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SUMMARY

The aim of this paper is to present the discrepancy between the "ideology" of the so-called (by Einstein) Mach's Principle, and Mach's own positivistic beliefs. Mach's philosophy and his criticism of Newtonian mechanics are also discussed.

The basic assumptions of the General Relativity Theory are inseparably connected with the "Cartesian program" of geometrization of physics. On the other hand, Einstein himself was greatly influenced by Spinoza's doctrine that the fundamental description of the Universe must necessarily be "closed". Precisely at this point Spinozian rationalism and the outcome of Mach's positivistic philosophy had also run alongside. The so-called Mach's Principle, according to which the mass of a test body is induced to it by all other masses present in the Universe, became to Einstein one of the main inspirations in creating the General Relativity Theory. However, as it is well known now, General Relativity is "Machian" only to a very small degree. It will not be out of place, therefore, to review here briefly some recent attempts to incorporate Mach's Principle into relativistic physics, as well as some critical arguments against them.

The Principle resulted from Mach's radically positivistic programme, whereas all recent attempts to construct a fully Machian theory appear to be based on some metaphysical (entirely non-empirical) reasons. In the author's view this remarkable situation may be considered as a yet another proof that the extreme positivism is only a hair-breadth apart from high metaphysics.

1. INTRODUCTION

In the last few years Mach's philosophy has become the subject of many studies undertaken not by professional philosophers, but by scientists working in different branches of physics. Yet when one considers the influence of Mach's thought upon the physics of our century, one is no longer surprised by this remarkable fact: surely, the scope and importance of Mach's philosophy for many branches of physics nowadays account for this otherwise peculiar interest. However, it is not very often met than physicists turn back toward their own past in order to establish historical regularities. Yet it appears that certain aspects of Mach's criticism on classical mechanics refer to the General Relativity Theory as well, and that a number of current theoretical works are thus stimulated by the so-called Mach's Principle.

Mach himself was an unhesitating positivist. This philosophy — albeit in the slightly modified version of neopositivism — greatly influenced physics of the beginning of our century. Quantum mechanics and the General Relativity Theory are generally considered as the greatest achievements of theoretical physics in those time. And according to the opinion of Professor Sachs "... the incompatible philosophical implications of the quantum and relativity theories are both contained in Mach's philosophy. One of these is in reference to Mach's acceptance of positivism as a necessary truth in natural philosophy. The other has to do with the Mach principle" [1].

In the present article problems connected with quantum mechanics are left aside; the article concentrates on the question, already posed by Professor Sachs, of the discrepancy between the "ideology" of the so-called (by Einstein) Mach's Principle and Mach's positivistic beliefs.

2. MACH'S PHILOSOPHY

Mach's thought was influenced by Kant's antimetaphysical attitude and Hume's empiric understanding of knowledge and meaning [2]. However, Mach constructed his own epistemology in certain aspects similar to that of Avenarius. After the elimination of all metaphysical hyposthases only "sensations" remain in human cognition from which, as from ultimate "elements", our knowledge of the world and of ourselves is built. One reads in the first chapter of *The Analysis of Sensations*:

> Colors, sounds, temperatures, pressures, spaces, times, and so forth, are connected with one another in manifold ways; and with them are associated dispositions of mind, feelings, and volitions. Out of this fabric, that which is relatively more fixed and permanent stands prominently forth, engraves itself on the memory, and expresses itself in language. Relatively greater permanency is exhibited, first, by certain complexes of colors, sounds, pressures, and so forth, functionally connected in time and space, which therefore receive special names, and are called bodies. Absolutely permanent such complexes are not. ...Further, that complex of memories, moods, and feelings, joined to a particular body (the human body), which is called the 'I' or 'Ego', manifests itself as relatively permanent [3].

The bodies are a subject for physics, the Ego a subject for phychology. However, the distinction between these two sciences depends not on the subject-matter, but on the line of our investigations. For instance, colour is a physical object when considered in connection with a source of light, temperature, and so on. The same colour is a psychological object when considered with respect to our sense of vision.

