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From Seventeenth-Century to Twentieth-Century Optics

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FROM SEVENTEENTH-CENTURY TO TWENTIETH-CENTURY OPTICS

During the Second World War there started a critical revision of the fundamental concepts of optics. This has been followed by a complete metamorphosis and a profound clarification on a philosophical basis involving questions of method that refer to the whole of modern science and going as far as to undermine convictions which have been held for centuries in regard to the philosophy of sciences. For this reason I do not think it will be out of place for me to set out the general lines of this development at an international level, such as that provided by the ICSU.

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The origins of optics are extremely ancient, and go back to more than two thousand years ago. But until the seventeenth century the development of optics was extremely slow and not very conclusive, because it came up against an extremely difficult obstacle which shattered the efforts of all the mathematicians and philosophers who tried to overcome it. They were not able to explain what is meant by "seeing". The theories thought out in ancient times and during the Middle Ages were unsatisfactory, and consequently the studies which are known today as optical studies were unable to proceed along rational lines. It is not until 1604 that we find the truly miraculous work with the modest title *Paralipomena ad Vitellionem* in which Johann Kepler for the first time provides the key to the mechanism of sight. This opened up a new era in the history of science.

Let us go a little into the details of this mechanism, because this will help us to see more clearly the transition from the optics which was born in the seventeenth century, in fact as a consequence of Kepler's ideas, to the optics of today, of the twentieth century.

Every material body (luminous or illuminated) is considered as an assembly of elements. Each of these elements on the surface of the body itself emits rays running in straight lines in all directions. Although in the general treatment of this subject an element of this kind is called "a point of light" it is necessary for us to drop this phrase and to replace it by the expression "radiating point". We shall see the reason for this very soon.

A cone of rays emitted by an element *S*, when it meets an eye, enters the pupil and is converted by the optical devices of the eye itself into another cone, which once again has as its base the pupil, but this time finds its apex at a point *P* on the retina. By repeating the same process for all the elements of the surface of the material body facing the eye we get a group of apices of cones of rays on the retina, which constitutes what we call today the "retinal image". If the object is point-shaped, like a star, the retina is stimulated in an extremely small area, which is usually referred to as a "point", naturally if we de not take into account any non-essential complications.

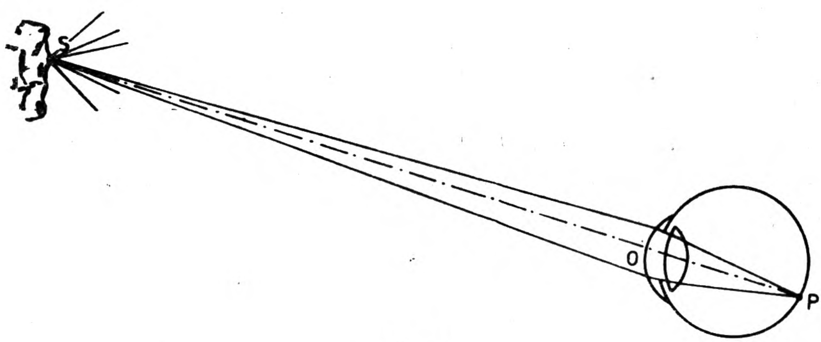

Fig. 1

But the process does not end on the retina: the function of the retina is to convert the radiation stimulus into a series of nervous impulses which travel along the optic nerves, pass through numerous complicated points of the brain and finally reach the occipital zone where the cortical area responsible for visual operations is located. From then onwards the process passes into the sphere of mental operations.

Unfortunately our ideas in this sphere are still very obscure, but in broad terms we may say that the process ends in the "representation" of the information received through the optic nerves by means of "phantoms", that is to say by means of coloured figures of light which are "located" in front of the eyes by the mind itself. When the operation is completed, the observer says that he "sees the material body". In other words, "seeing" means creating these coloured phantoms of light and locating them in front of us.

The line of reasoning put forward by Kepler was expressed in somewhat different terms, but this is essentially what it was. Among other things, he faced the problem of one of the most interesting processes concerning the mind, and tried to define the criteria by which the mind manages to locate the phantom it has created so that it is exactly in the position where the corresponding material body actually is.

