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ON THE HISTORICAL INTERPRETATION OF ARISTOTLE'S ANALYTICS

1. The application of methods of mathematical logic to historical research has not so far been as effective or fruitful as expected. This failure is best manifested now in the sceptical, if not negative, attitude of historians who at the start had welcomed these methods with enthusiasm. However, the question as to what are the potentials of logical theory for historical research in general and the history of science in particular still remains open. As a matter of fact, the problem seems to embrace two separate issues that can be briefly formulated as follows.

(1) Can one approach historical knowledge in a theoretical way like natural science theories which make it possible to translate the substance of the problems into a formal language and to present them in terms of numerical formulae?

In my personal opinion this problem can definitely be solved. History of human society or of its particular institutions is an objective process of natural history which obeys its specific laws in a perfect analogy to the processes that occur in nature. But, if converted to a form of theoretical knowledge thus understood, history ceases to be history in the conventional sense. It must abandon the whole description of particular facts and events. Such description of events will be subject to the application of the theory, but the latter cannot be a source of any specific knowledge about these events.

It is this circumstance which explains the historians scepticism or even resentment *vis-à-vis* the apparatus of mathematical logic. It would be a mistake, however, to deny the need for a theory of the historical process, though for sure, the creation of such a theory must not discredit historical science in the conventional sense of the term. Hence the second problem arises.

(2) Can the logical-mathematical tools be efficiently applied in historical research? The goal of historical science remains unchanged — it is to

describe and interpret historical events. This does not imply the objective of presenting knowledge in a strict mathematical-logical form. The logical apparatus is used as an analytical tool, but it should not be expected that the application of this tool to certain empirical material will yield a final historical conception of the events in question. It is merely an intermediate stage of historical research. The best what the logical analysis can do is to answer questions as to the essence and structure of the event. As for the question of why the event is such as it is, it can be answered only in historical terms.

Evidently, one need not prove that prior to answering a "why" question one must know all the "what's" and "how's". The mathematical-logical approach is quite often needed for the reconstruction of an event in the same manner as chemical, physical, biological, anthropological, linguistic, etc., methods of analysis that are regularly applied in historical research.

The tools of mathematical logic often prove particularly effective or even indispensable for the history of science. Indeed, the bulk of research in the field deals with the study of scientific knowledge. What is the essence and the structure of this knowledge? It is a question to be answered only by logical analysis that makes use of the latest accomplishments of logical science. In turn, this is connected with the need of using formal languages worked out on the basis of logical theory. Certainly, the use of any given formal language involves the adoption by the scholar of a certain corresponding theory of logical analysis¹. But this is a *sine qua non* of any research. Certain theoretical premises of each specific study appear in its framework as *a priori postulates*. The substantiation of these premises always surpasses the bounds of the study in question.

After this short preamble let me turn to the subject of this paper which notably, in my opinion, presents the second aspect of the application of the apparatus of mathematical logic to historical research.

2. Vast literature has been written concerning Aristotle's *Analytics*. The present article is, however, the first contribution on my part to the discussion on the subject. The reason for my undertaking the problem is twofold: first, I am not quite satisfied with any of the existing interpretations, and, second, it yields a most convenient opportunity to get an insight into the potential role of the methods of mathematical logic as applied to historical science. The good thing about *Analytics* is that this text has been repeatedly subjected to very thorough-going analyses.

I will not dwell here upon the book as a whole, but only on the theory of assertoric syllogism expounded in the *Prior Analytics*, while leaving

¹ N. Church, *Introduction to Mathematical Logic*, Moscow, 1960, pp. 16-17 (Russ. ed.).

aside the problems of model logic. Besides, the *Prior* and the *Posterior Analytics* will be compared.

According to what has been said before, I shall first try to discuss the essence of the assertoric syllogisms and only after that to answer the question as to its origins, i.e. its historical determinism.

The first question has been, to my mind, most aptly answered by Łukasiewicz in his *Aristotle's Syllogistics from the Standpoint of Modern Formal Logic*. One cannot classify this study into historical research for the sole reason that it deals with a phenomenon more than two thousand years old. In fact, Łukasiewicz could have carried out the same investigation on any contemporary theory.

Still, there are a number of points on which Łukasiewicz writes as a historian *par excellence*. A case in point is the discussion of the question as to why Aristotle in his syllogistics does not accept unique and empty terms for the most general terms (i.e. categories)², etc. On the whole, Łukasiewicz's study is a contribution to applied logic, not to history. But I believe that he has managed to formalize Aristotelian syllogistics in a most adequate fashion.