The aim of any scientific research is to establish all possible connections between sensations. "Bodies do not produce sensations, but complexes of elements (complexes of sensations) make up bodies" [3]. The world does not consist of "mysterious entities". The exploration of reality is reduced to the "analysis of sensations". As we see, the difficulties of the 20th century neopositivism, concerning the question of how subjective elements (the so-called elementary sentences) can originate intersubjectively meaningful (in order not to say objective) science, are rooted in Mach's own philosophy.

The Machian program (known in the history of philosophy as empiriocriticism) to clear all natural sciences of metaphysical superimpositions originates in the above-sketched philosophy. If one regards sensations as the ultimate elements of the world, may questions immediately reveal their metaphysical character. To quote Mach again:

> ... all that is valuable to us is the discovery of functional relations, and that what we want to know is merely the dependence of experiences on one another. It then becomes obvious that the reference to unknown fundamental variables which are not given (things-in-themselves) is purely fictitious and superfluous [3].

3. MACH'S CRITICISM OF NEWTONIAN MECHANICS

According to most recent views, any scientific theory should be analysed in full context of other theories ("science as a whole") and of their history [4, 5]. Also in this respect Mach should be considered as a predecessor of present day trends in the philosophy of science. Mach's criticism of classical mechanics is historical not only on account of his own interests, but also because he wanted to elucidate the logical principles and procedures used in "true" mechanics, in its actual development.

It follows from Mach's philosophy that no *a priori* knowledge is contained in mechanics, but only knowledge of the sense experience. Mathematical demonstration are no more than derivations of conclusions from sense stated facts. All entities which cannot be reduced to the sense data are "metaphysical obscurities". Such entities occur in classical mechanics; it seems as if Newton "had grown unfaithful to his resolve to investigate only actual facts" [6]. The first place among metaphysical concepts of this kind must be ascribed to Newton's absolute time and absolute space.

Absolute time and absolute space appear in Newton's considerations in connection with the law of inertia. If inertial forces act on a certain system of bodies, it means that this system of bodies moves absolutely. Absolute motion is referred to absolute space and measured by absolute time.

Besides, the law of inertia seems to convey some information about the bodies upon which no forces act. This situation, unacceptable to Mach, may be avoided by considering the law of inertia as a mere definition of force, which summarizes certain observational data. If one observes different motions on the surface of the Earth, one notices that no error is introduced by regarding Earth as being relatively at rest; in considering motions of planets one may regard the Sun as being at rest with respect to the distant stars, and so on. The law of inertia simply states that in certain physical situations certain "bodies of reference" may be regarded as being relatively at rest.

Then, what is absolute time?

> It is utterly beyond our power [answers Mach] to measure the changes of things by time. Quite the contrary, time is an abstraction, at which we arrive by means of the changes of things; made because we are not restricted to any one definite measure, all being interconnected. A motion is termed uniform in which equal increments of space described correspond to equal increments of space described by some motion with which we form a comparison, as the rotation of the earth. A motion may, with respect to another motion, be uniform. But the question whether a motion is in itself uniform, is senseless. With just as little justice, also, may we speak of an 'absolute time' — of a time independent of change. This absolute time can be measured by comparison with no motion; it has therefore neither a practical nor a scientific value... [6].

It is also impossible to assert anything meaningful about absolute space and absolute motion:

> When we say that a body *K* alters its direction and velocity solely through the influence of another body *K'*, we have asserted a conception that it is impossible to come at unless other bodies *A*, *B*, *C*, ... are present with reference to which the motion of the body *K* has been estimated. In reality, therefore, we are simply cognisant of a relation of the body *K* to *A*, *B*, *C*, ... If now we suddenly neglect *A*, *B*, *C*, ... and attempt to speak of the deportment of the body *K* in absolute space, we implicate ourselves in a twofold error. In the first place, we cannot know how *K* would act in the absence of *A*, *B*, *C*, ...; and in the second place, every means would be wanting of forming a judgment of the behaviour of *K* and of putting to the test what we had predicted, — which latter therefore would be bereft of all scientific significance [6].

If therefore absolute space and absolute motions "are pure things of thought, pure mental constructs" [6], how should one interpret the appearance of the inertial forces? Mach gave one possible answer to this question: the inertial forces must be measured in relation to all other bodies surrounding a given body. All masses filling the Universe supply the "background" with respect to which a local inertial system is determined. This background, however, is of a rather special character: "The crucial point here is a change in the role of the distant stars: from being marker-points for a frame of reference to becoming part of an interacting physical system" [7].

