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Did "algorism", [1] the medieval name for arithmetic based on decimal place-value Hindu-Arabic numerals, play any effective role in the European Renaissance of the fifteenth and the sixteenth centuries? If the answer is "yes", as many believe it is, the question of relationship of Indian mathematics, arithmetic in particular, with European Renaissance assumes immediate importance.

As to algorism as one of the active promoters of the Renaissance, one has only to look at the sudden appearance in quantity of printed arithmetical works, a good number of them in vernaculars, during the 16th century and the quick multiplication of their editions or reprints. Among the works in circulation in Italy, mention may be made of Cardano's *Practica arithmetice et mensurandi singularis* (Pavia 1501; Rome 1576) and Tartaglia's voluminous *La Prima Parte del general trattato di numeri e misure* (Venice 1556). In England, Robert Recorde's *The grounde of artes, teachyng the worke and practise of arithmetike* (London 1542) was reprinted 17 times before 1601 and Digg's *Arithmeticall militare treatise called Stratiotios* (London 1579) enjoyed considerable popularity. In Germany, the arithmetical movement was led by Jacob Köbel, author of *Rechenbiechlin* (Augsburg 1514), Stifel whose scholarly work *Arithmetica integra* (Nürnberg 1514) was reprinted several times, and by Christopher Clavius whose *Epitome arithmeticae practice* (Rome 1583) and its Italian translation (1586) were in extensive use

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[1] The words "algorism" and "augurim" (used by Chaucer) were originally derived from the name of al-Khwarizmi (9th century). Witness the use of the words *algoritmi, algorismi, algorismo* in the twelfth century Latin translations of al Khwarizmi's arithmetic or of other similar Arabic work. Henri de Mondeville, in his *Cyrurgia,* explained "algorism" as meaning arithmetic. Whereas computus dealing with Church calendar used Roman numerals, every algorismus used decimal place-value Hindu numerals (G. Sarton, *The Appreciation of Ancient and Medieval Science during the Renaissance (1450—1600)*, Phildephia 1955, p. 151.

among the Jesuit missionaries. In France where arithmetic attained great popularity in the 16th century, the leading works were Boissière's *L'art d'arythmétique contenant toute dimention... tant pour l'art militaire que pour autres calculations* (Paris 1554) and Pierre Forcadel's *L'Arithmétique* (Paris 1556—7); *Arithmétique entière et abrégée*, Paris 1565) and *Arithmétique démontrée* (Paris 1570). In the Netherlands, Gemma Frisius' *Arithmeticae practicae methodus facilis* (Antwerp 1540) scored a record by reappearing in print 60 times during the Renaissance. Simon Stevin's *L'arithmétic et la pratique d'arithmétique* (Leiden 1585) and its subsequent part appearing in the same year under a separate title *La disme* have been characterized as "the greatest arithmetical monument of Renaissance." [2] The convenient numerical symbolism of which arithmetic is a logical consequence proved to be a powerful tool for the development in due course of other branches of mathematics.

The genesis of this new movement has been traced to the activities, among others, of Adelard of Bath, John of Seville, Robert of Chester, Villedieu, Sacrobosco and Leonardo Pisano, all of whom were engaged —the first three in the twelfth and the last three in the thirteenth centuries, in transmitting Arabic mathematical knowledge in Latin translations to Christian Europe. At this time and up to a much later date, the abacus of Gerbert was in general use for all calculations and counting purposes. Adelard of Bath (c. 1142) who was an abacist to start with and became an algorist later was probably the earliest Latin exponent of Hindu arithmetic, trigonometry and astronomy through his translations of al-Khwārizmī's mathematical and astronomical works. In 1126, he translated from Arabic into Latin the astronomical tables of al-Khwārizmī in the version of the Spanish astronomer Maslama al-Majritī. He is also believed to have translated *Liber ysagogarum Alchorismi in artem astronomicam magistro A. compositus*, an arithmetical work attributed to al-Khwārizmī. [3] But the earliest Latin version of al-Khwārizmī's arithmetic *Algoritmi de numero Indorum*, of which the Arabic original is lost, is due to an unknown translator. [4] This work containing about 5000 words, treats at length of numeration, using in most cases Roman numerals, and also

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[2] G. Sarton, *op. cit.*, p. 157. *La disme* is of special importance as in it is discussed Stevin's discovery of decimal fractions.

[3] G. Sarton, *Introduction to the History of Science*, II, p. 167—68. Also see C. H. Haskins in *The English Historical Review*, XXVI, p. 494.