Still using as a basis the idea of a point as the source of radiation, Kepler easily deduced that the *position* of the stimulated point on the retina indicates to the mind the directions from which the rays have come. All that is then left to do is to determine the distance at which the radiating point is situated from the eyes.

The first idea which Kepler expounded in this connection is one which is still well known today: since we have two eyes, the fact that we can rotate them so that the two lines of vision pass through the radiating point (and when this happens we have the "fusion" on the two phantoms separately created by each eye) enables the mind to carry out a "triangulation", that is to say to measure the parallax at which the interpupillary base is seen from the radiating point. Today we generally say that the "convergence" of the eyes makes it possible for the observer to estimate the distance between the eyes and the object which is being looked at.

However, Kepler very rightly observed that an observer, even if he only uses one eye, is able to see in front of him the figures distributed over a depth. This means to say that even with one eye the mind is in a position to measure the distance of the radiating points from the eyes. It was necessary to explain how this could be. Kepler looked for a triangle which would make it possible to carry out a triangulation even using a single eye, and he found it: the triangle which has its apex at the radiating point S and has as its base the diameter of the pupil. He called it the *triangulum distantiae mensorium,* that is to say the *distantiometrical triangle*. In this way he was able to enunciate the rule: *an eye sees a point of light at the apex of the cone of rays which arrive at the pupil.*

Today we add to this line of reasoning a number of other pieces of information relating to the intensity of the point of light and its colour. Kepler did not say anything about this, but what he did say already amounts to a great deal. In his own day and age it was something little short of miraculous.

The repercussions of Kepler's theory were enormous. On the basis of this theory of the mechanism of sight an entirely new system of optics was built up and it is precisely this system which I have called the optics of the seventeenth century. This theory made it possible to apply geometry, and therefore also algebra, to the study of optical phenomena, and at that time this constituted an absolute novelty which opened up boundless horizons.

The first application which Kepler himself made of his theory was to explain why images are seen behind plane mirrors, a phenomenon which had been well known for thousands of years but which no one had as yet succeeded in explaining fully.

Yet the most noteworthy consequence of Kepler's theory was in fact the definition of the concept of an optical image, not only in the case of plane mirrors but in the case of any optical system whatsoever.

The fact that when one looks into a curved mirror, or even through a lens, figures are seen which are more or less similar to the material objects but are of different dimensions and in positions in which the objects certainly are not, had been well known for an extremely long time, but no one had ever been able to give a proper explanation of this fact. It was only Kepler who was able to place the ideas on the road which they have followed from then until today.

If we think once again of the radiating point S which sends its rays to a lens L (or to any other optical system), it can happen that the emergent rays constitute a new cone which still has the lens as its base but has its apex at a point I, which is different from S. That is to say these rays converge at I and then diverge once again. I is therefore the apex of a cone of rays, just like a material radiating point. If the rays coming from I meet an eye, they are concentrated in a point-shaped retinal image and therefore, *on the basis of the rule of the distantiometrical triangle,* the observer *must* locate the point of light at I and not at S.

The point of light which the observer sees at I is in fact the one which is called the *image* of the point S produced by the optical system in question.

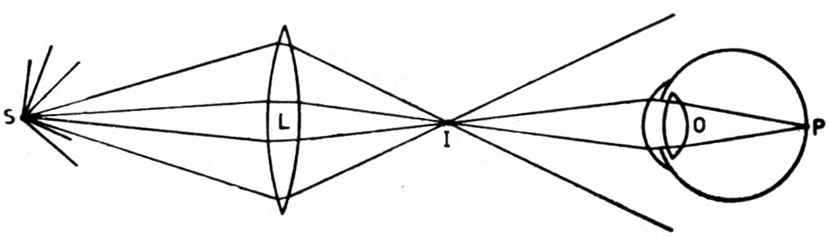

Fig. 2

This line of reasoning seems to be flawless. Nowadays throughout the whole of optical science this is the only line of reasoning which is repeated. However, it is done in a much simpler manner, and it is very important for us to note the difference. We simply say: the point S sends its rays to the optical system; the latter deviates them so as to form a new cone with its apex at I; the point I is the image of S produced by the optical system. No one cites the rule of the distantiometrical triangle and it is extremely difficult to find anyone who knows of it and is aware that the definition of an optical image is derived from it.

