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Some Considerations on the Role of the Mediaeval Postulates to Base Scientific Cognition of Mathematics

Organon 8, 123-136

1971

Artykuł umieszczony jest w kolekcji cyfrowej Bazhum, gromadzącej zawartość polskich czasopism humanistycznych i społecznych tworzonej przez Muzeum Historii Polski w ramach prac podejmowanych na rzecz zapewnienia otwartego, powszechnego i trwałego dostępu do polskiego dorobku naukowego i kulturalnego.

Artykuł został zdigitalizowany i opracowany do udostępnienia w internecie ze środków specjalnych MNiSW dzięki Wydziałowi Historycznemu Uniwersytetu Warszawskiego.

Tekst jest udostępniony do wykorzystania w ramach dozwolonego użytku.

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SOME CONSIDERATIONS ON THE ROLE OF THE MEDIAEVAL POSTULATES TO BASE SCIENTIFIC COGNITION ON MATHEMATICS *

One day I reached for a book in the hope that it might drag me away from the problems of the mediaeval science. The theme awoke my interest tremendously and I was affected even stronger at once by the way the subject had been treated by the author. But the book did not divert me from my work. On the contrary: the problems of my work drew nearer, and the longer I read the book, the more I felt as if it were written for the specific purpose of helping me to deal with my work better than heretofore. Actually, of course, the author of the book [1] intended simply to meet the needs of students and teachers of mathematics—to aid the former in the development of their own potentialities and to show the latter how to develop most effectively the abilities of their students. The third aim of the author, certainly not the least in importance, has been to rouse the curiosity of those who are interested in methods leading to original inventions and discoveries. I am neither a student nor a teacher of mathematics but I am deeply interested in the *ars inveniendi*, not only in the literal sense of this expression as the art of making scientific discoveries, but also in a more general sense — as the art of correct reasoning, as the skill of most adroitly approaching all questions, as the ability to resolve problems—not only mathematical problems. Even first of all problems that are not mathematical. Because I am not a mathematician. And that's why, what struck me most forcibly when reading G. Polya's book, was the thought that great benefits could derive from it to one interested in historical sciences, to one whose knowledge of math-

* This article gives a general outline of the problems to be discussed in a monograph, bearing the same title, now being prepared by the authoress.

[1] The book referred to is B. G. Polya, *How to Solve It*, Polish translation, Warszawa, 1964.

ematics is limited to what he learned in the secondary modern school. Every step in my work demands the solution of some problem; now and again I make affirmations which must be demonstrated. The application of the general modes of procedure appropriate for the solution of mathematical problems and for the demonstration of mathematical propositions could facilitate my work considerably and, what is more important, could ensure for my conclusions the certitude and truth. "The certitude without doubth and ...truth without error..."—are these not the words of Roger Bacon? Of course, the meaning of these words is not the same as the meaning of my statement. And the aims are different—what I am interested in is the application of the rules of mathematics to my own narrow section of research, when the mediaeval philosopher thought of the application of mathematical rules and achievements to practically all departments of human activity. But in both instances the basic thought is the same: it is the wish to make use of mathematics as of an infallible model of the truly scientific thinking, the most perfect model attainable by human minds. So it happened that a book whose reading had been expected to divert me from my work actually originated the theme of this article. Mathematics as the model of thinking—the role which it was to play in scientific research according to mediaeval scholars who were seeking new methods—these were the questions worthy of a scrupulous investigation. The fact that I am not a mathematician was judged by me to be rather advantageous. For I believe that anyone representing the exact sciences would have difficulties in dealing with this kind of problems. He would naturally be more interested in the concrete mathematical knowledge and the creative potentialities of the scientist who is being investigated, and—should he find such a scientist lacking in special value from this point of view—he would probably be inclined to regard as of little worth the general statements of such an author on the significance and role of mathematics because such statements contribute nothing of importance to the history of mathematical disciplines. For me, on the other hand, such statements are of utmost interest, while the mathematical knowledge and the contribution to the history of mathematics of the authors whose texts are examined are relatively less important. I find the ground for this attitude of mine in the firm belief that a truly outstanding mathematical mind will devote all efforts to mathematics as such and will be fully absorbed in the solution of mathematical problems, not paying more than a very limited dose of attention to other problems. The interest in other problems could develop much easier with philosophers possessing considerable taste for mathematics but who are able to look on it as if from outside and in this way to see better its value for the scientific cognition in general. This view is supported, I think, by the mediaeval texts. Roger Bacon may be cited in evidence for this, since he never contributed anything truly original to the development of pure mathematics but reached many a valuable and independent methodological conclusion based on mathematical concepts, summed up in his assertion that mathematics was "the door and key of the sciences and things of this world". [2]

