates of return can be treated as a part of stochastic process realization. For each time period $t$ there is a certain random variable $X_t$ possessing a specific probability distribution. The process, that is the run of the portfolio rates of return is strictly stationary if the combined distributions of variables $X_{t1}$, $X_{t2}$, ..., $X_{tk}$, are the same as the combined distributions of variables $X_{t1+\tau}$, $X_{t2+\tau}$, ..., $X_{tk+\tau}$ where is any integer, that is they do not depend on shift in time (GRUSZCZYŃSKI, PODGÓRSKA 2000). Usually the stationarity according to moments, i.e. the so-called weak stationarity is the subject of studies. It assumes that the expected value and the variance are finite and independent of the time while the covariance between observations from two periods depends only on the distance between those observations. Distributions are considered identical and the process stationary if the average and the variance do not change dismissing other properties of the distribution such as, e.g. asymmetry, concentration or presence of thicker tails.

The issue of stationarity of a time run for variable $y_t$ can be tested using the stationarity test (process integration) such as DF test (Dickey-Fuller test). In this test the equation having the following format is analyzed (CHAREMZA, DEADMAN 1997):

$$\Delta y_t = \delta y_{t-1} + \xi_t \quad (t = 1, ..., n) \tag{8}$$

where the hypotheses are: $H_0 : \delta = 0, H_1 : \delta < 0$. Rejection of the zero hypothesis in favor of an alternative one means absence of the so-called unilateral elements. That process is the integrated zero degree process I(0), i.e. it is stationary. This procedure allows testing the so-called weak stationarity only. In case of self-correlation of the random component resulting in ineffectiveness of MNK estimators the ADF test (Augmented Dickey-Fuller test) is applied. In the regression equation the delayed increase of the dependent variable is additionally included, that is:

$$\Delta y_t = \delta y_{t-1} + \sum_{i=1}^{k} \delta_i \, \Delta y_{t-i} + \xi_t \quad (t = 1, ..., n) \tag{9}$$

The testing method is identical as in the DF test. It should be pointed out that in both tests testing equations with a free expression or a trend can occur.

Variability of the time run can also be tested by applying statistical, non-parametric consistency tests such as, e.g. Kolmagorov-Smirnov test (K-S). That test serves verification of the hypothesis that two populations have the same distribution $[H_0 : F_1 (x) = F_2 (x)]$against the alternative hypothesis that two samples come from two different populations $[H_1 : F_1 (x) \neq F_2 (x)]$. The test statistic has the format of (SOBCZYK 2000):

$$\lambda = D_{n1n2} \sqrt{\frac{n_1 n_2}{n_1 + n_2}} = \sup | F_{n1} (x) - F_{n2} (x) | \sqrt{\frac{n_1 n_2}{n_1 + n_2}} \tag{10}$$

where:
$F_{n1} (x) - F_{n2} (x)$ represent the distribution function of a characteristic in the first and the second sample respectively. As opposed to stationarity tests, the K-S test verifies the conformity of the distribution "shape" in the analyzed samples. As a consequence it detects all differences in asymmetry, concentration and other characteristics of the distribution.

## Results

The studies covered 70 largest and most liquid companies continuously listed at Warsaw Stock Exchange during the covered period of two years, i.e. from January 1, 2004 until December 31, 2005. The portfolio analysis, because of time required for computations and low flexibility as well as liquidity of the portfolio generally applies to long-term analyses (TARCZYŃSKI 2002). For that reason use of quarterly rates of return for the covered securities and building portfolios of them was proposed. To simplify the analyses, dividends were disregarded in computation of the rates of return. Rates of return were computed as relative increases in prices of stocks according to the formula:

$$z_{it} = \frac{n_{i,t+s} - n_{it}}{n_{it}} 100\% \tag{11}$$

where:
$s$ – length of the investment process expressed in days, $n_{it}$ – listed value of $i$ asset at the moment $t$, $n_{i,t+s}$ – listed value of $i$ asset after s days of investing started at moment $t$.

The number of time units ($m$), for which rates of return are registered depends on the number of listings and length of the investment period:

$$m = n - s \tag{12}$$

where:
$n$ is the number of listings.