It has been necessary for us to go right back to the origins in order to find the hidden significance of the definitions in use today.

For the evolution of Kepler's rule has in fact been a very strange one. A philosophical deformation has taken place which probably Kepler himself had not foreseen. For once it has been admitted that the mind of the observer *must* locate the point of light at the apex *I* of the cone of rays emerging from the optical system it was no longer necessary to repeat every time that this location was effected by the mind on the basis of the rule of the distantiometrical triangle. Consequently no one spoke any more of this rule and of the physio-psychological intervention involved, and the definition of the image was restricted to the few words given above.

This is what happened within the sphere of physicists and mathematicians, that is to say optical scientists engaged with optical instruments. Of course it goes without saying that the physio-psychological part of sight was and still is a subject of study by physiologists and psychologists. It is a well-known fact that the tremendous development in science during the last three centuries has led to specialization and consequently to the formation of watertight compartments, the occupants of which are unaware on what is happening in the next. In particular this has happened between those engaged in the physical and mathematical study and those concerned with physiological and psychological studies.

Most important of all, the field of physics and mathematics has felt the guiding influence of the positivist attitude from the seventeenth century onwards, with an explicit disdain for anything which was not definitely "objective" and "independent of the observer". The rule of the distantiometrical triangle was warmly welcomed by the new current of philosophy, precisely because it enabled physicists to talk of images independently of the observer. But once the effects had been obtained, it was necessary to forget Kepler's rule and it was also necessary to forget the wonderful and fundamental contribution of Kepler to the foundation of modern optics. To have remembered all this would have been harmful to the new ideological stream. If optics was to become a completely physical science, it was necessary to forget that the definition of an image had been based on considerations of a psychological character, namely the representation of luminous stimuli in terms of luminous and coloured phantoms. It was also necessary to forget the rule of the distantiometrical triangle, because this had been evolved in order to explain the mental location of the phantoms.

That the "point of light" should be located at the apex of the cone of rays reaching the eye has been considered as a self-evident fact, which was not even worth a few words in justification.

It was in this way that the optics of the seventeenth century was born and developed miraculously. The bringing of optical phenomena into the field of mathematics made it possible to build up a theory which has held all men of science in its thrall because of its organic and perfect nature, so that this theory has been regarded as definitive, even as one of the very pillars of modern science. The rules and conclusions of optical theory have been considered everywhere as the perfect representation of physical reality, a theoretical representation which by now is above any criticism and is more worthy of trust and confidence than experience itself.

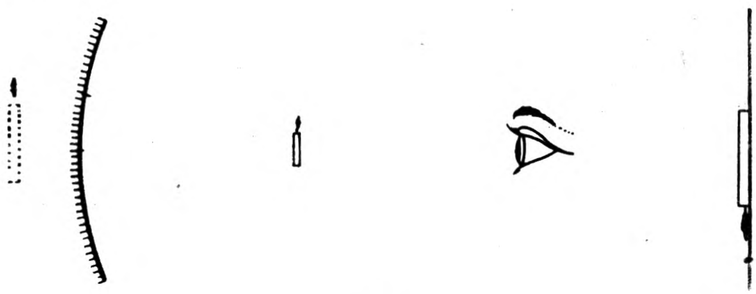

Fig. 3

For in actual fact experience has not at all times shown itself in agreement with the theory, and as far back as in the seventeenth century Barlow and Berkeley raised their voices to put forward reservations regarding the correspondence between theory and experience in a number of particularly discordant cases. But the faith of the new mathematicians, who every day became more numerous and more enthusiastic over the new optics, buried these reserves under a pall of oblivion.

It was in this way that the principle was founded that optical theory constituted scientific truth and that if in some cases experience showed itself to be capricious, the fault lay with the observer and this did not in any way invalidate the solidity of the theoretical edifice: experimental errors and shortcomings of various kinds could always be invoked to explain the failure of experiments when the judge felt sure *a priori* that right was on the side of the theory.