The history of mathematics generally speaking does not pay much attention to the Middle Ages taking it for granted that, after the achievements of the antiquity, they are a period in which mathematical thought had been restricted mainly to making compilations. Of course, even in the Middle Ages it is possible to find names of persons distinguished in the history of mathematics, such as Leonardo Fibonacci, Nicole Oresme, or Regiomontanus—whose studies clearly show a degree of ripe scholarship and independence—but these exceptional men rather sharply contrast with their contemporaries who, as a rule, did not go beyond purely practical objectives, particularly those connected with the growing commercial interests. It is not suprising, therefore, that in practically all European manuals on the history of mathematics the period of European mediaeval history is altogether omitted. [3] Thus, it could appear that the study of the mediaeval mathematical conceptions might now serve no other purpose than—commendable, of course—satisfaction of the curiosity concerning the roads and devious paths traversed by the human mind before it reached the present heights of abstraction. In spite of all that, and contrary to opinions held by the historians of mathematics, it is just in the Middle Ages where we can pick even now quite a few most interesting questions, particularly actual today in view of the current postulates of the integration of science and penetration of mathematics into an ever growing number of fields of human activity. And I do believe it to be worth undertaking proper research work that one might show mathematics in the mediaeval texts as the foundation of all scientific studies, as a model of scientific thinking, as a specific exponent of the existing reality—and to point out the postulates advanced at that time of applying mathematics to practical life. Naturally, this paper makes no pretence to be an attempt to present fully the questions mentioned above. Neither shall the problem find a totally comprehensive exposition in the monograph I am now working on. I shall endeavour to give here but a sketchy outline of some selected questions which I propose to discuss at length in the intended publication.

It was usual to regard it as self-evident that more or less until the time of Francis Bacon the concept of the unity of sciences had been de-

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[2] *Opus maius*, ed. J. H. Bridges, Oxford, 1900, I, 4, p. 97.

[3] As a valuable exception one should mention probably the only book in the literature of this subject which treats synthetically the history of mathematics in the Middle Ages, written by an eminent Soviet historian of mathematics, A. P. Juszkiewicz: *The History of Mathematics in the Middle Ages*, Polish translation, Warszawa, 1969. However, the methodological function of mathematics is discussed here rather inadequately.

termined in its essence by religion and reduced to the idea of uniting all sciences in the service of God. Such an opinion does not seem to be based on the consideration of all aspects of the idea of scientific universalism in the Middle Ages. This idea derives indeed from the thought that the glory of God and the eternal salvation are the final goals of all human activity, the scientific activity not excepted—because the theological universalism predominant at that time postulated it. But many thinkers of the Middle Ages—especially the representatives of the new orientation toward the study of nature—not questioning the above general goals, were vividly interested in "particular" aims and concrete objectives of the scientific disciplines which could be turned to the service of society at large and of individuals. Such interests were the product of the new socio-economic and cultural conditions which took shape when the mediaeval mode of life was at its highest, that is, in the twelfth and thirteenth centuries. A growing demand for an effective advancement of knowledge was the foundation on which new scientific method was to appear. This method—one for all sciences—was to guaranty to all disciplines the absolute certitude and truth of their conclusions and to facilitate further studies. The postulate of a single scientific method for all branches of scientific cognition becomes, in a sense, a second cause of the idea of the unity of sciences, this time based on the methodological universalism. Being theoretically tied as regards its origin and the final goal with the theological universalism, the methodological universalism is autonomous and independent from the former for all practical purposes. The method which should unite all disciplines was to be founded on experiment on the one hand and, on the other hand, on mathematics, the most perfect among the sciences because it commands the formal demonstration which, being made use of in other sciences, will ensure for them an accurate and precise mode of reasoning.