[4] The MS was discovered in the Cambridge University Library by Prince Baldassare Boncamagni and transcribed and printed by him under the title *Trattati d'aritmetica*, Rome 1857, which also included John of Seville's *Liber algorismi de pratica arismetrice*. See also Suzan Rose Benedict, *A Comparative study of the early treatises introducing into Europe the Hindu art of reckoning*, University of Michigan 1914.

discusses fundamental operations with integers and fractions. The use of Roman numerals and the current abacus terms, e.g., *caracter, erigere, levare,* indicate the translator's unfamiliarity with the new arithmetic based on decimal place-value numeration. John of Seville's (c. 1135) *Liber algorismi,* although based on al-Khwārizmī's arithmetic, was not a verbatim translation, but utilized other Arabic sources. It contains discussion of the extraction of roots and summation of series, absent from the *Algoritmi* which it strikingly resembles in the general mode of treatment.

Robert of Chester (c. 1141) introduced the study of algebra in Latin Europe by his translation of al-Khwārizmī's *Hisāb al-jabr wal-muqābala,* which bears the impress of Hindu algebraic thought.[5] He also revised al-Khwārizmī's astronomical tables in Adelard's version of the meridian of London and was possibly the first to translate the Arabic *jaib,* derived from the Sanskrit *jīva,* into *sinus.*

Villedieu's (Alexandre de Villedieu, d. 1240) *Carmen de algorismo,* composed in hexameter, closely followed John's *Liber algorismi.* In this work zero signifying nothing was treated as one of the Hindu numerals, whose number was stated to be "twice five." Translations of *Carmen* into English, French and Icelandic, several commentaries produced on it in many languages and large number of MS copies found in the libraries of Europe bear testimony to the influence it exerted and the wide circulation it enjoyed in the thirteenth century. The work possibly played an important role in the diffusion of Hindu numerals in Latin Europe.[6]

Villedieu's contemporary John Sacrobosco, author among others, of the arithmetical treatise *Algorismus vulgaris* "contributed powerfully to the diffusion of the Hindu numerals" in Latin Europe.[7] The work, designed to be a practical exposition of the decimal place-value numeration for use in the universities, dealt with, besides the four fundamental operations, the methods of extraction of square and cubic roots and arithmetical series. Its great popularity for about three hundred years is evident from the large number of MS copies available in the libraries of Europe, and from several editions after it was first committed to printing in Strassburg in 1488. Despite his great service in popularizing the Hindu art of reckoning he was largely responsible

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[5] L. C. Karpinsky, *Robert of Chester's Latin translation of the Algebra of al-Khowārizmī*—with an introduction, critical notes and an English version, Macmillan, 1915. Solomon Ganz, upon whom Aldo Mieli largely depended for his comments on Arabic algebra (*La Science Arabe,* Leiden 1939) is of the opinion that al-Khwārizmī's algebra was derived from some ancient Babylonian or Iranian source ("The Origin and development of the quadratic equations in Babylonian, Greek and early Arabic algebra," *Orisis,* III, pp. 405—557).

[6] G. Sarton, *Introduction...,* II, p. 617.

[7] *Ibid., II,* p. 618.

for the later mistaken medieval view that the Arabs were the inventors of the science of calculation.

Another date often taken to be the starting point of European mathematical renaissance is 1202 in which year appeared Leonardo Pisano's arithmetical classic *Liber abaci*, containing probably the first complete exposition in Latin of Hindu and Muslim arithmetic and of decimal place-value numeration. Through another work *Flos* Leonardo also initiated, in anticipation of Bachet de Meziriac (1581—1638), the study of intergral solutions of indeterminate equations of the first and the second degree, in which the Hindus had excelled from the time of Āryabhata (c. A.D. 499) and Brahmagupta (c. 628) and of which the origins have been traced to the *Śulba-sūtras.* [8]

The process of transmission of Hindu arithmetic to Latin Europe through Arabic translations and summaries is closely associated with the transmission of Hindu astronomy, of which mathematics formed an integral part. Yet the importance of the latter was relegated to the background due largely to this system being superseded by the more sophisticated methods of Ptolemy. Nevertheless the initial and early source of inspiration of Arab astronomical renaissance never died out completely and received a new lease of life in Spain at a time when the Eastern Arabs were revising their astronomical tables after the *Almagest.* Kennedy, in his survey of Islamic astronomical tables, [9] has listed 21 *Zījes* (including 5 in the supplementary list), whose computations were either based on or influenced more or less by the Indian *Siddhāntas.*

