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LEIBNIZ'S ALGORITHMIC INTERPRETATION OF LULLUS' ART

In 1966 we call to mind not only the 250th anniversary of Leibniz's death but, at the same time, the tercentary of his *Dissertatio de arte combinatoria*, a work in which this young scientist, then aged 20, made his debut and, at the same time, revealed the significant roots of his genius. What we call "combinatorics" is a mathematical discipline, requiring for its operation none of the means surpassing the scope of antique mathematical knowledge. Even so, it is characteristic that it was not until the *Grand Siècle* of the Baroque, that combinatorics which demands no more than the four fundamental rules of arithmetic, was conceived as a new mathematical discipline. In the framework of Aristotle's antique logic it had been possible to picture operations of thinking by operations of calculations; however, antique thinking was unable to the final step in abstraction, and to admit logical relations not only between complete logical subjects but between relations as well by calculating with relations in the same manner as with numerals. The arithmetic of antiquity was not yet able to pass on to the arithmetic of relations, that is, to what is called combinatorics. In his 1666 dissertation Leibniz even pointed out the logical superiority of relations over relating subjects; this dissertation calls attention not only to his calculating machines of a later date but, likewise, to the notion of the logical operator in general. And it does not seem to be a haphazard occurrence that neither the followers of the numeral system nor practical scientists among the mathematicians invented calculating machines for the use of merchants, but this was rather done by the philosophers and logicians like Leibniz and Pascal, by the use of their *Ars combinandi*. Statistics also is an achievement of the 17th century; this branch of science, called *Ars conjectandi*, was developed by Jakob Bernoulli, also a pure mathematician.

Fig. 1. 3rd and 4th logical Figure of Raymond Lullus
(from a MS of the 14th century)

Again it might not be by accident, that Pascal's machine was inferior to that of Leibniz, because it was unsuitable for carrying out the last of the four fundamental rules, the division; Pascal's *Théorie des combinaisons* still lacked the notion of an operator which, even when lacking any concrete contents, would maintain its logical existence.

Leibniz was probably the one thinker of the Baroque who, while most consciously adhering to traditional thinking, aimed at converting, science from logically static subjects and predicates of its scholastic past into a science of functions of variable values of the positivistic future, without philosophical obstacles. His 1666 writing still bore predominantly scholastic features: quoting his predecessors he attempted to automatise thinking and, in this particular case, calculating. Not only did Leibniz point to Neper's (Napier's) rhabdology dating from 1617, by which this Scotch peer, known to us as the founder of logarithmic calculus by means of his "rods" or "bones" (mechanical devices for carrying out arithmetical operations), was the first to supersede the abacus by automatizing calculations. Leibniz also dedicated a number of chapters on logic to the legendary Catalonian monk Ramon y Lull, who as early as in the 13th century attempted to automatise logical operations by means of his rotating discs. A manuscript found by Couturat among the 80,000 papers of the heritage left by Leibniz mentions: "Raymundus Lullus also dabbled in mathematics; he hit upon the notion of the 'science of combinations.' This Lullus' art would undoubtedly be somethink beautiful, were it not that the fundamental expressions he uses like: goodness, greatness, duration, force, wisdom, will, virtue, fame, are vague and merely suitable for speaking of truth, not for detecting it."

Leibniz by no means passes censure on Lullus' intention as an attempt of automatizing processes of thinking by machinery; however, he merely recognizes Lullus' "Figures" to be inadequate as logical apparatus. For instance, in his third Figure Lullus depicts a model intended to illustrate, how the 9 absolute predicates are to be combined with the 9 relative-predicates of his logical *Alphabetum,* to form judgments, and how—in Lullus' belief—one passes from the general to the specific. However, first one has to know, by which intermediate notion the predicates of these judgments are to be combined with their subjects. This Lullus tries to achieve by mechanizing, in his fourth Figure, the arrangement of his 9 absolute and 9 relative predicates in such manner, that by rotating the two inner circular discs there can pass along each of the 9 subdivisions of the outer disc the 9 subdivisions of the inner disc, and that the subdivisions of the intermediate disc can operate as intermediate notions of judgments.