The most decided champion of the mathematical method (jointly with the experimental method) in the Middle Ages had been undoubtedly Roger Bacon who, in his works, now and again expressed his deep belief in the weight and significance of mathematics as a model of scientific thinking, and in its immeasurable practical utility.[4] He was not alone in these considerations. One should look for texts expressing thoughts identical with, or similar to those of Bacon—first of all among the representatives of the Oxford school, taking Robert Grosseteste as

[4] The following works of Roger Bacon contain most of the material on this subject: *Opus maius* (ed. J. H. Bridges, vol. I-III, Oxford, 1900); *Opus minus* and *Opus tertium* (ed. J. S. Brewer, in the *Rerum Britannicarum Medii Aevi Scriptores*, vol. 15, London, 1859); *Communia Naturalia* (ed. R. Steele, *Opera hactenus inedita Rogeri Baconi*, fasc. III, *Liber primus communium naturalium Fratris Rogeri*, Oxford, 1911) and *Communia mathematica* (ed. R. Steele, *Opera hactenus inedita Rogeri Baconi*, fasc. XVI; *Communia mathematica Fratris Rogeri*, part I and II, Oxford, 1940).

the principal exponent of this type of opinions and—going back—in the school of Chartres which turned to Plato and the Pythagoreans. Undoubtedly, the Platonist and Pythagorean schools of thinking were the source of all concepts connected with the importance and methodological functions of mathematics. Such is the opinion of the author of the fundamental work on the mathematical method, L.Brunschvicg, [5] who asserts that the Pythagoreans were the first to appreciate the real significance of mathematical cognition.

As a parenthetic remark, it is worth mentioning that the two basic works [6] which, judged by their titles, should have contained information of considerable value to one interested in the mathematical method in the Middle Ages — bring disappointment when their content is read. For in L.Brunschvicg the mediaeval period of history, and in M.Grabmann the mathematics — are not to be found. L.Brunschvicg omits the period of the Middle Ages because he believes that after the achievements of the Pythagoreans it was really only Descartes with his idea of the *mathesis universalis* who opens the road for the modern triumph of mathematics. M.Grabmann, on the other hand, mentions the mathematical method only on a couple of pages of the first volume, when he analyses the theological writings of Boethius, [7] apart from that dedicating his efforts primarily to a deep study of the dialectical method. This is no proof, in my opinion, of the lack in the Middle Ages of material for the history of the mathematical method—though, of course, it provides additional evidence in support of the old truth that peculiar interests and personal convictions of authors affect the scope of, and the treatment of the subject in their books.

To return to the Pythagoreans—and exceptionally valuable text in connection with the discussed problem is the brief exposition of the Pythagorean conceptions concerning the universal significance of the number, included in the work of Sextus Empiricus *Against the Logicians.* [8] It may be worthwhile to analyse the respective fragment of this work.

Considering the criteria of truth Sextus Empiricus affirms that according to the laws of nature things of a given nature are perceived by senses of a similar nature. Thus, the sense of vision has the same nature as light and because of this fact our eyes are able to perceive light and whatever is connected with it. On the same principle our

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[5] L. Brunschvicg, *Les étapes de la philosophie mathématique*, Paris, 1947.

[6] In addition to the above-mentioned work of L. Brunschwicg I refer here to M. Grabmann, *Die Geschichte der scholastischen Methode*, vol. I–II, Berlin, 1957. This work covers the period, chronologically, up to the beginnings of the thirteenth century.

[7] *Ibid.*, ch. III: *Die scholastischen Methode in den Opuscula sacra des Boethius*, pp. 163–77.