During the analyzed time period the highest average rate of return for an individual company was 34,54%. The analysis considered seven assumed rates of return ($\gamma$ = 1, 5, 10, 15, 20, 25, 30%). The study period was divided into two yearlong samples (2004 and 2005), where each of them contained 221 quarterly rates of return. On their bases the significance of changes in the distributions of the rates of return for the considered portfolios was analyzed. For that purpose the K-S test was applied for two samples and the ADF test, for which the empirical format is:

$$\Delta R_{t\gamma} = \mu + \alpha t + \delta_0 R_{t-1,\gamma} + \xi_t \ (t = 1, ..., 442); \ (\gamma = 1,5,10,15,20,25,30) \tag{13}$$

where:
$\Delta R_{t\gamma}$ – represents the increase of the rate of return for the portfolio determined for the assumed rate $\gamma$ during the period $t$. The results of application of the above tests are presented in table 1. For low assumed rates of return the variance of the distribution of the rates of return for SEM portfolios was higher than for Markowitz portfolios. This is indicated by high K-S test values and low absolute values of ADF test.

Table 1
Kolmogorov-Smirnov test of the significance of differences between rates of return distributions test and the test of unit root for the effective portfolios designed on the basis of minimizing variances and covariances during the period of 2004-2005

| $\gamma$ | Portfolio type | K-S test statistics value | ADF Test version with constant |
|---|---|---|---|
| 1 | Markowitz | 0.798 ($p$>0.1) | -6.282[a] |
| 1 | SEM | 6.083 ($p$<0.001) | -1.913 |
| 5 | Markowitz | **0.756** ($p$>0.1) | -6.348[a] |
| 5 | SEM | 2.943 ($p$<0.001) | -2.601[c] |
| 10 | Markowitz | 2.091 ($p$<0.001) | -4.759[a] |
| 10 | SEM | 4.509 ($p$<0.001) | -1.826 |
| 15 | Markowitz | 3.511 ($p$<0.001) | -3.199[b] |
| 15 | SEM | 4.992 ($p$<0.001) | -1.978 |
| 20 | Markowitz | 6.895 ($p$<0.001) | -1.916 |
| 20 | SEM | 6.181 ($p$<0.001) | -1.540 |
| 25 | Markowitz | 7.316 ($p$<0.001) | -1.614 |
| 25 | SEM | 7.316 (p<0.001) | -1.547 |
| 30 | Markowitz | 7.610 ($p$<0.001) | -1.470 |
| 30 | SEM | 7.410 ($p$<0.001) | -1.439 |

*Source:* own computations.
*a, b, c* – process integrated I(0) at the significance level equal to: $\alpha = 0.01$; $\alpha = 0.05$; $\alpha = 0.1$ respectively.

SEM portfolios, with one exception for the assumed rate $\gamma = 5\%$, are characterized by non-stationarity of the rates of return distributions. In case of high rates of return assumed, distributions of both Markowitz portfolios and SEM portfolios change over time statistically significantly. A particularly higher stability of the rates of return distribution for Markowitz portfolios as compared to SEM portfolios becomes visible for the assumed rate of return $\gamma = 1\%$. Graphically that situation is presented in figures 1-4.

Variability of the distributions for the analyzed portfolios over time shows that portfolios effective during one period will not be effective during the following period. Construction of portfolios on the basis of the proposed models is justified making the assumption that the investor acts under conditions of risk, i.e. that the investors know the distributions of the rates of return. Variability of the distribution over time causes that the investor operates under conditions closer to the conditions of uncertainty that conditions of risk.

Tests applied in the study, in case of portfolios minimizing variance for the rates of return at 10% and 15% gave ambivalent results as concerns variation of the distributions of the rates of return for those portfolios. Those differences result from treatment of time run stationarity in the ADF test in the narrow sense only, i.e. testing the differences concerning the basic parameters of the