The *Ars magna* of Lullus' art became not only the watchword of the *Rinascimento* humanists; it continued to be considered the *Nuova Scienza* in general—until the time when it suffered its baroque disfigurements in Rosenkreuz's *Rota Mundi* which was believed to contain everything conceivable by science.

However, it happened to be Leibniz who, still in school, confounded his teachers by the assertion that, alongside of the scholastic table of predicaments by means of which concepts can be composed into true judgments, there must also exist a corresponding table of predicamental judgments by means of which judgments can be composed into true conclusions—so that there must be in existence a thought alphabet of predicaments, by the use of which all kinds of truths can be expressed.

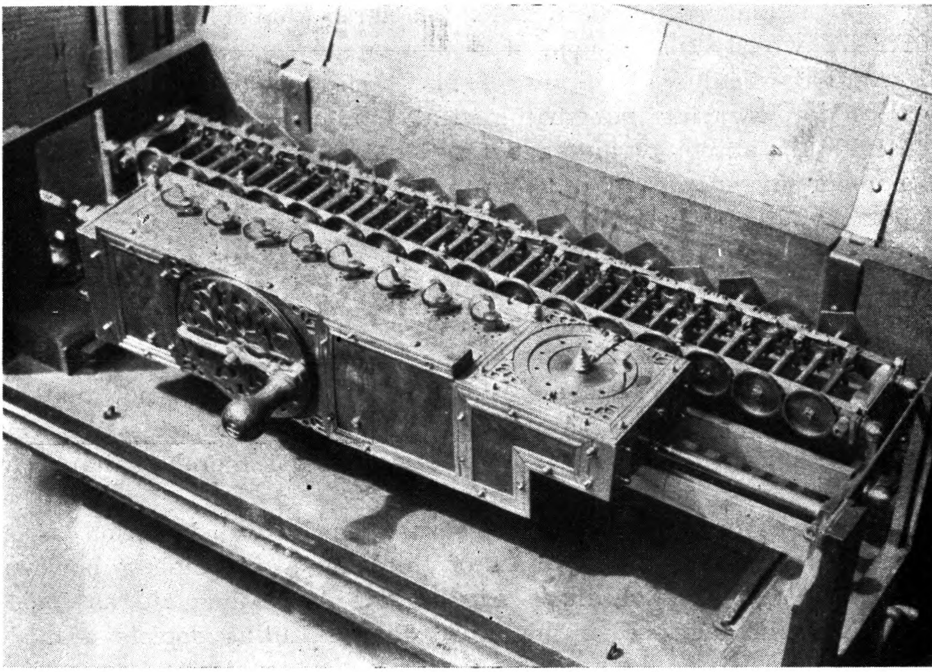


Fig. 2. The original of Leibniz's calculating machine is held by the Lower Saxonian Provincial Library at Hannover (phot. L. v. Mackensen, 1966)

In youthful Leibniz this concept, put forth by Lullus, flourished with a significant root: to wit, as the concept of a logical automatism, which would exceed the previous range of definitions and advance algorithmically to new stages. From the time of Al-Khwārizmī's 9th century textbook on algebra, the art of calculating is called algorithm, because the Latin translators muddled the incipit of the Arabic name in *Dixit Algorizmi* ... However, by several nominalists this term suffered in the 14th century a characteristic variation: Nicolas d'Oresme's book *Algorismus proportionum* already introduced fractional exponents of roots, so that this abstract formulation of root extraction is older than the radical sign of the Cossists. Even so, d'Oresme's formulation is derived from a typically algorithmic process: the potentials which originally had been defined exclusively for integer exponents, now became new inverse values, that is, radicands, in consequence of a merely formal transition from numbers to fractions.