Things are still at this point in a great part of the scientific world, but a revolutionary wave is rapidly advancing and is growing greater every day. This revolution aims at bringing things on to a much more rational and realistic level.

I have started off by giving this historical summary of the origins of the optics of the seventeenth century because an historical study of these foundations, taken in conjunction, with a long experimental and technical application of the laws of optics, has shwon that the value which they posses is in fact of a conventional and provisional nature and that they do not at all merit the entire confidence which physical and mathematical cricles have placed in them up to now. We can in fact talk of a true collapse of the optics of the seventeenth century.

The way in which one has arrived at this conclusion which is surprising to most of those concerned in this subject would be interesting and would also serve to demonstrate its inner significance. Unfortunately it would take rather long and it would therefore be better to summarise the fundamental features in the shortest possible space.

Looked at logically, the line of reasoning may be set out as follows: the basis of the optics of the seventeenth century is the rule of the distantiometrical triangle. Now a careful experimental examination shows that this rule *is hardly ever borne out.* Consequently, the optics of the seventeenth century are devoid of an experimental basis and hardly ever fall in line with experience.

In other words, the rule of the distantiometrical triangle must be regarded as a "working hypothesis", a hypothesis of incalculable value at the time it was enunciated; but despite this fact, it has not changed its nature. The optics of the seventeenth century, consisting as it did of a set of rules and laws *deduced from the rule of the distantiometrical triangle,* is not a physical science but a mathematical science.

We should not be at all surprised if many readers, when brought face to face with affirmations of this kind, were to express their incredulousness in no uncertain terms. But before giving a final judgment it would be best to examine carefully the proof put forward on either side and in favour of each view. In fact everyone, when doubt is cast upon the rules of the science of optics as generally known, goes back in his thoughts to the experiments which were shown to him when he attended school and college and which convinced him that he was learning scientific truths. Were not these experiments true? And if they were true, did they not prove their point?

In those experiments there generally was some special element which was carefully worked out with a view to obtaining the desired result: they were not general experiments. The general rule which can be enunciated today, on the other hand, states that *figures seen by the eye* hardly ever correspond with those calculated by the rules of geometrical optics. The discrepancy between the calculated image and the image as seen may even be enormous.

It is usual to cite the case of the plane mirror as the one which lends itself to an obvious demonstration of the above-mentioned rules. Nevertheless, care is taken to carry out the experiment by having the mirror rather near to the observer and to the object, because if the experiment is repeated by carrying out the observation in mirrors which are at least a few metres away from the observer it is easy to observe that the image of the objects is no longer seen in a symmetrical position in relation to the reflecting surface.

But the divergence between theory and practice becomes truly enormous when the observations are carried out in curved mirrors, especially concave mirrors. If we wanted to give a list of examples in which experimental results do not agree with the elementary theory of the real and virtual images provided by a concave spherical mirror, we would have to fill many pages with them. But I think it will be sufficient for us to mention one example to demonstrate how experience differs very widely from theory.

Let a concave spherical mirror with a radius of curvature of about 50 cm be arranged a few metres away from a vertical wall, and let the optical axis be arranged horizontally. Let a source of light, such as a candle or an electric light bulb, be located near to the focus of the mirror in such a way that an enlarged and inverted real image of it is projected on to the wall opposite, just as theory says. But if at the same time an observer places himself in the path of the rays travelling to the wall and if he looks towards the mirror he will see another image of the source of light, behind the mirror, that is to say, virtual, the right way up and also slightly enlarged. No theory provides for this image. There is no theory in existence which justifies the simultaneous presence of a real and a virtual image, because the conditions which lead to the formation of one of them exclude the conditions which lead to the other.