[8] Sextus Empiricus, *Against the Logicians*, I, 93–109, Polish translation, Warszawa, 1970.

sense of hearing catches sounds. And in the same manner the mind comprehends the essence of reality, because the mind is of the same nature as the essence of the universal beings. The Pythagoreans proclaim that the number is exactly this essence of being, since in the Universe reigns the perfect harmony of numerical proportions. The most perfect number is 10 because it is the sum of 1, 2, 3 and 4, while these in their turn reflect anything that exists. For one corresponds to a point, two—to a line, three—to a surface and four—to a solid. According to the Pythagoreans—says Sextus Empiricus—it is impossible to comprehend anything in existence without the notion of the number. All corporeal things have dimensions, composite things comprise quantity—and these are comprehensible only mathematically. And even incorporeal things are expressed in numerical categories—as, for example, the time which we divide into years, months, days and hours. All means by which we facilitate our daily life and which make our technical skills more effective, such as measures, weights, the monetary system with the rules of doing commerce, with loans, bills of exchange and the like—are reducible to numerical relations. Likewise all human skills, such as the plastic arts or architecture, are based on certain proportions and these are founded on numbers. Thus it is clear that actually the real essence of everything in existence is the number—cognizable by the human mind thanks to the affinity of its nature to the nature of the number. So much Sextus Empiricus. This is a capital text, actually good enough to serve, as it is, for a conspectus of the main problem of a paper on the role and significance of mathematics. The one, mathematical nature of the existing universe gives ground for postulating one only method of cognition: the mathematical method. The universe is perfect and perfection is beautiful—mathematics, therefore, is also the exponent of beauty (Plato will speak later on the beauty of a straight line [9]). Mathematics is also the base of our daily life, of all human arts and skills. Now, all these are the ideas we can find again in the Middle Ages, enriched by the trends and thoughts of many centuries and, which is most important, Christianized.

Neoplatonist, Augustinian and Arab philosophic and scientific thought evolved various aspects of the Pythagorean philosophy of the number and jointly had prepared the ground on which, when scholastic learning attained its summit, blossomed new postulates for bringing mathematics into the methods of scientific cognition. For Augustine, the Christian philosopher of perhaps the greatest consequence in the Middle Ages, the mathematical truths were the firm foundation of all knowledge. Augustine expressed this opinion repeatedly in his works, but probably most convincigly and vividly in the following passage "A. So

[9] Plato, *Phileb* 51c, Polish translation, Warszawa, 1958.

now you know for certain that to form a figure of straight lines at least three lines are necessary. Would you retract from your opinion if a proof were found contradicting it? E. Should anyone demonstrate to me evidently the falsity of this opinion, I shall lose all hope for the possibility of acquiring any knowledge."[10] The above sentences have for us an unexpectedly modern tinge, quite free from the Pythagorean—Platonist mysticism. Yet it is known that mediaeval thought throve predominantly on Augustine's mysticism, on his theological conceptions, his withdrawal from nature. The Augustinian theory of illumination weighed heavily for centuries on Christian philosophy. It is interesting to observe, nevertheless, how the Augustinian system, thoroughly idealistic, permeated by religious elations, turning its back on nature—and undisputably conservative in character when scholastic philosophy reached its summit—generated premisses for new, original conclusions concerning a scientific method. Obviously, an important role in this process played the works, hitherto unknown to mediaeval scholars, which contained practically the entire heritage from the antiquity and from the Arabs. And it was exactly the Augustinian theory of illumination in conjunction with the Arabic metaphysics of light that blossomed out, in the Oxford school of the thirteenth century, into the theory of the multiplication of species—the theory which contributed much to the high appraisal of the utility of mathematics for the study of reality. According to this theory the light is the substance of reality and is therefore the basis for explaining the world. And the light can be known only mathematically—the laws of the diffusion of light are defined by geometry. For this reason the only right method of scientific cognition is the mathematical method, which is certain and infallible. No doubt, it is one of the many paradoxes in the history of human thought that the germs of new ideas often come into existence in surroundings saturated with old ideas and seemingly incapable of creatively affecting the human mind—while new systems are frequently born with a hidden stigma hampering the free development of an independent searching thought.