In this view, mass appears to be not an intrinsic property of a given body, its own "measure of resistance against an accelerating force", but a property induced to a given body by all other masses in the Universe. Newton's famous experiment with the rotating bucket [8] yielded a negative result because the contribution of the bucket walls to the inertial forces is negligibly small as compared with that of distant stars. Since only relative motions exist, it makes no differences whether a body rotates with respect to distant celestial bodies, or whether all celestial bodies rotate with respect to a given body. It is worth to quote here Mach's reasoning in full length.

> Let us now examine the point on which Newton, apparently with sound reasons, rests his distinction of absolute and relative motion. If the earth is affected with an absolute rotation about its axis, centrifugal forces are set up in the earth: it assumes an oblate form, the acceleration of gravity is diminished at the equator, the plane of Foucault's pendulum rotates, and so on. All this phenomena disappear if the earth is at rest and the other heavenly bodies are affected with absolute motion round it, such that the same relative rotation is produced. This is, indeed, the case if we start *ab initio* from the idea of absolute space. But if we take our stand on the basis of facts, we shall find we have konwledge only of relative spaces and motions. Relatively, not considering the unknown and neglected medium of space, the motions of the universe are the same whether we adopt the Ptolemaic or the Copernican mode of view. Both views are, indeed, equally correct; only the latter is more simple and more practical. The universe is not twice given, with an earth at rest and an earth in motion; but only once, with its relative motions, alone determinable. It has, accordingly, not permitted us to say how things would be if earth did not rotate. We may interpret the one case that is given us, in different ways [6].

4. EINSTEIN'S PHILOSOPHICAL PRESUPPOSITIONS

It is almost a rule that underneath all greatest scientific achievements, consistent in constructive criticism as well as in critical creativeness, a certain philosophical doctrine is imbedded, if only, which usually is the case, as an initial inspiration. As it has been shown, Mach's criticism of classical mechanics resulted from his positivistic views. Einstein's new theories, in turn, were inspired by the rationalistic tendencies of their author.

The intricate problem of the influence of different philosophies on Einstein's contribution to contemporary physics still awaits more detailed investigation. Some valuable remarks concerning this question may be found in the book of Kuznetsov (especially in Chapter V, VI, VIII, and IX), [9]. In Kuznetsov's opinion, the origin of Einstein's philosophical inclinations may be traced back to the rationalism of Descartes and Spinoza.

According to Descartes, the very essence of material bodies has to be identified with the extension of bodies, understood as the ability of a body to exist in space. A body may lose all its properties with no harm to its existence, but having lost its extension it ceases to be a body. Precisely, it is geometry which is involved in the extension. Every change of the extension may be reduced to a replacement in space, i.e., to a mechanical motion. Therefore, the science of nature ought to be expressed in terms of no other branch of mathematics but geometry (more geometrico) and in terms of no other branch of physics but mechanics (more mechanico). The mechanics of Descartes, however, is purely kinematic and does not contain any concept that cannot be understood in terms of geometry and time.

The basic assumptions of the General Relativity Theory are inseparably connected with the idea of the geometrization of physics. Moreover, the "Cartesian program" becomes more forcible (in a certain sense) as, owing to the space-time concept (introduced by Minkowski), all mechanics, not only its kinematic part, had become a branch of geometry, already at the stage of Special Relativity.

The essential premise of Spinozian philosophy seems to be his definition of substance: "Substance is that which is in itself and is conceived through itself; it can be conceived independently of the conception of anything else" [10]. From this definition it immediately follows that the Substance has all attributes usually ascribed by philosophers to the Absolute. So "God" an "Nature" are two names of one Substance--Absolute.

The pantheistic views of Spinoza have led him to pure rationalism. If the Substance has to be "conceived independently of the conception of anything else", the science of nature must be deductive in character and must form, in a sense, a closed system. "Spinoza's deductive system is presented as geometrical because Euclidean geometry provided for him, as for his contemporaries, the paradigm case of an axiomatized system. As with all such systems, the axioms and the primitive terms can be understood only in terms of the propositions which are consequently derived, just as the consequent propositions must be referred back to axioms and primitive terms. Furthermore, when such a system is an attempt to set forth the total character of the universe, the system takes on a specially self-enclosed character" [10].