His results undoubtedly deserve more attention. His major conclusion is that the syllogism of conventional logic essentially differs from the Aristotelian syllogism. Whereas in traditional logic the syllogism is an inference rule, Aristotelian syllogism is a proposition. Łukasiewicz notes that implications in which the consequent is the conjunction of two propositions called premises, and the antecedent is a proposition called conclusion, that is in the form

$$\alpha \text{ and } \beta \rightarrow \gamma$$

Nevertheless, Aristotelian syllogistics is not a form of propositional calculus for it is a theory of implications considering the structure of elementary propositions. Aristotle distinguished four types of propositions he made use of (apart from indefinite and unique) in his syllogistics: general affirmative, general negative, partial affirmative, and partial negative. All these propositions have the subject-predicate structure. Generic relations, which Aristotle considered so important, have been proved by Łukasiewicz to play no role in syllogistics. The latter is built by means of a content-descriptive system of axioms; once certain axioms are presumed, theorems are proved without any reference to generic relations. The subject-predicate relation in the proposition is a special kind of relation, similar to the "more than"/"less than" relationship in mathematics. This enables Łukasiewicz to assert that Aristotelian logic is a theory of the relations *A*, *E*, *I*, *O* in the sphere of general terms³. It may be pre-

² J. Łukasiewicz, *Aristotle's Syllogistics from the Standpoint of Modern Formal Logic*, pp. 40-41, see also Chapter 2.

³ Łukasiewicz, p. 50.

sumed that for Aristotle this pattern of thought was important so that the initial syllogisms (Barbara and Datisi) were convincing from a psychological view-point⁴. Certainly, a reference to generic relations proves nothing, but provides the psychological motivation for the acceptance of the axiomatic system.

Further details of the structure of Aristotelian syllogistics do not concern us here.

It should be emphasized again that Aristotelian syllogistics is a theory built on propositions that are always true if some of them (Barbara and Datisi) are assumed to be true. This theory is valid (and built) only in relation to the propositions of subject-predicate structure. The best reference for the reader interested in further proof and substantiation of this statement is provided by Łukasiewicz's book.

It is noteworthy that there is a substantial difference between the theory of Aristotle and the system of Łukasiewicz which is a reconstruction of the former. Łukasiewicz himself points to this difference in par. 35. The system presented by Łukasiewicz is a formalization of the substantial logical theory of Aristotle⁵. As Curry aptly noted, "... there is a fundamental difference between a formalized substantial theory and the original substantial theory. The fact is that a proof or any other discussion on formal theory remains true independent of its interpretation"⁶. The term "formal" here has the sense of the term "formalized". But interpretation was essential for Aristotle. If he had not presumed an interpretation for his syllogisms, he would have been unable to develop his theory so rigidly. And this notwithstanding the fact that all the theory is developed not in concrete terms but with variables.

One example will suffice to show the importance of the foregoing. Łukasiewicz formalized and furnished with axioms not only the object of his study (this has been largely done by Aristotle himself), but also the tools of reasoning about this object. For that purpose he had to modernize syllogistics. "Since Aristotelian exposition was not quite correct in certain points", Łukasiewicz writes, "I was obliged to make up for these flaws, for instance, for certain insufficiency of the proofs by *reductio per impossibile* or rejection by concrete terms"⁷. This modernization was necessitated by the fact that Aristotle reasoned about the objects of the theory in interpretative terms; that is, visualizing a certain concrete interpretation, and therefore he could afford to make use of concrete terms in some of the proofs. Naturally, this way of argumentation is beyond the com-

⁴ Łukasiewicz, p. 90.

⁵ While the theory of Aristotle belongs to the field of formal logic, it is expounded in interpretative terms. Hence, a theory of formal logic needs not necessarily be formalized.

⁶ H. Curry, *Foundations of Mathematical Logic*, Moscow, 1969, p. 36, (Russ. ed.).

⁷ Łukasiewicz, p. 189.

petence of logic. But this trait is peculiar to Aristotelian syllogistics — the latter is not a theory of proofs, but a theory of propositions. In this connection, the reproach that syllogistics is "... a limited system inapplicable to all kinds of reasoning, for instance, to mathematical proofs" ⁸ is in itself unreasonable for this system does not deal with reasoning at all. But it is still a logical theory in so far as it is a theory of true propositions of a special type. Aristotle himself asserted that syllogism is not an inference rule, but a proposition. In the *Prior Analytics* he wrote: "... syllogism, that is an utterance in which from assertion of something a certain thing different from that asserted necessarily follows..." ⁹ This statement must be understood as follows: if one has the conjunction of two premises assumed to be true, then, considering the subject-predicate structure of the elementary clauses and under the condition that the two premises have only three terms, one can build, according to certain rules, a true implication in which one must construct only the antecedent. As to the consequent, it is already given by the conjunction. So, indeed, Aristotle did attach very much importance to the doctrine of syllogism and to the syllogism as such, but this belongs, as he would say, to the domain of opinion, not knowledge.