The only way to get the proper picture of mathematics in the Middle Ages as a discipline indispensable for a truly scientific cognition of the world is, naturally, one which leads through an accurate analysis of the mediaeval texts. The choice of the texts from which to start the analysis is not an indifferent matter, because the adoption of a particular point of departure often affects the direction in which the work is going to evolve; it might change the placing of emphasis and influence the perspective in which the questions are viewed. In our case, I think, we should start by getting acquainted with the mediaeval

[10] Augustine, *De quantitate animae*, VIII, 13, Polish translation, Warszawa, 1953.

classifications of sciences. The task of classifying the sciences is usually undertaken when it becomes necessary to order anew the accumulated knowledge — whether on account of the increased tempo of the accumulation, or under the influence of new intellectual trends for which old moulds no longer suffice. This is exactly what had happened in the twelfth and thirteenth centuries. The classifications then made were, by and large, compilations reflecting more or less accurately the average contemporary scientific thought, but also, as they took into consideration new sources, foretelling (though not always in a direct way) novel trends, often such as were actually to become prevalent many years later. A careful scrutiny of the place occupied in these classifications by mathematics may greatly facilitate further research work on the significance in the Middle Ages of the postulates for increasing the range of uses of mathematical sciences. The position in which mathematics was placed among other sciences by the authors of the various classifications would reflect more or less the average scientific opinion of the period. It will therefore supply a specific scale by which to appraise the attitude to mathematics of various authors and in texts other than those concerned with the classification. The average scientific opinion will become a zero grade on our scale, something like the zero on the Celsius' thermometer. "Above" will be placed original, new thoughts, "below"—statements which are but an echo of the old theories and convictions.

Several classifications of the twelfth and thirteenth centuries should be reviewed as examples. Even a brief and, naturally, rather superficial analysis of selected texts should give us an interesting picture of the changes which the position of mathematics among other mediaeval sciences had been subjected to.

Among the classifications of the twelfth century two are especially noteworthy: those of Hugh of St. Victor and of Dominicus Gundissalinus. Hugh of St. Victor, using exclusively Latin sources, divides the whole knowledge into theory, practice, mechanics and logic—and the theory he subdivides into theology, mathematics and physics. [11] This is a typical Aristotelian division, in which theoretical sciences are arranged in a sequence according to the increasing degree of abstraction: from physics as the least abstract science, to the most abstract among sciences—metaphysics. Mathematics which, according to Hugh of St. Victor, treats diverse kinds of quantity is divided into four disciplines in conformity with the scheme of the *quadrivium:* arithmetic, music, geometry and astronomy. "Magnitudinis vero alia sunt mobilia, ut sphaera mundi, alia immobilia ut terra. Magnitudinem ergo quae per se est, arithmetica

[11] Hugh of St. Victor, ***Eruditionis didascalicae,*** ed. J. P. Migne, PL 176, 752 and 765.

speculatur, illam autem quae ad aliquid est: musica. Immobilis magnitudinis geometria pollicetur notitiam. Mobilis vero scientiam astronomicae disciplinae peritia sibi vindicat. Mathematica igitur dividitur in arithmeticam, musicam, geometriam, astronomiam." [12] With evident gusto Hugh of St. Victor explains on each occassion the origin of the name of the particular science (mostly indicating that the name derives from Greek) and he gives also the further divisions of each of the four mathematical disciplines. [13] But the most important among his affirmations is this: that mathematics, jointly with logic, should be learned before one begins to study physical sciences, because mathematics and logic are instrumental to the cognition of reality: "Quia enim logica et mathematica priores sunt ordine discendi quam physica et ad eam quodammodo instrumenti vice funguntur, quibus unumquemque primum informari oportet antequam physicae speculationi operam det: necesse fuit, ut non in actibus rerum, ubi fallax experimentum est, sed in sola ratione, ubi inconcussa veritas manet, suam considerationem ponerent, deinde ipsa ratione praevia ad experientiam rerum descenderent." [14]