This precisely is the point which had so strongly impressed Einstein's philosophical imagination. Certainly, Spinozian self-enclosed, pantheistically understood nature was considered by Einstein both in a more physical and a more methodological manner. The physical sense of Spinozian doctrine presented above "implies that the fundamental description of a physical system must be necessarily closed. That is to say, the concept of a free, non-interacting bit of matter can only relate, according to this principle, to an asymptotic approximation in which its mutual coupling with the rest of a physical system is arbitrarily weak — but is never'off" [1]. Methodologically, this served as a directive to construct a physical theory of such a self-closed system containing the entirety of all physical "bits of matter" and fields. From the logical point of view relativistic cosmology is an application of Einstein's theory of gravitation to the Universe as a whole. In the real fact, however, the inspirations came precisely in the reverse way; it was the theory of gravitation which had resulted from Einstein's cosmological intuitions.

Similarly, in Einstein's work — thanks to his genius and intuition — had Spinozian rationalism blended with the outcome of Mach's positivistic philosophy, namely with the postulate according to which the mass of a test body is induced to it by all other masses present in the Universe. This postulate, called by Einstein Mach's Principle, became for him one of the main inspirations in creating the General Relativity Theory.

5. MACH'S PRINCIPLE AND GENERAL RELATIVITY

The historical role of Mach's Principle in the early development of General Relativity (especially in Einstein's works) is quite well known. It is not out of place, however, for the sake of the article's exhaustiveness, to recall if only the most essential facts.

Einstein explicitly enumerates three — according to his opinion, independent — assumptions, upon which the General Theory of Relativity is based, in the paper *Prinzipielles zur allgemeinen Relativitätstheorie* [11]. These assumptions are: (a) Principle of Relativity, (b) Principle of Equivalence and (c) Mach's Principle. The first, as formulated there, appears to be *de facto* only the principle of general covariance, and the last is expressed in the following manner: "G-field is entirely determined by the masses of bodies. Mass and energy, according to conclusions of Special Relativity, are essentially equivalent; formally, energy is described by the symmetric energy tensor, i.e. G-field is defined and determined by the energy-momentum tensor" [12].

One finds a remarkable explanation in the footnote: „Up to now I have not distinguished between the Principles (a) and (b); this, however, had led to misunderstandings. The name 'Mach's Principle' is chosen because this principle appears to be a generalization of Mach's statement that inertia ought to be reduced to the interaction of bodies" [11].

There are a few other statements in the same paper worth quoting: "From Mach's Principle it follows, in accordance with the gravitational field equations, that no G-field can exist without matter. It is evident that postulate (c) is strongly connected with the problem of spatio-temporal structure of the Universe as a whole, since all masses participate in creating the G-field" [11].

These words — written a year after the publication of Einstein's first work on cosmology [12] — confirm the opinion of Mc Crea, namely, that it is Mach's Principle to which we owe the existence of relativistic cosmology. "Einstein was apparently [writes Mc Crea] much attracted by Mach's concept that local inertial properties may be determined by the contents of the Universe in the large. He wanted to discover whether this feature is necessarily incorporated in the theory of general relativity. In order to do this he obviously needed to apply the theory to the Universe in the large, that is to construct a cosmological model" [13].

The field equations of General Relativity are partial differential equations; consequently, the G-field cannot be determined solely by them without the imposition of suitable boundary conditions. It seems natural to postulate, with a suitable choice of a reference frame, that at spatial infinity components of the metric tensor g_{ik} ought to tend to their Minkowski's values. From the point of view of Mach's doctrine, however, essential difficulties arise here: "... if we adopt this view, [Einstein himself confesses] we fail to comply with the requirement of the relativity of inertia. For the inertia of a material point of mass m (in natural measure) depends upon g_{ik}: but these differ but little from their postulated values, as given above, for spatial infinity. Thus inertia would be influenced, but would not be conditioned by matter (present in finite space). If only one single point of mass were present, according to this view, it would possess inertia, and in fact an inertia almost as great as when it is surrounded by the other masses of the actual universe" [14].