Such is the general outline of the results of a logical analysis of the essential part of the *Prior Analytics*. A century ago these results could not have been obtained for we had first to discover the propositional calculus, predicate calculus, to develop the theory of proof and to gain a profound understanding of the essence of the axiomatic method. All these sections of logical theory, however, had only begun their vigorous development late in the 19th century. It can be therefore asserted with good reason that Aristotelian logic has not been adequately understood prior to the 20th century. But no such historical interpretation of Aristotelian syllogistics as would conform to the present stage of its analysis has been given as yet.

3. It is a common view that Aristotelian logic in general and the theory of assertoric syllogism in particular is a generalization of the practices of argumentation adopted by ancient science, specifically, by mathematics. The following statement can serve as an example of this view. Van der Waerden says: "When Aristotle put together all the rules of logic, he merely systematized the regularities that he had found in the argumentations of the mathematicians and philosophers who had lived before him. A majority of his examples are borrowed from mathematical textbooks of the day. It is obvious that those textbooks in their logical reasoning fol-

⁸ *Ibid.*

⁹ Aristotle, *Prior Analytics*, Moscow, 1952, p. 10 (Russ. ed.).

lowed the examples to be found in works of great mathematicians; the reverse process is unthinkable. Hence, the thinking of Greek mathematicians long before Aristotle must have conformed to very strict requirements" ¹⁰.

The truth of the last two sentences of this quotation can hardly be disputed. The two first ones, however, apparently seem erroneous. First, it is unfair to say that Aristotle collected rules of logic. His theory, as has already been mentioned, is neither a theory of inference nor, hence, of rules. He does make use of the conventional patterns of argumentation in his reasoning and proofs, but his theory is not about these patterns.

The second sentence is simply wrong. The logical doctrine proper is set forth by Aristotle in the *Prior Analytics*. Its volumes contain but six references to mathematics. Most of his examples used for syllogisms are not mathematical at all but taken from colloquial speech and based on common sense.

Moreover, even the few mathematical examples he mentions give evidence not for but against the opinion that Aristotelian logic is a generalization of the norms of reasoning pre-existing in science (i.e. in mathematics).

A closer look at these examples will suffice. Two of them refer to the problem of commensurability of the diagonal ("diameter" in Aristotle's terminology) and the side of a square. There are other two examples about parallel lines, one about the sum of angles of a triangle, and one more about the equality of angles in an isosceles triangle. And that is all. Thus, essentially, there are not six, but four examples. But even these four have nothing to do with syllogistics, as shall presently be shown. The first example is given in the following context: "Indeed, all [syllogisms] that are built by reduction to the impossible deduce the false, but they prove the originally true, proceeding from a [certain] assumption, since from an assumption [of a situation] contradictory [to the originally assumed] there follows something impossible as, for example, when one proves the incommensurability of diameter [with the side], because, if one admits their commensurability, then odd would be equal to even. Thus, that odd is equal to even is here deduced syllogistically, whereas that diameter [with the side] is incommensurable, is proved as proceeding from an assumption, since in case of the assumption [of a proposition] contradicting [the originally accepted], the false follows" ¹¹.

The statement of Aristotle's that the conclusion about the equality of odd and even can be derived by a syllogism from the assumption of the commensurability of diagonal and side is erroneous. In effect, inside this proof one has repeatedly to resort to the method of demonstration by the

¹⁰ Van der Waerden, *Otwakende Watensheep*, Moscow, 1959, p. 215 (Russ. ed.).

¹¹ Aristotle, *Analytics*, p. 70.

contrary which, as Aristotle himself acknowledged, is an unsyllogistic technique.

The second example is used to demonstrate that one of the premises in a syllogism must be general. Aristotle shows that the theorem of the equality of angles at the base of an isosceles triangle could not be proved without assuming three general statements. This is certainly true. But I, as one, definitely fail to understand how this theorem can be proved by pure syllogisms without reference to a different logical system — propositional logic and logic of relations.