The work of Dominicus Gundissalinus *De divisione philosophiae* comprises, in a general outline, similar ideas to those included in the work of Hugh of St. Victor. However, Dominicus Gundissalinus makes use of Arabic sources as well as of Latin ones. His work is based first of all on two works of Alfarabi: the *De ortu scientiarum* and *De scientiis.* This is very pronouncedly noticeable from the way he distinguishes the particular mathematical disciplines. Dominicus Gundissalinus divides the entire human knowledge into theoretical and practical branches. Logic, grammar and rhetoric—coming first in the order of knowing—are, in his opinion, subservient in character as introductory disciplines to the study of philosophy. The theoretical philosophy includes physics, mathematics and theology. [15] Mathematics is, according to Dominicus Gundissalinus, an abstract science whose subject—matter is quantity abstracted from corporeal bodies. Though in reality there are no lines, surfaces, circles, triangles etc. existing separately from bodies—mathematics, nevertheless, examines them as beings abstracted from matter and only as such, assuming conventionally their objective existence. Arithmetic, music, geometry and astronomy are strictly parts of mathematics, while one counts also to mathematical sciences optics, the science of weights and the science of inventions. Mathematical sciences should be studied after physical sciences: here, the order of cognition agrees

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[12] *Ibid.*, 755.
[13] *Ibid.*, 755–7.
[14] *Ibid.*, 758–9.
[15] A general characteristic of the entire division of knowledge Dominicus Gundissalinus gives in the prologue to his work *De divisione philosophiae*, ed. L. Baur, *Beiträge zur Geschichte der Philosophie des Mittelalters*, IV, 2–3, Münster 1903, pp. 3–19.

with the doctrinal order, because our senses perceive first the form together with the matter and only later our intellect can attain the perception of the form abstracted from the matter.[16] Within each of the mathematical disciplines Dominicus Gundissalinus distinguishes its theoreticaol part which considers the basic principles of this discipline, and the practical part which teaches how these principles could be applied to practical ends. For instance, the science of weights (*scientia de ponderibus*) treats the theoretical principles of heaviness and weight but deals also with the instruments which serve to raise heavy things and transport them from place to place.[17] The science of inventions (*scientia de ingeniis*)—which closes the list of mathematical sciences —is worthy of attention "Scientiae ergo ingeniorum docent modos excogitandi et adinveniendi..."[18] The aim of this discipline is to utilize the theoretical principles of all mathematical sciences for diverse useful purposes, such as the construction of measuring implements, musical and optical instruments, tools for bricklaying, carpentry and other "mechanical arts".

The characteristic feature of the classification of sciences expounded by Dominicus Gundissalinus is his insistence on the great importance of the utility of each of the discussed disciplines. The same attitude, possibly even toned up, prevails also in the classification deriving from the first half of the thirteenth century, written by Michael Scot.[19] His division of philosophy is a stereotype: the theoretical and practical branches, the theory comprising physics, mathematics and theology. The subject-matter of mathematics is quantity, continuous and discontinuous, and—in accordance with the kind of quantity considered— mathematics is divided into arithmetic, music, geometry and astronomy. The mathematical science should be studied after physics because, in the opinion of Michael Scot, the sensory perception proper to physical sciences which consider the form in corporeal bodies is prior in the order of knowing to rational cognition on which mathematics is based.[20] Evidently, in these speculations Michael Scot does not differ from his predecessors. But his argumentation concerning the practical branches of knowledge is noteworthy. He divides the practical philosophy into

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[16] *Ibid.*, pp. 28–35. The author discusses mathematics at first in a general way, presenting—according to his wont observable in the whole work—its subject-matter, its genus, division, aim, its particular manner of reasoning and the like questions, and then proceeds to analyse thoroughly, one after another, each part of mathematics (*ibid.*, pp. 90–124). Almost one third of the whole work of Dominicus Gundissalinus is given up to mathematics.

[17] *Ibid.*, pp. 121–2.

[18] *Ibid.*, p. 122. The whole science of inventions is expounded on pp. 122–4.

[19] The work of Michael Scot on the classification of sciences has been preserved only in fragments included in the *Speculum doctrinale* of Vincent of Beauvais. These fragments were edited by L. Baur as an appendix to the *De divisione philosophiae* of Dominicus Gundissalinus, *Beiträge...*, IV, 2–3, pp. 398–400.