Introducing the so-called cosmological constant Einstein was able "to regard the universe as a continuum which is finite (closed) with respect to its spatial dimensions" [14]. Thereby the infinity, with all its difficulties implicit in the presence of boundary conditions, was entirely abolished.

This solution would be impossible without assuming a non-zero (positive) value of the cosmological constant, According to Einstein's opinion of those days, the introduction of the cosmological constant had guaranteed two "Machian properties" of the model: (1) "The cosmical constant was related in a simple way to the mean density of matter in the Universe; in a general sense, therefore, it did realize Mach's expectation of a relationship between local physics and the contents of the cosmos" [13]. (2) Field equations for the empty space ($T_{ik} = 0$) have no solutions.

Einstein believed that these two properties are a consequence of the postulate: "there can be no inertia relatively to 'space' but only an inertia of masses relatively to one another" [12]; which may be considered as another formulation of Mach's Principle. As it is well known, the new cosmological solution found by de Sitter [15] almost immediately after *Kosmologische Betrachtungen* were published, had completely destroyed Einstein's illusions. The compromising, anti-Machian property of de Sitter's world is its emptiness, the density of matter in the model appearing to be equal to zero.

The necessary and sufficient condition for a space to be a flat one is that twenty independent components of the Riemann curvature tensor should vanish ($R_{iklm} = 0$), whereas Einstein's field equations for the empty space postulate only ten independent components to vanish ($R_{ik} = 0$). One should therefore expect the local curvature of space-time to be only partially affected by the global distribution of matter. As Bergman writes: "... the gravitational metric field is capable of steering the material bodies immersed in it, but the converse does not hold. The vacuum field possesses degrees of freedom not tied to the motion of any massive sources" [16].

Indeed, it is well known that according to the predictions of General Relativity: (1) a body experiences an acceleration if nearby bodies are accelerated; the accelerating force acts in the same direction as the acceleration of nearby bodies; (2) inside a rotating body a field of Coriolis force is generated. The first effect was stated by Einstein himself [17], the second is known as the Thirring-Lense effect [18]. Both effects are very small, which proves that General Relativity is Machian to a very small degree. To what degree precisely still remains a question and a subject of heated discussions. For details of these discussions the reader will be best advised to refer to the review paper by Reinhardt [19].

6. EINSTEIN VERSUS MACH

Constantly bearing in mind that General Relativity appears to be Machian only in a very limited sense, it may be of interest to relate here the arguments propounded by Sachs [1] which concern certain differences between Einstein and Mach when it comes to Mach's Principle.

(1) Einstein's view of the material universe requires a description in terms of a fundamental set of field equations, whereas Mach regards the system of the world in terms of spatially isolated interacting pieces of matter.

(2) Mach's review remains in agreement with Newton's action-at-a--distance postulate, whereas the concept of action-at-a-distance is incompatible with Einstein's use of the continuous field concept.

(3) In the General Theory of Relativity many coupled fields are mapped into one space-time coordinate system, whereas in Mach's "pieces of matter" system each piece of matter entails a different coordinate system. It is so because every piece of matter is clearly distinguishable in terms of its own space-time trajectory.

(4) According to Mach's doctrine, the Universe forms a "closed system" only with respect to inertial interactions; in Einstein's field approach, a suspicion must arise that not only intertial (or gravitational) fields but also other types of fields (electromagnetic, strong and weak interactions)

must enter this "closed system". Einstein's faith in such an universal interaction (and in the possibility of its geometrization) had motivated all his attempts to find a unified field theory.

7. SOME ATTEMPTS TO INCORPORATE MACH'S PRINCIPLE INTO RELATIVISTIC PHYSICS

An ideology connected with Mach's Principle is still alive in contemporary physics in spite of all previous failures of its realization. There are two main trends in all attempts to incorporate Mach's doctrine into the frame of relativistic physics: (1) one can try to restrict Mach's Principle in such a manner that General Relativity contains it automatically; (2) the other one can try to generalize the General Theory of Relativity in order to include in it Mach's original ideas.

The attempts of the first kind are best exemplified by interpretations proposed by Wheeler and by Sciama and his co-workers respectively.