Occurences of this kind are also met with in many instances in everyday life and although it is not usual to take any notice of them they are nevertheless very revealing. An ordinary table-spoon is a concave mirror which may have in front of it numerous objects, including some with a high degree of illumination, such as burning electric light bulbs, windows *etc.* According to the classical theory, the images of all these objects will be formed in front of the spoon a few centimetres away from the reflecting surface and they are reduced in size in relation to the objects themselves. If these images were actually seen at the place demanded by the theory, the spoon would have to appear absolutely chock-full of images and in order to see whether in fact the spoon was full or not it would be necessary to decide to make use of the sense of touch. In fact, nothing of the kind happens at all. Every one of us sees the spoon completely empty and we see the surface of it varying in brightness to different extents, that is to say, we see the images of the objects in front of it on its reflecting surface, despite the fact that this does not at all fall in line with the theory.

Similar and even more varied findings could be repeated when the observation is carried out through lenses, whether convergent or divergent. There is no need of precision laboratories in order to show the tremendous discrepancy which exists between what you can see and what is required according to the theory. Among the infinite number of experiments which can be carried out even with a modest magnifying glass, it is sufficient to note that the enlarged image is almost always seen in the same plane as the material object. It is nothing more than a commonplace to say that if one observes a leaf on a bench using a magnifying glass "one sees the leaf magnified", but it is still seen on the plane of the bench. Theory, on the other hand, would want the image further away from the lens than the leaf, and this distance could even be as much as infinity. No one has ever observed anything of the kind.

Thus a short-sighted person equipped with concave lenses should only see images between his lenses and their respective foci. Since the lenses used by myopic persons have a focal length which rarely exceeds one metre and very frequently is as little as a few decimetres, if the geometrical rules were complied with the observer would have to see everything greatly reduced in size and everything contained in a sphere having a radius of a metre or less. No one has ever seen anything of the kind.

We could carry on like this *ad infinitum*. Moreover, if things do not go well with simple mirrors and lenses, it goes without saying that they will go worse still when dealing with more complex optical systems such as telescopes, microscopes and the like. The examples we have already given, although restricted in number, are so forceful that they are bound to be sufficient to shake the blind faith held for several centuries in the theory of geometrical optics.

Instead of increasing the number of examples of the contrast between theory and experiment, I think it would be more helpful if we were to analyse the causes, because this will show any reader who wishes to check the matter experimentally the way in which he can be sure to achieve this end.

Let us start off by showing the trick which is used for carrying out the demonstration experiments by means of which it is possible to convince the public — in the form of schoolchildren — that the laws of optics are perfectly in line with physical reality. The trick consists in projecting the real images on to a screen. This trick is not called by this name and, in fact, it is not shown as an important part of the experiment, so as to leave one believing, as a result of this silence, that it is merely a matter of a simple detail of execution and one of no material importance.

Kepler had so well understood that things were not like this that he went so far as to coin two different names for the pictures projected on to the screen (which he called *picturae*) and those seen without a screen (which he called *imagines rerum*). This essential difference has been completely forgotten today, thus committing two serious errors of logic:

(1) The images on the screen are those known as "real". The "virtual"

images cannot be projected on to a screen but must always be seen by looking with the eye facing the optical system. Moreover, the general rule is that the verifications, including the numerical ones, should only be carried out in regard to the real images, and when it has been confirmed that they are in accordance with the rules and formulae it is deduced from this that the formulae are correct and in order and that therefore they are also valid for the virtual images, despite the absence of experimental confirmation.

(2) No mention is made of the fact that the images which are seen without a screen never correspond with those seen on the screen. By maintaining silence regarding this difference in behaviour under the two different types of experimental condition one implicitly gives the idea that the difference just does not exist, or that the rules which are valid for images observed on a screen are also generally valid, thus making a completely gratuitous extrapolation in logic which runs absolutely counter to experimental evidence.

To anyone who has not made a fundamental study of these problems the decisive part thus played by the modest and innocent screen on to which the images are projected may seem rather strange. But the reason for the importance of this screen will be obvious when the validity of the rule of the distantiometrical triangle has been thoroughly gone into, and in this way it will be possible to show the actual mechanism by which the luminous and coloured phantoms are located.