The situation is similar with the other mathematical examples. Moreover, there is an additional indirect argument against the notion that syllogistics has been “extracted” from mathematics or science in general. It has already been mentioned that when talking about proofs in syllogisms Aristotle surpassed the framework of syllogistics by using rules that do not belong to it as metarules. In the second volume of the *Prior Analytics* he examines one such rule and asserts that the two implications “if α , then β ” and “if not- α then β ” cannot be both true since they lead to the conclusion “if not- α , then α ”. Łukasiewicz notes that this assertion of Aristotle's is wrong. But the important point here is not that Aristotle has made a mistake: the truth of this conclusion is intuitively very doubtful. The point I want to make here is this: In the 7th volume of Euclid's *Elements* a theorem is proved by means of this kind of argumentation. Considering the fact that this volume is “... a fragment of ancient mathematics”¹² that Euclid has included *in toto* in his *Elements*, it becomes obvious that Aristotle did not at all need to scrutinize mathematical argumentation to build his syllogistics. Moreover, he appears to be incompetent in this field of problems, but I do not see any reason to blame him for this. In effect, Aristotle did not undertake a study of the procedures of proof in science from a logical standpoint, and, therefore, could not have been expected to be familiar with all the subtleties of these procedures.

The questions arise: What was the actual intention of Aristotle in constructing his syllogistics? What kind of material enabled him to shape his theory?

4. In an attempt to answer them it is necessary to part with logic and enter the domain of history.

An outline of the general situation in the city of Athens at the time of Aristotle would greatly exceed the scope of my paper. Yet I must at least emphasize the special importance attained then by the art of rhetoric thanks to the political developments in Athens since the Peloponnesus War. It is not a mere coincidence that the majority of great Greek ora-

¹² Van der Waerden, *op. cit.*, pp. 155–161.

tors lived in the 4th century B.C. Rhetoric served to exert political influence on the citizens of Athens and thus came to be a means of arriving at important political decisions.

At the time of Aristotle, the Athenian society had already entered the path of its downfall: the deterioration and final disappearance of noble civic aspirations was accompanied by a lack of political principles and the corruption of the citizens. In this context, the striving to understand the essence of the art of eloquence was very acute. A demagogue could lead the politically immature masses to disaster — by the sole art of his eloquence. Demosthenes warned of this repeatedly. An interesting passage from his third speech against Philip proves this. Stating that it is not enough to oppose Philip by military action, he goes on to say: "... but we should also hate with all our heart and mind the orators who speak for him before you... But this, I swear by Zeus and the other gods, you will be unable to do, and you do not want it, but you have fallen into such folly or insanity..., that either for the sake of controversy, or out of jealousy, or for fun, or for whatever occasional reason, you let corrupt men speak... and you laugh when they scold some person. But this is not the whole horror, although this is horrible in itself. It is that you have given these people an opportunity to run political affairs even with more carelessness than the orators who defend you"¹³. I indulged in this long quotation, for it is a graphic illustration of the importance of eloquence attained at the time of Aristotle.

What is eloquence? Plato answers this question in his *Gorgias*. When Socrates asks Gorgias what eloquence is, the latter answers:

Gorgias. It is truly the greatest wealth and it gives a man both freedom and power over other people, each in his city.

Socrates. What is it, after all?

Gorgias. The ability of convincing by word the judges in the court, and the councillors in the council, and the public in the convention and in any other gathering of citizens".

They proceed to discuss the meaning of the word "convincing".

Socrates. Do you think that it is the same thing — 'to learn' and 'to believe', 'knowledge' and 'belief', or that they differ somehow?

Gorgias. I think, Socrates, that they are different.

Socrates. You think right, and here is the proof. If some person would ask you Gorgias, can a belief be true and false? you would answer, I suppose, that it can.

Now, what about knowledge? Can it be true or false?

Gorgias. By no means.

Socrates. So it is clear that it is not the same thing... But now, conviction is carried both by those knowing and those believing. May we then establish two kinds of conviction: one giving belief without knowledge, the other giving knowledge?... Which conviction is created by eloquence at court and in other

¹³ Plato, *Coll. Works*, vol. I, Moscow, 1968, pp. 264–265 (Russ. ed.).

gatherings...? The one that engenders belief without knowledge or the one that gives knowledge?

Gorgias. Clearly, Socrates, the one that engenders belief...

Socrates. Hence, an orator at court or at some other gathering does not teach what is just and what is not but merely implants belief. Well, indeed, the crowd could not really grasp so important things in so short a time¹⁴.

This is enough to understand the general purpose of rhetoric. The two main features of interest for the purpose of this paper are that: (1) eloquence convinces, but it produces belief and not knowledge; (2) the former needs not necessarily be connected with ill will or hypocrisy on the part of the orator: just "... the crowd could not grasp so important things in so short a time". In other words, the very conditions under which the speech is delivered compel the orator to adopt this way of convincing his audience.