Wheeler reformulates Mach's Principle understanding it to be "a boundary condition to select allowable solutions of Einstein's equations from physically inadmissible solutions" [20]. Physically admissible solutions are solutions which result from the initial data consisting of a closed and regular 3-geometry, the rate of its change in time, density, and flow of mass-enrgy in this closed and regular hypersurface. Its closure is needed to avoid difficulties with boundary conditions (for the constraints equations), quite analogous to those encountered by Einstein himself in his first cosmological paper. According to Wheeler, the physically admissible solutions "determine the geometry of space-time, past, present and future, and thereby the inertial properties of every inertial test particle" [20, 21].

The Machian program proposed by Sciama and elaborated independently by Lynden-Bell and by Altshuler, and further by Sciama, Waylen and Gilman, is also based on regarding Mach's Principle as a certain selection rule in General Relativity. According to the above-mentioned scholars, only those solutions are Machian which are entirely source--generated. In order to find such solutions one must represent the gravitational potential at any point as the sum of a volume integral representing the contribution from sources outside the volume and from any source-free part of the potential. In every Machian (and, consequently, physically admissible) solution the surface integral must vanish if the considered volume tends to infinity [22].

In both interpretations presented above, Mach's Principle is understood in a very special manner, which has little in common with Einstein's original inspirations, prompting him in the creation of the theory of General Relativity. It is worth to mention that a considerable improvement of Sciama's approach have been recently elaborated by Raine [28].

The best known attempt of the second kind of approach is the Brans--Dicke theory. In this theory the existence of a long-range scalar field (weakly coupled to the trace of the energy-momentum tensor) is postulated. The value of the scalar field at any point is supposed to be determined by an integral over the mass distribution. The ordinary General Relativity may be considered as a special case of Brans-Dicke theory (with the scalar field equal to zero everywhere). It seems highly improbable, however, that a theory could be Machian if any special case of this theory should lead to typically anti-Machian situations [23].

In this paper's selective approach many other attempts of incorporating Mach's Principle into contemporary phisics have necessarily been omitted [24, 25, 26].

8. "THE ELUSIVE PRINCIPLE OF MACH"

The work of Lynden-Bell prompted Mc Crea to criticize the philosophy underlying all interpretations of Mach's Principle [27]. All interpretations postulate certain interconnections — although understood in a different manner — between the space-time geometrical structure and the matter contents of the Universe. Such interconnections are difficult to discuss because "if we try to say what we mean by matter and by space-time we must employ a precisely formulated theory. Then what we really are asking, it seems, is whether in the theory it is sensible to say that what the theory calls matter generates what the theory calls space-time" [27].

Mc Grea explains this idea in a more detailed way: "The entity we here [in General Relativity] call geometry is one single entity that is a model of a considerable amount of phisics. One feature of the geometry gives the result of the operations that an observer in the model would call measuring the density of matter; another feature gives what an observer would find when he says he is observing or measuring the inertia of matter, and so on. But within the strict bounds of usage of the theory it is not sensible to say that there is a region of the model in which no matter is present because the theory does not distinguish between space--time and matter. There is the one inclusive entity and it is not even permitted to say that matter is an aspect of this entity, only that the results of certain operations within the model, that we call density of matter, stress and so on, are aspects of the entity" [27]. Consequently, maintaining that space-time must be caused by the matter "seems to be saying that the model must be caused by the model. It is hard to see what this could mean" [27].

One may, of course, make some approximations in General Relativity, and perhaps it would be possible in some approximative way to assert "that 'matter' having zero density and stress does not contribute to the gravitational field but does contribute to the inertial field. For some metaphysical reasons, physicists (and Dr Berkeley) may regard this as a defect of the theory" [27]. Thus Mc Grea is inclined to talk about "the elusive principle of Mach" [27].

Mc Crea's opinion is not isolated among physicists. However, one has to admit that this "elusive principle" has played a positive role in the development of relativistic physics. As Reinhardt says, concluding his review-paper: "If one decides to bury Mach's principle, as the author is inclined to do, it should be buried with honours, since it has stimulated so much research and thereby produced a gamut of important results" [19].