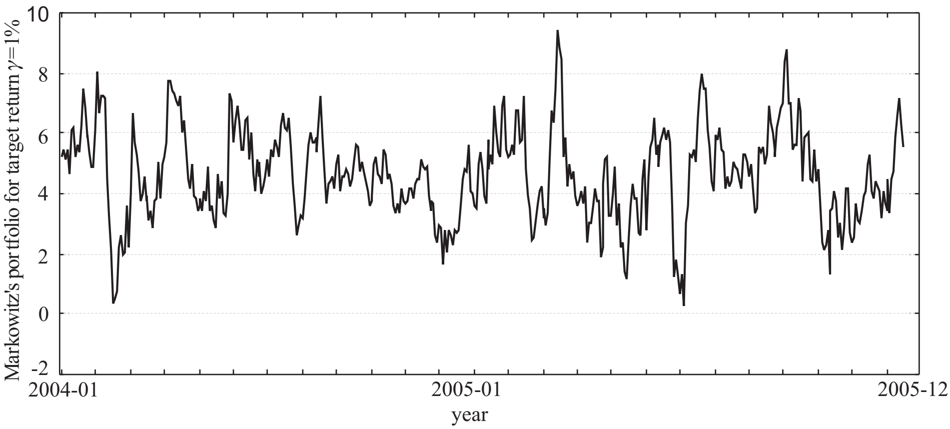


Fig. 1. Rates of return for the portfolio minimizing variance (Markowitz portfolio), for the assumed rate of return $\gamma = 1\%$ during the period of 2004-2005
*Source:* Own work.

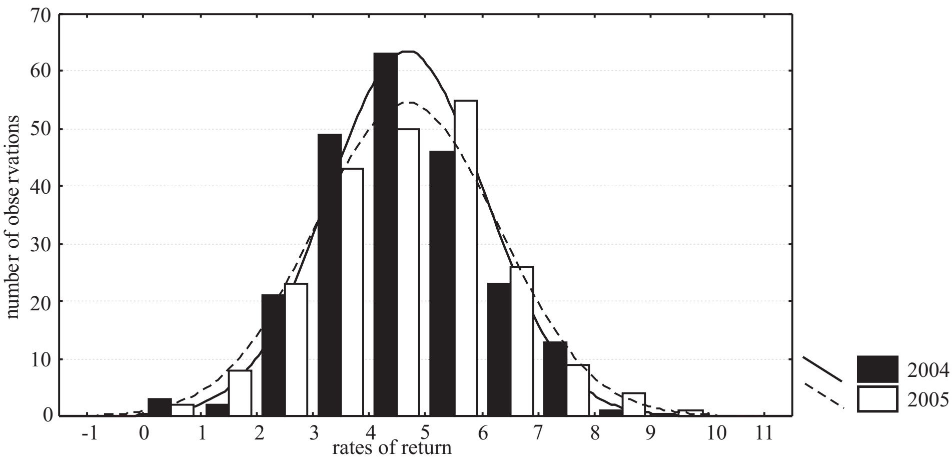


Fig. 2. Distributions of the rates of return for Markowitz portfolios for 1% during 2004 and 2005
*Source:* Own work.

distribution. The above test does not consider the issue of the distribution shape, and in particular its consistency with the normal distribution. The K-S test applied for two samples tests all the differences between distribution functions of the distributions compared. Figure 5 presenting the histogram of the rates of return for Markowitz portfolios at $\gamma = 10\%$ represents the graphic expression of that situation. It shows major differences in the distribution density functions for two samples although the ADF test gave grounds to reject the zero hypothesis and in favor of the alternative hypothesis indicating

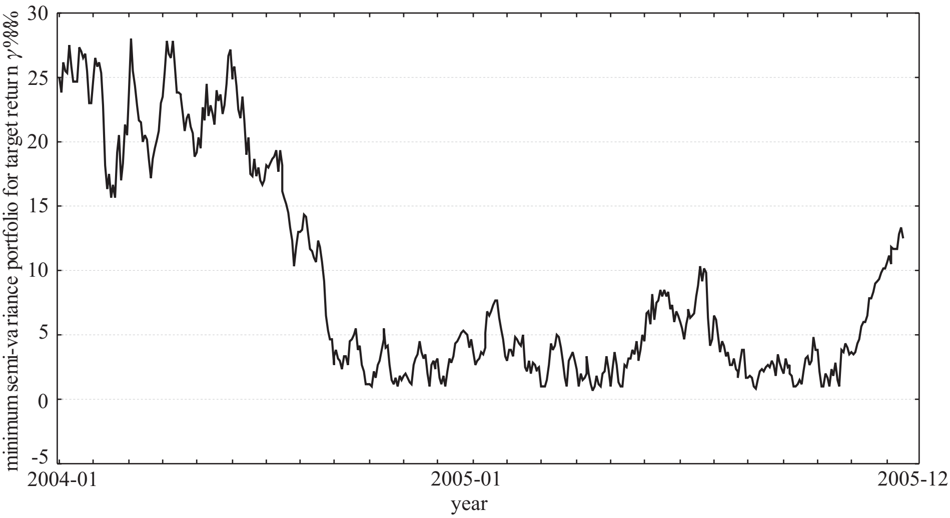


Fig. 3. Rates of return for the portfolio minimizing semivariance for the assumed rate of return $\gamma = 1\%$ during 2004-2005
*Source:* Own work.