[20] Michael Scot (see footnote 19), pp. 399–400.

three parts corresponding to the three parts of the theoretical philosophy. Mathematics corresponds to the second part of the practical philosophy "... quae adinventa est ad similitudinem doctrinalium, ut negotiatio, carpentaria, fabrilis, cementaria, textoria, sutoria, et aliae huiusmodi multae quae spectant ad mechanicam et sunt quasi practica illius." [21] The skills mentioned turn to their benefit the theoretical knowledge of the various mathematical disciplines, in this way improving their artefacts and making them easier to produce.

L. Baur considers the thirteenth century classification of Robert Kilwardby, contained in his work *De ortu et divisione philosophiae*, to be the best among the mediaeval classifications. [22] However, taking into account only the division of mathematics and its place in the whole classification—there is nothing particularly in Kilwardby, when we compare his ideas with the classifications discussed heretofore. We find with him the same considerations on the subject-matter of mathematics: the diverse kinds of quantity; the same position of mathematics in the list of theoretical sciences, between physics and theology, as befits the scheme of the increasing degree of abstraction. Robert Kilwardby utilized in a large way the Latin and Arabic sources, the form of his classification is finished, but it does not seem to contain any truly original thoughts.

On the contrary, a really original division of science can be found, I am convinced, in the works of Roger Bacon. His division is not of the type discussed above, because Bacon never wrote a separate work on the classification of sciences, like his predecessors. What he did was to adopt a certain order of all disciplines when he planned his encyclopaedia on sciences. But he is worth mentioning for the sake of comparison. Bacon divides all sciences into four principal parts. Part one comprises grammar and logic—the disciplines instrumental in character, subservient to the other sciences. They give the knowledge of the rules of thinking and of correctly expressing thoughts. The second part consists of mathematical disciplines. The third part contains the physical sciences, and the fourth—metaphysics and moral science. [23] Bacon places mathematics before the physical sciences because, he maintains, one should begin all studies by learning mathematics. Mathematics, in his opinion, which considers all kinds of quantity, is the most simple science, containing truths inborn to men. It is natural for human cognition to proceed gradually from easier subjects to more difficult ones. Bacon retains the division into four parts also for mathematics, which

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[21] *Ibid.*, p. 399.

[22] L. Baur gives the contents of, and discusses the work of R. Kilwardby in the *Beiträge...*, IV, 2–3, pp. 369–75.

[23] For an extensive analysis of the classification of sciences in Roger Bacon see M. Frankowska, "*Scientia*" *w ujęciu Rogera Bacona* ("Scientia" as Interpreted by Roger Bacon), Wrocław, 1969.

he divides into arithmetic, geometry, astronomy and music. However, following Alfarabi's division of sciences, he sees two principal parts in mathematics: a general part, treating the elements and basic principles of mathematics as a whole—this part is a species of introduction to the second part, which is divided into the four particular disciplines mentioned above. Each of the particular disciplines is, in its turn, divided, in two indissoluble parts: the theoretical and the practical part. The theoretical disciplines are indissolubly united with their practical counterparts "quoniam vero speculativa completur per suam practicam, et evidentius per eam apparet, et e converso, ideo conjungam quamlibet practicam cum sua speculativa correspondente." [24] Thus, the theoretical arithmetic considers diverse kinds and properties of numbers, while the practical arithmetic transfers these considerations to the concrete problems connected with commerce, bank accounts, modes of lending money on interest, etc., and even teaches various games based on arithmetical principles. [25] Roger Bacon devoted considerable space in his works to the discussion of various mathematical disciplines. He attached great importance to mathematics as the means of scientific cognition and asserted that demonstrations and the manner of reasoning proper to mathematics, when applied to other sciences make possible a more efficient conduct of research work and guaranty the certitude and truth of their conclusions. The utility of mathematics, according to Bacon, is even more many-sided, since its range includes not only scientific cognition but also the practical activities of men, undertaken for the benefit of the state and the Church.