9. IN PLACE OF CONCLUSIONS

The so-called Mach's Principle resulted from Mach's radically positivistic programme: all physical "entities" ought to be reduced to "pure sensations". Yet the history of science, sometimes produces remarkable surprises for the philosopher. All attempts to create a fully Machian theory, as has been demonstrated, are based on some metaphysical (entirely non-empirical) reason. In the author's opinion, this unexpected fact may be considered as a yet another proof that extreme positivism is very often only a hair-breadth apart from high metaphysics.

FOOTNOTES

[1] M. Sachs, „Positivism, Realism, and Existentialism in Mach's Influence on Contemporary Physics", *Philosophy and Phenomenological Research,* **30,** 403, 1970.

[2] Cf: P. L. Gardiner, *19th Century Philosophy,* The Free Press, New York, 1969, p. 367.

[3] E. Mach, *The Analysis of Sensations* (translated by C. M. Williams), Dover Publ. Inc., New York 1959, passim.

[4] T. S. Kuhn, *Struktura rewolucji naukowych,* PWN, Warszawa, 1968.

[5] L. Geymonat, *Filozofia a filozofia nauki,* PWN, Warszawa, 1966.

[6] E. Mach, *The Science of Mechanics* (translated by T. J. Mc Cormack), Open Court Publishing Comp., Chicago 1902, passim.

[7] H. Goenner "Mach's Principle and Einstein's Theory of Gravitation", *Boston Studies in the Philosophy of Science,* **4,** 200, 1970.

[8] For details of this experiment see: J. M. Cohen, M. D. Cohen, "Mach's Principle and General Relativity", *The Physics Teacher,* 7, 241, 1969.

[9] B. G. Kuznetsov, *Albert Einstein* (Polish translation from Russian), PWN, Warszawa, 1966.

[10] *The Encyclopedia of Philosophy* (ed. by P. Edwards), The Macmillan Comp. and The Free Press, New York-London 1969, vol. 7.

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[12] A. Einstein, "Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie", *Sitzungsber. der Preuss. Akad. d. Wissenschaften,* **1,** 142, 1917.

[13] W. H. Mc Crea, "The Cosmical Constant", *Quart. Jour. of Royal Astr. Soc.,* **12**, 140, 1971.

[14] A. Einstein, *Kosmologische Betrachtungen...,* English translation in: *The Principle of Relativity,* Dover Publ. Inc., p. 176–188.

[15] W. de Sitter, "On the Relativity of Inertia, Remarks Concerning Einstein's, Latest Hypothesis", *Proc. Akad. Wetensch. Amsterdam,* **19,** 1217, 1917.

[16] P. G. Bergman, "Ernest Mach and Contemporary Physics", *Boston Studies in the Philosophy of Science,* **4**, 69, 1970.

[17] A. Einstein, *The Meaning of Relativity,* Princeton University Press, 1955.

[18] See, for instance: M. Heller, "Rotating Bodies in General Relativity", *Acta Cosmologica,* in press.

[19] M. Reinhardt, "Mach's Principle — A Critical Review", *Zeitschrift für Naturforschung,* **28a**, 529, 1973.

[20] J. A. Wheeler, *Geometro-dynamics and the Issue of the Final State,* [in]: *Relativity, Groups and Topology,* New York-London, Paris, 1963, 315.

[21] For critical remarks concerning this interpretation see: M. Heller, "Some Remarks about Two Trials of Mach's Principle Realization", *Acta Phys. Pol.,* **B1**, 123, 1970.

[22] For the history of this interpretation, critical remarks and references to the original papers see: M. Heller, „Dzieje pewnego rozumienia zasady Macha", *Kwartalnik Historii Nauki i Techniki* (in Polish), **19**, 59, 1974.

[23] For critical remarks concerning Machian properties of Dicke-Brans theory, and references to the original papers of Brans and Dicke, see: M. Heller, *art. cit., Acta Phys. Pol.,* **B1**, 123, 1970.

[24] V. A. Wynne, G. H. Derrick, *The Representation of Matter in the General Theory of Relativity,* Il Nuovo Cimento, **17B,** 122, 1973.

[25] M. Sachs, "On Positive-Definiteness of Intertial Mass, Nonpositive-Defeniteness of Electromagnetic Coupling and the Mach Principle from General Relativity", *Il Nuovo Cimento,* **53B**, 398, 1968. There find references to other works of this author.

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