What was thus the manner of convincing adopted by such speeches? Leaving aside the process of its very delivery, which is important but irrelevant to our subject, it must be stressed that the ancient theoreticians of rhetoric distinguished in a speech between the argumentation and the refutation. The former was divided into two types: simple (unsophisticated) and artificial. Simple argumentation referred to facts, documents, precedents. Artificial argumentation was built on the basis of selected arguments and relations between these. Refutation was to be built according to the same pattern. Taking into account the peculiarities of eloquence, artificial argumentation is to be understood as furnishing an apparent proof, being in essence a procedure of convincing to form belief.

The question arises whether and, if so, how knowledge (the truth) could be combined with the process of convincing and not with the process of proof. This ought to be considered only as a theoretical issue since from the practical standpoint of rhetoric this is no problem at all. In practice the orator resorts to all kind of ruses, sophisms, direct deceit so as to gain his ends by any means. The question is, however, whether it is possible to solve the problem objectively, irrespective of the art of the orator.

An analogy will help to comprehend the issue fully.

The capitalist society is never free of deceit, roguery, and robbery of a peculiar sort. However, when Marx studied capitalism he wanted to find the laws of the functioning of capital under the condition of "fair" relations between the economic agents. A similar problem arises with the analysis of the art of eloquence.

It was this problem that Aristotelian syllogistics intended to solve theoretically. His syllogisms, as it has been mentioned, are neither proofs

¹⁴ *Ibid.*, pp. 267-268.

nor inference rules, but propositions. But they are propositions of a peculiar kind. They are true by force of their structure of form. But they are also convincing precisely by force of their structure. It takes no hard work to prove one's point if one has to deal with the proposition of the form: If all A's are B's and all B's are C's, then all A's are C's. How many such propositions can one build in one's speech under certain specified conditions; are they all equally evident? If not, can one reduce the non-evident ones to the evident form so as to expand the class of evident propositions? These are the problems that are solved inside the theory.

Thus the realm explored by Aristotelian syllogistics was not scientific argumentation, but the art of eloquence. This strict logical theory, the first in the Western civilization, strived toward the establishment of a theoretical understanding of the common sense expressed in everyday colloquial language. Accordingly, Aristotle took his empiric material from the speech of his contemporaries. As Radtsig noted in his thorough paper entitled *Demosthenes as Orator and Statesman*, artificial argumentations are arranged in the form of enthymemes and leading questions¹⁵. Besides, "A most convincing argument [in a speech — B. G.] is a reference to the common opinion or a popular proverb"¹⁶. Proverbs are for the most part also enthymemes. Hence, Aristotle did not need to invent the syllogism. He just had to give it its theoretical interpretation.

Such is the historical background of the Aristotelian syllogistics, the analysis of which has been possible solely due to the logical research accomplished by Łukasiewicz.

5. Naturally, this interpretation contradicts the views held by Aristotle himself, especially as propounded in his *Posterior Analytics*. But there is nothing unusual about it, such things happen to theoreticians quite often. Schroedinger, our contemporary, famous for his research in physics, also wrote an interesting book *What is Life from the Standpoint of Physics?* In that book he tried to apply the quantum theory not only to biology, but also to the social behaviour of man. This does not detract from the value of the quantum theory or from the merit of Schroedinger as one of its fathers.

The same was the case with Aristotle. The way he writes about syllogistics in his *Posterior Analytics* is strikingly contradictory to the spirit of assertoric syllogistics. Generally, in contrast to the *Prior Analytics*, the later book is indeed devoted to the analysis of science and scientific knowledge. It obviously deals with other problems than the

¹⁵ S. I. Radtsig, *Demosthenes as Orator and Statesman*, [in]: Demosthenes, *Speeches*, p. 468 (Russ. ed.).

¹⁶ *Ibid.*, p. 470.

first book; it is not a continuation but an altogether different book, and Aristotle was right in calling it "the other".

Paradoxical as this may sound, the *Posterior Analytics* is not a logical treatise. It is a book on methodology of science and scientific cognition. As such, it presupposes a certain "metaphysics" (in Aristotelian sense), whereas the *Prior Analytics* does not deal with metaphysical questions at all. How it occurred then that Aristotle, eager as he was to build the logic of science (as witnessed by his striving to make use of the findings of the *Prior Analytics* for analysis of science), still fell short of his goal? The reason for this failure may lie in the extraordinary complexity of science viewed as a logical phenomenon. Its analysis requires a very sophisticated conceptual apparatus of logical and the corresponding techniques (formalized language). Undoubtedly, Aristotle was a pioneer in creating a logical theory as such. But the developmental stage necessary for a logical analysis of science was reached by logic not earlier than by the end of the previous century.