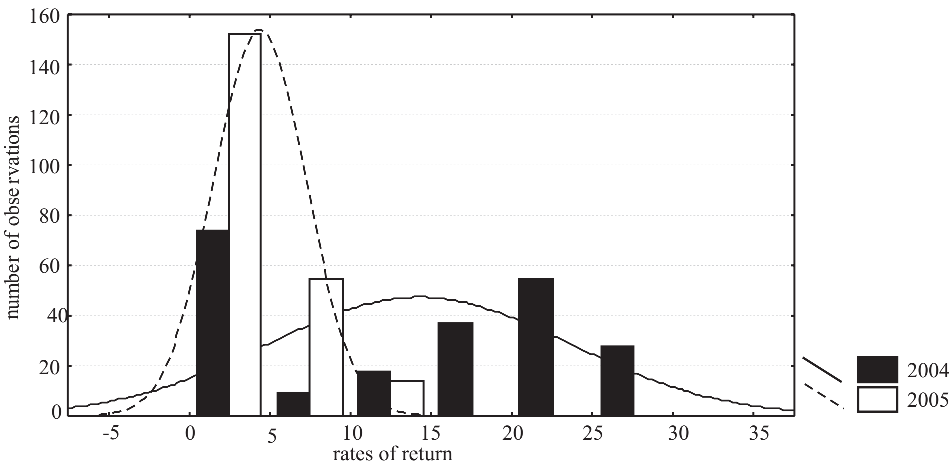


Figure 4. Distributions of rates of return for SEM portfolios for 1% during 2004 and 2005
*Source:* Own work.

stationarity of the rates of return run for that portfolio over time. In 2004 the distribution of the rates of return for that portfolio did not differ significantly from the normal distribution, however, in 2005 that difference was significant because of, e.g. double modality of the distribution visible in the figure.

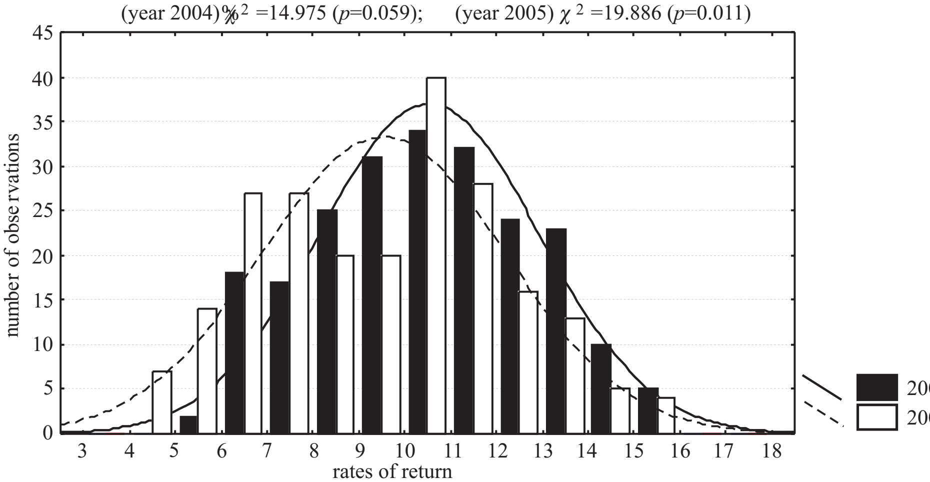


Fig. 5. Distributions of rates of return for Markowitz portfolios for the assumed rate of 10% during 2004 and 2005

*Source:* Own work.

Detailed analysis of the distributions obtained for both the entire sample and the two sub-samples was the next step of the studies. The basic characteristics of the distributions and the decision of consistency test $\chi^2$, verifying consistency of the distribution tested with the normal distribution, are presented in table 2.