Kepler was an extremely great mathematician, but he possessed the mentality of a mathematician which thinks in terms of abstract magnitudes endowed with absolute accuracy. In practice it is never possible, to count on absolute accuracy because this is equivalent to infinite precision and things infinite do not fall within the compass of human possibilities. Magnitudes which are smaller than the limits of sensitivity of the means of observation are without value and it is just as though they did not exist. Thus, when Kepler discovered the distantiometrical triangle he restricted himself to confirming its existence and naturally reached the conclusion that the mind must make use of it in order to determine the distance of the radiating point from the eye. But he did not ask himself what were the magnitudes of the extremely acute angle of this triangle and whether it was reasonable to assume that a human organ could appreciate this magnitude and make use of it an operation as important as seeing the outside world. This is even more remarkable when we remember that the minimum distance of the radiating point may be about twenty centimetres from the eye (in the case of a normal, young eye) and the maximum distance may be enormous. In order to give the rule a realistic character, Kepler should have asked himself "What is the smallest angle the eye can perceive?" and "What is the maximum distance for the radiating point if the angle is still to be greater than this minimum?"

If Kepler had posed this problem and had been able to solve it (which is very doubtful in view of the time when he lived), he would not have enunciated the rule of the distantiometrical triangle. For he would have found that the efficacy of such a means for measuring distances from the eye ceases to exist at all for distances of more than about 4 m and that even for a distance of 1 m the roughness of the method is such that it must be regarded as inefficient. Consequently the rule of the distantiometrical triangle is devoid of any physical and physiological basis.

Nevertheless, it was extremely fortunate that Kepler did not pose himself this question and consequently enunciated his rule, which he then applied with such success. Had things been otherwise, the optics of the seventeenth century would not have been born (or at any rate it would not have been born then, and no one can tell when it would have been born).

Nevertheless, even if it is fortunate that we have had this extremely valuable rule available there is no point in going too far. We must attribute to it its correct value and not overvalue it. In other words we must regard it, as we said earlier on, as an excellent working hypothesis. But we must not claim that it tells us everything. It tells us what it can, and if we do not wish to put a stop to scientific development at this point but wish to proceed as far as possible, we must do what we can to devise a science of optics which is not based on the distantiometrical triangle.

It is this new style optics which we have called the "optics of the twentieth century".

This is a science of optics which draws a clear distinction between the radiating point and the point of light. It assigns a physical nature to the former and a mental nature to the latter. It is a science which regards its fundamental purpose as that of determining the laws by which an observer locates the point of light in the apparent space when his eyes have been stimulated by the energy emitted from a radiating point.

The optics of the seventeenth century have many elements to offer for solving this fundamental problem, but not all that is necessary. It is a matter not only of a study of physics, as it has been considered up to now, but a study of a threefold nature: a study of the physics of the stimulus is followed by a physiological study of the response of the receptive organ and this in turn is followed by a psychological study of the representation of the apparent world. This apparent world is not the real world, but differs from it very greatly and it possesses distinctly subjective characters which therefore differ from one observer to another, in such a way that the fundamental task enunciated above is followed by one which is more important still from both the philosophical and practical points of view, namely that of "determining what really exists in the outside world by deducing it from what the observer sees in the apparent world".

A study of this kind has radically transformed the nature of optics, bringing into prominence the intricate way in which this is all bound up with a knowledge of the mechanism of our mind. It has been necessary to conclude that it is only within the compass of the mind that one can talk of light, colour and shape and that it has been a harmful error to call the radiating point a point of light because the two things are generally distinct from one another and must be given different names. Likewise, it has also proved erroneous to use the word "light" to designate the radiant energy or radiation which travels from the radiating points to the eye, because this energy is neither luminous nor coloured, and we can only talk of light and colour when we start considering the effect of such radiations on the mind of an observer.

But if we now wished to go into the details of the new optics of the twentieth century, we would have to set out a long and detailed exposition which does not belong in the present article. My intention has been to give a quick glimpse of the reasons which have led from the optics of the seventeenth century to the optics of the twentieth century, or to call the attention of philosophers and those concerned with optics to the existence of the rule of the distantiometrical triangle, its importance in present-day science and the fact that it is by nature purely conventional and provisional and a very long way from reality.

By now the optics of the twentieth century have developed largely in pioneer circles and are now slowly but inexorably conquering the whole of the scientific world. Anyone who has learned to think along the new lines can feel all the mediocrity and insufficiency of the old mode and can see with extreme ease the errors of method and of reasoning which we had became accustomed to committing with such offhanded carelessness.