The technique of an algorithmic creation of new mathematical forms developed in Leibniz to a high degree of artistry; because the *Ars Lulliana* is meant to be less a logistic *Ars demonstrandi* than rather a scientific *Ars inveniendi*. It should be remembered, that in 1637 Descartes had by his *Méthode nouvelle* subjected geometry to algebraic calculation; in 1670 Leibniz, who later was to create the notion of "function", already demanded in his *Physica nova*—a work devoted more definitely to the future than his baroque 1666 dissertation—that Galilei's dynamics should be subjected to Descartes' algebraic calculations. This latter scientist had steadily declined to understand Galilei's dynamics; the reason was, that the notion of a *variable* transgresses the barrier of *Clare et distincte* of the Cartesian finite coordinate

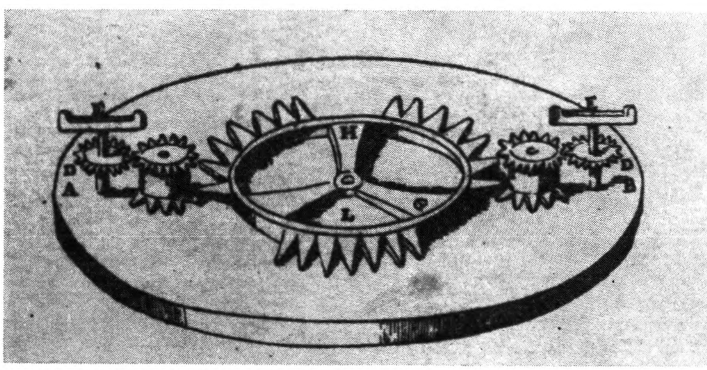


Fig. 3. Leibniz's watch as pictured in the *Journal des Savants*

system. On the contrary, Leibniz demanded a calculation that would also deal with variables as well as with finite values—a way of thinking preposterous to antiquity.

However, Leibniz did not find his new solution within the theorems of Galilei's dynamics, where the problem of velocity is explained as the tangential limit position of the secant between two points infinitely close to each other on the curve of velocity, as a year previously Newton had discovered by his fluxion theory. It so happened, that Leibniz's intellect had to be sparked at the point when the formally logical predominance of a constant relation over the infinitely disappearing relationed subjects became more clearly visible than in the problem of tangents. Leibniz arrived at his success in 1673 when he was shown, by Huyghens, the writings left by Pascal: drawing at the tangent of a circle from two adjoining points the parallels to the axes of the circle, the small triangle created at the tangent must always equal the larger triangle formed by the radius, the ordinate and the subnormal. Even so, Leibniz perceived something "que l'auteur Pascal n'avait pas vu": that this axiom is valid for all curves, not only for the circle, and that from point to point of the curve this triangle changes, so that it is possible from these changes to define the shape of the curve; and that, for this reason, this triangle is justly called the *Triangulum characteristicum.*

Because therefore the tangential triangle, be its sides as small as may be, always resembles the large triangle—this does not apply to Galilei's tangents, though—Leibniz, the champion of *Ars combinatoria,* takes the important step: in view of the logical principle of a relation being constant, he continues to write the limit value of this constant relation even then as quotient, when the logical material subjects of this relation, i.e. the sides of the triangle, turn into zero. And while, from the Cartesian point of view, these values are no *extensiva,* thus