The average rates of return ($\bar{z}_p$), minimum and maximum for all portfolios are higher in the first sub-sample than in the second one. Those differences increase with the increase of the assumed rate of return $\gamma$ and achieve higher values for the SEM portfolios than for the Markowitz portfolios. At the same time SEM portfolios have higher average rates of return, minima and maxima than the Markowitz portfolios. Because of those parameters of the distribution SEM portfolios should be considered superior to Markowitz portfolios.

Because of semivariance SEM portfolios are safer than Markowitz portfolios. Only because of the variance Markowitz portfolios are better than SEM portfolios. Those results mean that in SEM portfolios, as compared to Markowitz portfolios the dispersion of the rate of return around the average is larger while the diversions below the assumed rate of return are smaller and those above it are larger. For those reasons SEM portfolios should be considered superior for the investors because at a lower risk of obtaining the rate of return lower than assumed they create the opportunity for high rates of return.

The majority of the portfolios hade the right sided skew distributions. Few examples of left sided asymmetrical distributions are characterized by a relatively low strength of asymmetry (asymmetry coefficient from -0,22 to -0,03).

Table 2
Selected characteristics of the rates of return distributions for portfolios during the years 2004-2005

| $\gamma$ | Portfolio | Year | $\bar{z}_p$ | Min. | Max | $s_p^2$ | $ds_p^2\ (\gamma)$ | $A$ | Consistency test $\chi^2$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | M | **2004-05** | **4.64** | **0.29** | **9.42** | **2.27** | **0.0029** | **0.03** | Normal |
| | | 2004 | 4.63 | 0.38 | 8.03 | 1.93 | 0.0031 | -0.06 | Normal |
| | | 2005 | 4.64 | 0.29 | 9.42 | 2.61 | 0.0027 | 0.09 | Normal |
| 1 | SEM | **2004-05** | **9.23** | **0.74** | **27.95** | **71.44** | **0.0002** | **0.88** | Not consistent with n. |
| | | 2004 | 14.14 | 1.00 | 27.95 | 86.65 | 0.0001 | -0.14 | Not consistent with n. |
| | | 2005 | 4.33 | 0.74 | 13.25 | 8.19 | 0.0004 | 1.05 | Not consistent with n. |
| 5 | M | **2004-05** | **5.00** | **0.72** | **9.75** | **2.28** | **1.11** | **0.07** | Normal |
| | | 2004 | 5.00 | 0.94 | 8.35 | 1.87 | 0.92 | -0.03 | Normal |
| | | 2005 | 5.00 | 0.72 | 9.75 | 2.71 | 1.32 | 0.12 | Normal |
| 5 | SEM | **2004-05** | **10.47** | **4.81** | **20.76** | **16.13** | **0.0002** | **0.51** | Not consistent with n. |
| | | 2004 | 11.48 | 4.99 | 20.76 | 19.25 | 0.0001 | 0.29 | Not consistent with n. |
| | | 2005 | 9.45 | 4.81 | 19.95 | 11.03 | 0.0005 | 0.45 | Not consistent with n. |
| 10 | M | **2004-05** | **10.00** | **4.34** | **15.88** | **6.58** | **3.34** | **-0.02** | Not consistent with n. |
| | | 2004 | 10.49 | 5.28 | 15.88 | 5.68 | 2.03 | -0.05 | Normal |
| | | 2005 | 9.51 | 4.34 | 15.74 | 7.02 | 4.67 | 0.10 | Not consistent with n. |
| 10 | SEM | **2004-05** | **16.73** | **5.81** | **47.16** | **92.24** | **0.51** | **1.56** | Not consistent with n. |
| | | 2004 | 21.14 | 5.81 | 47.16 | 136.53 | 0.33 | 0.67 | Not consistent with n. |
| | | 2005 | 12.33 | 6.45 | 20.79 | 9.37 | 0.69 | 0.42 | Normal |
| 15 | M | **2004-05** | **15.00** | **2.80** | **25.75** | **26.59** | **13.37** | **-0.03** | Not consistent with n. |
| | | 2004 | 16.87 | 6.01 | 25.75 | 24.74 | 6.85 | -0.22 | Not consistent with n. |
| | | 2005 | 13.13 | 2.80 | 23.76 | 21.49 | 19.95 | 0.01 | Not consistent with n. |
| 15 | SEM | **2004-05** | **17.08** | **5.13** | **45.75** | **81.14** | **10.55** | **1.41** | Not consistent with n. |
| | | 2004 | 21.73 | 6.23 | 45.75 | 108.85 | 5.27 | 0.60 | Not consistent with n. |
| | | 2005 | 12.43 | 5.13 | 20.13 | 10.33 | 15.88 | -0.03 | Normal |
| 20 | M | **2004-05** | **20.00** | **-2.44** | **49.73** | **137.52** | **58.36** | **0.43** | Not consistent with n. |
| | | 2004 | 27.95 | 10.00 | 49.73 | 100.52 | 7.23 | 0.15 | Not consistent with n. |
| | | 2005 | 12.05 | -2.44 | 29.35 | 48.05 | 109.81 | -0.09 | Not consistent with n. |
| 20 | SEM | **2004-05** | **20.00** | **3.33** | **53.63** | **166.95** | **49.45** | **1.12** | Not consistent with n. |
| | | 2004 | 27.73 | 5.48 | 53.63 | 196.44 | 21.15 | 0.25 | Not consistent with n. |
| | | 2005 | 12.27 | 3.33 | 23.47 | 18.09 | 77.99 | -0.03 | Not consistent with n. |
| 25 | M | **2004-05** | **25.00** | **-3.45** | **83.01** | **426.56** | **147.62** | **0.90** | Not consistent with n. |
| | | 2004 | 38.84 | 5.39 | 83.01 | 406.59 | 40.07 | 0.20 | Not consistent with n. |
| | | 2005 | 11.16 | -3.45 | 28.57 | 63.63 | 255.85 | 0.08 | Not consistent with n. |
| 25 | SEM | **2004-05** | **25.00** | **-0.56** | **83.91** | **448.32** | **143.80** | **1.02** | Not consistent with n. |
| | | 2004 | 39.15 | 3.23 | 83.91 | 454.78 | 45.49 | 0.18 | Not consistent with n. |
| | | 2005 | 10.85 | -0.56 | 26.36 | 41.65 | 242.76 | 0.20 | Not consistent with n. |
| 30 | M | **2004-05** | **30.00** | **-13.73** | **131.96** | **1135.18** | **337.72** | **1.25** | Not consistent with n. |
| | | 2004 | 52.24 | 0.30 | 131.96 | 1184.76 | 83.11 | 0.56 | Not consistent with n. |
| | | 2005 | 7.76 | -13.73 | 31.50 | 97.02 | 593.87 | 0.24 | Normal |
| **30** | **SEM** | **2004-05** | **30.00** | **-13.66** | **133.19** | **1140.46** | **337.19** | **1.27** | Not consistent with n. |
| | | 2004 | 52.01 | -0.72 | 133.19 | 1210.82 | 87.51 | 0.57 | Not consistent with n. |
| | | 2005 | 7.99 | -13.66 | 32.38 | 101.59 | 588.40 | 0.29 | Normal |