The starting point was the preparation of the Arabic version of *Az-Zīj as-Sindhind* (c. 770) of one of the Indian *Siddhāntas,* possibly the *Brāhmasphuta Siddhānta* of Brahmagupta. This work, through revisions and refinements by subsequent authors such as al-Fazāri, Ya-'qub ibn Tāriq, al-Adamī, al-Quasim, al-Khwārimzī, al-Hasan ibn Misbāh, an-Nairīzī, al-Majritī, as-Saffār and others exerted a profound influence, first among the Eastern Arabs and subsequently among the Western Arabs of Spain. Maslama al-Majritī (fl. 1000) of Cordova edited al-Khwārizmī's tables which, as already mentioned, have survived in the Latin translation of Adelard of Bath. [10] The *Zīj* compiled by Abu al-Qāsim Asbagh... ibn as-Samh of Granada (c. 1010) and the collection of tables *Mukhtasar az-Zīj* by Ahmad bin 'Abdallah... ibn

[8] S. N. Sen, "Study of Indeterminate Analysis in Ancient India," *Proceedings of the Tenth International Congress of History of Science,* Herman 1964, pp. 493—497.

[9] E. S. Kennedy, "A Survey of Islamic Astronomical Tables," *Transactions of the American Philosophical Society,* 46, 1956, pp. 123—177.

[10] O. Neugebaur, "The Astronomical Tables of Al-Khwārizmī—Translation with commentaries of the Latin version edited by H. Suter supplemented by Corpus Christi College MS. 283," *Historisk-filosofiske skrifter udgivet af Det Kongelige Danske Videnskabernes Selskab,* Bind 4, nr 2, København 1962.

as-Saffār of Cordova (c. 1100) were based on the *Sindhind*. Some of the Arab astronomers like ibn Yūnis and al-Battāni who chose to compile astronomical *Zījes* after Ptolemy had also access to *Sindhind Zījes* and occasionally used Hindu parameters.

Some of the special features of Hindu astronomy which were in this way incorporated in Islamic *Zījes* are the zero meridian of *Ujjayinī* which assumed the name *Arin* in Arabic, the era of Kaliyuga (February 17, 3102 B.C.) which became the "era of Flood" the Hindu planetary theory, the tables of sines (R = 150) and the tables of solar declinations ($E = 240°$), and the methods of spherical trigonometry. The sine and the solar declination tables given by al-Zarkālī (c. 1050) in the *Toledan tables,* for example, in which the norm R = 150 is used, correspond to those met with in Brahmagupta's *Khandakhādyaka.*[11] In 1951, Lynn Thorndike published and translated an anonymous fifteenth century Latin MS Ashmole 191 II, in which computations were made for the geographical latitude of Newminster, England for the year 1428. The study of the astronomical parameters and tables given in the unsuspecting MS has revealed the characteristic features of Hindu astronomy and another interesting instance of transmission as late as the fifteenth century possibly through Arabic intermediaries[12]. The Newminster MS begins the era of Flood from the year 3102 B.C. in keeping with the beginning of the Kaliyuga. To find the geographical latitudes, the typical Hindu method of determining the length of the equinoctial noon-shadow has been given. The sine functions have been used instead of the Greek chord, and the values have been tabulated for R = 150, a norm used in the *Khandakhādyaka* and in the *Toledan tables.*

The meeting of the two kinds of trigonometrical functions, the Hindu sine of the *Sūryasiddhānta* and the *Khandakhādyaka* and the chord of the *Almagest,* in the astronomical *Zījes* no doubt stimulated interest in this new mathematical technique which was fully exploited by the astronomers and translators engaged in compiling the *Alfonsine tables.* Continuance of this interest by Levi ben Gerson and Richard Wallingford in the 14th century paved the way for the great trigonometrical work of Regiomontanus, *De triangulis omnimodis* (1476).

In such circuitous and unexpected ways elements of Indian arithmetic, algebra, trigonometry and methods of astronomical computations did find a place in the great revival of learning in Latin Europe from the thirteenth to the fifteenth century, upon which the Renaissance was ultimately based.

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[11] O. Neugebaur, "The Transmission of Planetary Theories in Ancient and Medieval Astronomy," *Scripta Mathematica,* 22, 1956, pp. 165—192.

[12] O. Neugebaur and Olaf Schmidt, "Hindu Astronomy at Newminster in 1428," *Annals of Science,* 8, 1952, pp. 221—228.