I will conclude my remarks by pointing out some of these errors.

What an observer sees in an appearance of a mental nature, as we have shown in the foregoing pages. Every one of us who is not blind surrounds himself with phantoms created by his own mind on the basis of information reaching the brain *via* the optic nerves, that is to say from the eyes which have been stimulated by outside physical agents. We may also say that we are dealing with a dream built up as a result of the action of these agents. What we see, therefore, is a function of physical, physiological and psychological elements. How has it been possible, when studying this problem, to build up a purely physical science, as is generally believed? The analysis which we have made in the foregoing pages has demonstrated the underhand way in which this has been done: the psycho-physiological elements have been regarded as constant. Obviously once it has been assumed that the psycho-physiological mechanism functions in a constant and perfect manner (as is assumed by the rule of the distantiometrical triangle), the phenomenon becomes solely the function of the physical variable and is therefore a physical phenomenon. But it is also evident that this mode of procedure must be called a method only if it is used explicitly in regard to the light of the sun for the purpose of studying one at a time the factors involved there. However, it becomes a deception and an error when it is thought or allowed to be thought that psycho-physiological factors have no real influences there. And today we are actually witnessing this strange phenomeneon: that many proofs are being put forward as physical experiments when they are in fact psychological experiments. A careful overhaul along these lines of the experimental paraphernalia of optics has brought about the collapse of an unbelievable number of rules which had hitherto been regarded as definitive even in the most highly reputed circles.

The fields in which this error of a philosophical nature has brought about the strangest results are those of photometry and colorimetry. These are, in fact, two sciences which lay claim to be unadulterated branches of physics and yet which, in the ultimate analysis, aim at coping with the laws dealing with two purely mental phenomena such as light and colour.

There is quite a lot that could be said on this point, but I have to cut short here so as to be able to pass on to another brief comment regarding the method employed on an extremely wide scale in the optical science of the seventeenth century. I mean the part played by mathematics, whether geometrical or analytical.

There can be no doubt that mathematics constitutes a means of investigation of tremendous power and that when it is possible to express a subject in terms of mathematics, that subject immediately has great prospects of rapid development. Nevertheless, we must not overdo it even in this approach. Let us give everything what is deserves, but no more.

What happened in the case of the optics of the seventeenth century is of very great interest in this connection. As we have already said, the rule of the distantiometrical triangle made it possible to apply geometrical constructions and calculations to the study of optical phenomena, thus bringing order into the desert wastelands of mediaeval optics, which had hitherto defied all efforts to apply mathematics, precisely because it was so heavily loaded with experiments of a psycho-physiological nature. This was indeed a great and meritorious service performed by Kepler, the rule and mathematics, because it enabled the science of optics to make a truly miraculous jump ahead.

But here again the fault has been to overdo things. It was forgotten that geometrical constructions and algebraical calculations are nothing more than reductions to a simpler form which, although they can be extremely useful, are not a perfect representation of reality and that the last word always rests with experimental evidence.

What happened in this connection a couple of centuries ago is extraordinary. Mathematical theory was raised to the level of indisputable truth, transcending all criticism. Experimental evidence came to be regarded as confirmatory and only worthy of consideration when it substantiated the results of calculation. When this confirmation was lacking, doubt was not cast upon the validity of the calculations. No one expressed doubts to the effect that these calculations — although they may have been extremely rigorous from the purely formal point of view — may have started off from premises which had been oversimplified. Instead, the experimental worker was accused of lack of skill or ability and he was invited to modify his procedure or means of investigation so as to arrive at the result foretold by the calculations.

That is to say, unless it was found possible to eliminate tacitly and universally the observation of those experiments, such as the one mentioned above involving the spherical mirror in which images are seen which no mathematical theory has ever been able to take into account.

We have to come round to thinking along these lines if we are to explain how it could possibly have taken more than three centuries to wake up to the fact that the rule of the distantiometrical triangle was nothing more than a "working hypothesis" which was absolutely devoid of any experimental foundation.