*Source:* Own computations.

In the majority of cases SEM portfolios are more right sided skew than Markowitz portfolios. Right-sided asymmetry is demanded by the investors (JAJUGA, JAJUGA 1998), so they will prefer SEM portfolios.

The rates of return distributions for low values of the assumed rate of return (1% and 5%) are consistent with normal distribution in case of Markowitz portfolios only, as opposed to SEM portfolios. For the higher rates of return assumed, with few exceptions, the distributions of all portfolios differed statistically significantly from the normal distribution. It should be pointed out that because of the differences in the level of distributions asymmetry, deviation from normal distribution is larger for SEM portfolios than for Markowitz portfolios, which is confirmed by not presented, detailed results of the consistency test.

## Conclusion

Studies on variability of rates of return distributions over time based on the ADF run integration test and K-S consistency test showed that rates of return distributions for the designed portfolios, in particular for high assumed rates of return, were characterized by high variation. The phenomenons of variation of distributions over time as concerns both the distribution parameters and shape are unfavorable for the investor as effective portfolios structured on the basis of historical data can loose their properties in the future (between purchase and sale). In case of analyzed portfolios, the K-S test proved to be a more stringent method of verification of persistence of the rates of return distributions over time than the ADF test.

Considering the selected distribution parameters, SEM portfolios were superior to Markowitz portfolios but they showed a higher variation of distributions over time.

Translated by JERZY GOZDEK

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