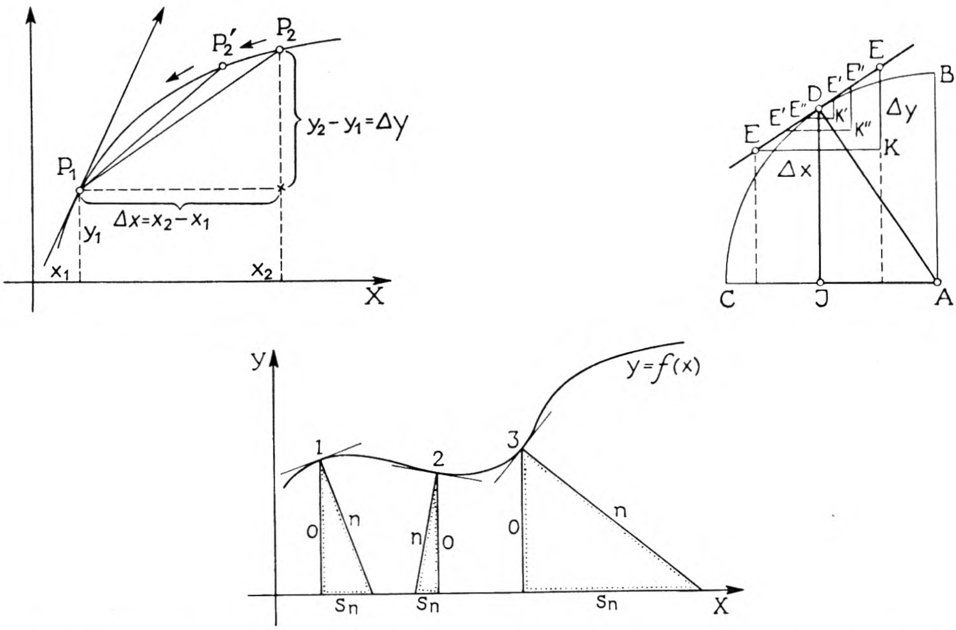


Fig. 4. The *Triangulum characteristicum* mentioned by Galilei and Leibniz

without meaning, Leibniz revives them as inextensive, in other words, intensive values on the basis of the relative trend of his thinking. Thus, for example, to him Galilei's inextensive yet intensive indivisible of velocity is, as $\frac{ds}{dt}$ a true quotient of the infinitely small distance ds to the infinitely small time dt. As soon as the two *infinitesimalia,* the logical material of this relation, turn into zero, in the scholastic unity of *Forma* and *Materia* (i. e., *subjectum*) the form becomes infinitely dense, creating in this manner a new subject: the derivative.

More than Newton, Leibniz adhered to the Cartesian-Lullian tradition, but less than Newton to the tradition of the Nominalist school of Oxford and Paris; in late medieval times these schools implicitly already made use of the concept of a function in their graphical presentation of nature's law—a procedure earlier anticipated by Guido d'Arezzo with his scale of tones. There exist several "codices" dated from the 12th and 14th centuries, where already the ecliptical latitudes of planets were shown graphically as functions of lengths, that is, of time. It does not seem by haphazard, that this probably oldest relation, known already, though empirically only, to the Babylonians—because the Greeks did not attempt to express this relation by a variety of geometrical models—became the basis for graphically indicating natural processes during the Middle Ages; this is evident from the fact, that in *Mensurae formarum* the later Cartesian ordinates were called *Latitudines* and the later Cartesian *abscissae Longitudines*. Leibniz chose to define his concept of functions within the system of Aristotle's scholastics. And it is only by the application of the Lullian algorithm to the concept of functions, that the century's yearning for the *Scienza Nuova* was fulfilled. Leibniz asserted: "Functio est continuatio omnium variationum formarum." In the late Scholastic sense of the word, Galilei called velocity *Forma motus*. Therefore, by continuity in time of the change of all forms of velocity, Leibniz solved the process described by the function. In this manner physics—a special case of metaphysics—denotes the distance travelled as the product of process:

$$s = \int v\,dt = \int \frac{ds}{dt}\,dt$$

However, for Leibniz the differential quotient is a true quotient—this is the proper *Metaphysique du calcul infinitesimal*—so that dt can be cut out and thus the logical, and the more so, the arithmetic identity $s = \int ds$ results.