Roger Bacon evidently makes extensive use of both the Latin and Arabic sources. Like Dominicus Gundissalinus, he relies mostly on the classification of Alfarabi. The evidence of his having drawn from the above sources is clearly visible, but Bacon certainly contributes much of his own original thought to the considerations he expounds. Bacon's presentation of the distinction between the theoretical and practical parts of particular disciplines is consistent and decided beyond compare with any earlier similar effort. The stress he puts on the application of mathematical theories in practice and the firm assertion that theoretical considerations for their own sake make no sense—though both ideas derive from affirmations of his predecessors—were for the first time advanced so clearly and so forcibly. The grasp of the wide range of the utility of mathematics is also found for the first time expressed in such a large way in the opinions of Roger Bacon. This, of course, is the result of Bacon's special interest in the methodological function of mathematics with regard to other sciences.

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[24] Roger Bacon, *Communia mathematica*, p. 39.
[25] *Ibid.*, p. 47.

It is interesting to note how even a superficial review of several selected classifications of sciences—such as the one made here—helps to bring out the course of the changes in the position of mathematics among other disciplines. For Hugh of St. Victor, who relied exclusively on the Latin sources, like for Aristotle, mathematics is a science on a higher level of abstraction than physics; it treats various kinds of quantity, abstracted from corporeal bodies. Hugh of St. Victor, however, is aware of the methodological function of mathematics and places it—in the order of cognition—before physics. Though the methodological aspect disappears from later classifications, which drew freely from the Arabic sources, yet the practical utility of mathematical disciplines commes to the fore. The science of invention of Dominicus Gundissalinus and the parts of the practical philosophy in Michael Scot testify to this fact. In Roger Bacon both aspects appear together. The practical application of the achievements of mathematical disciplines and the methodological role of mathematics in relation to other sciences are presented by him on the same plane, and he discusses both problems in a considerably more mature way than his predecessors ever did. For instance, the practical parts are set with their theoretical counterparts more logically—without linking theoretical mathematical knowledge in any way with bootmaking or weaving, as we saw it done in Michael Scot.

The analysis of the mediaeval classifications of science could thus undoubtedly make a good starting point for a further scrupulous analysis of the texts, for the purpose of preparing a synthetic picture of the role played by the postulates put forward in the Middle Ages for assigning to mathematics the universal methodological function in the scientific cognition. Naturally, in the scientific cognition first of all. It seems, however, that the picture would not be complete should it fail to include at least an outline of the importance of mathematics in the everyday life of this period of history, at least an idea of how it was reflected in the arts, literature, music. One should remember that mathematics in the Middle Ages was not only a science indispensable for acquiring knowledge, not only a discipline of enormous practical utility, but also a science reflecting the ideal of beauty. Mathematics is a beautiful science indeed, and it is worth showing that in the Middle Ages people had been aware of this fact. To avoid being accused of idle talk, I propose to finish this article by quoting a twelfth century tale of a marvellous robe made by four fairies:

> L'uevre del drap et le portret.
> Quatre fées l'avoient fet
> Par grant san et par grant mestrie.
> L'une i portrest Geometrie,
> Si come ele esgarde et mesure,

Con li ciaus et la terre dure,
Si que de rien nule n'i faut,
Et puis le bas et puis le haut,
Et puis le lé et puis le lonc...
Et la seconde mist sa paine
An Aritmetique portreire,
Si se pena mout de bien feire,
So come ele nonbre par sans
Les jors et les ores del tans,
Et l'eve de mer, gote a gote,
Et puis aprés l'arainne tote
Et les estoiles tire a tire,...
La tierce oevre fu de Musique,
A cui toz li deduiz s'acorde,
Chant et deschant, et son de corde,
De harpe, de rote et viële.
Ceste oevre fu et buene et bele;
Car devant li gisoient tuit
Li estrumant et li deduit.
La quarte qui apres ovra,
A mout buene oevre recovra;
Car la mellor des arz i mist.
D'Astronomie s'antremist
Cele, qui fet tant mervoille,
Qui as estoiles se consoille
Et a la lune et au soloil;...[26]

[26] Chrétien de Troyes: *Erec et Enide*; *La robe des quatre fées*, in: *Antologia poezji francuskiej* (An Anthology of French Poetry), ed. by J. Lisowski, vol. I, Warszawa, 1966, pp. 46–50.