This logically formal identity was apt to become the source of the whole material essence of physics. Because now there were only to be established the differential laws—a matter to be assigned to the physicists; afterwards it was the mathematician's task to solve them. And should he be unable to do the integrating operations, computing machines would attend to this with any desired accuracy. And this indeed happened to be fundamental task of Newton's physics: in practical use, Newton's calculus of fluxions operated by the development of functions into infinite sequences. Here came to light the fundamental difference in the invention of the calculus between Newton and Leibniz. The notorious controversy on priority between these two scientists was essentially a contest as to the correct notation of the infinitesimal calculus. Both of them combined the attitudes adopted by Descartes and by Leibniz, and this synthesis was bound to result automatically in the infinitesimal calculus, because just as much as Descartes repudiated Galilei's continuous variables, Galilei—being a *Rinascimento* Italian—shrinked from an algebraization of geometry, which already had been condemned by Plato. And yet, while Newton accomplished this synthesis as a physicist, Leibniz did it as the algorithmian. In the latter's work the fundamental theorem of inverse calculus achieved its full expression in its formal perfection: if a function is to represent the "continuatio variationum functionis $f = \int df$," this should operatively be written:

$$1 = \int d.$$

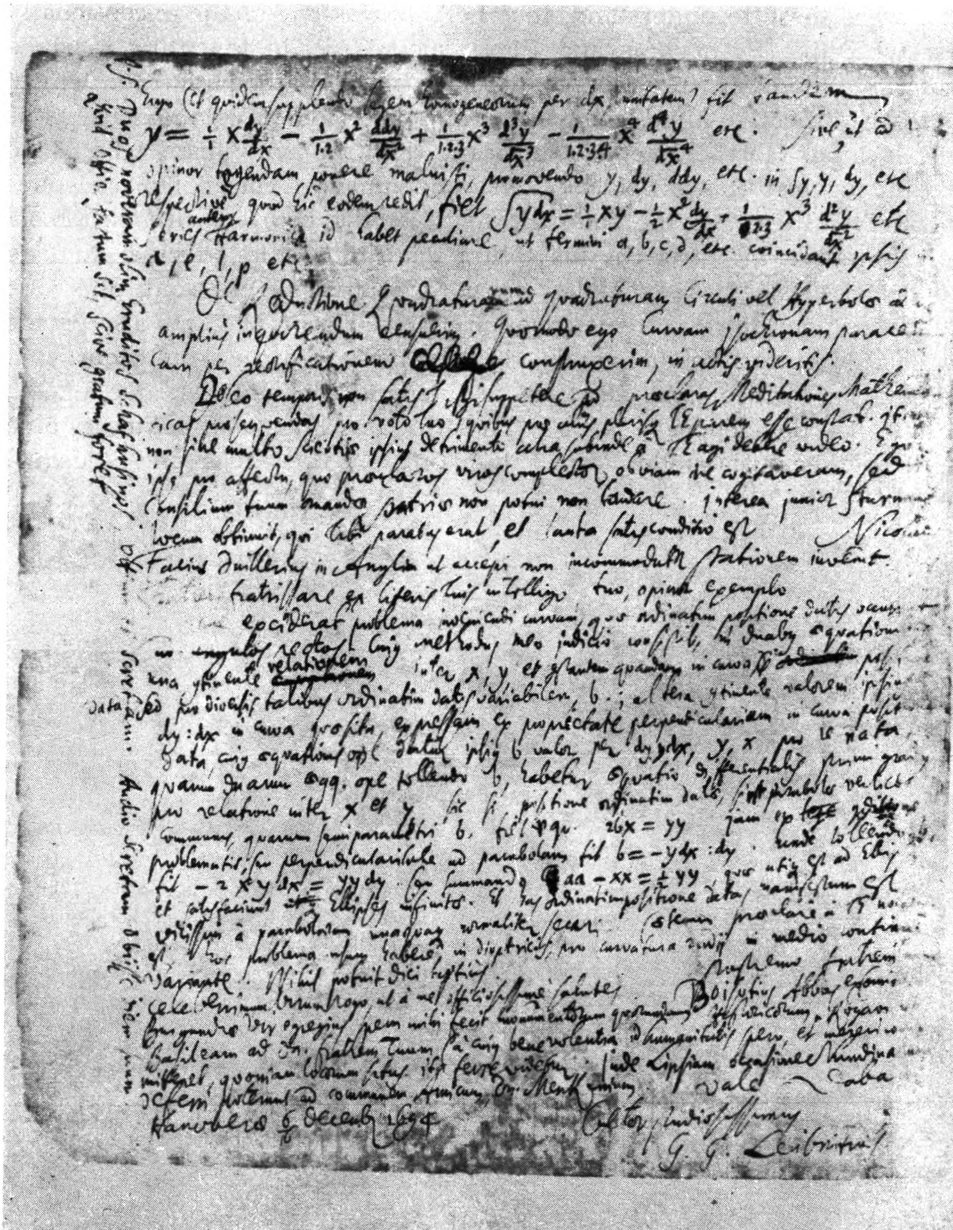
By introducing this relation Leibniz has completely shattered the framework of antique mathematics; because now one can write $d^{-1} = \int$ and algorithmically develop the full sequence

$$\ldots\ldots d^{-3},\ d^{-2},\ d^{-1},\ d^{0},\ d,\ d^{2},\ d^{3}, \ldots\ldots$$

This sequence of higher differentials and integrals impressed the Cartesians merely as a metaphysical monstrosity: how to imagine the existence of an infinite sequence of values, all mutually infinitely large, or infinitely small?

Even more so: while the Newtonians discovered in the problem of interpolation the Taylor series as an approximate polynomial of higher order, Leibniz deduced the Taylor series from his $\int = \mathrm{d}^{-1}$ formula, by using this formula for the n-th product-differentiation $d^n\ (uo) = (du + do)^n$.

Admittedly the n-th differential of a product can be written symbolically as the binomial development for the sum of the first differential. Therefore, one can consider $\int f\, dx$ to be the -1 derivative of the product of f and dr—meaning that $(df + d^2x)^{-1}$ must be symbolically developed into an infinite binomial sequence. This then was indeed, as Johann Bernouilli enthusiastically claimed, the "series universalissima, quae omnes integrationes exprimit," that is: $\int f\, dx$. To be sure, here the coefficients, under development towards higher derivations, were still functions; however—as pointed out by Pringsheim—this Leibniz-Bernouilli sequence can be transformed into the Taylor series. The development of the Taylor series materially completed the full scope of the theory of functions, because it also embraced the complex and, from a historical point of view, even became the foundation of the universal theory of functions. By this achievement Leibniz the algorithmian attained his supreme triumph: with a minimum of mathematical form he expressed a maximum of mathematical contents.


$$y = \frac{1}{1} x \frac{dy}{dx} - \frac{1}{1.2} x^2 \frac{ddy}{dx^2} + \frac{1}{1.2.3} x^3 \frac{d^3y}{dx^3} - \frac{1}{1.2.3.4} x^4 \frac{d^4y}{dx^4} \text{ etc.}$$

$$\int y\,dx = \frac{1}{1} xy - \frac{1}{2} x^2 \frac{dy}{dx} + \frac{1}{1.2.3} x^3 \frac{d^2y}{dx^2} \text{ etc.}$$

Fig. 5. Leibniz's letter to J. I. Bernoulli (dated Dec. 16, 1694) containing the "Taylor series"

It is characteristic, that until into the 19th century Leibniz was looked upon as a plagiarist of Newton, reputed—at best—to have merely invented a more suitable form of notation. Due to this, Leibniz's prestige had greatly suffered among the professional philosophers. However, how were these to understand, that Leibniz was one of the exceptionally few who, like Pythagoras and Plato, succeeded in inventing a whole esoteric philosophy in order to describe their mathematics for which, in their respective time, the vocabulary, later on in common use, was still lacking. Here it seems significant that the only man, who made a stand against this official secular condemnation voiced by academies and universities, was the chief ideologist of the *Esprit positif de l'École Polytechnique* in Paris. In his *Cours de Philosophie positive* Auguste Comte asserts: "Des exemples de nature aussi divers sont plus que suffisants pour faire nettement comprendre en général l'immense portée de la conception fondamentale de l'analyse transcendante, telle que Leibniz l'a formée (and here we feel tempted to add: "et que l'École Bâloise l'a developpée"), et qui constitue sans aucune doute la plus haute pensée à laquelle l'esprit humain se soit jamais élevé jusqu'à présent."