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Jan Łukasiewicz - a Historian of Logic

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JAN ŁUKASIEWICZ—A HISTORIAN OF LOGIC

Professor Jan Łukasiewicz is known, first of all, as an author of the most eminent achievements in the field of logic. His merits in this field enable us to include him into the group of the greatest logicians in the world. It is to be remembered, however, that the scientific activities of Łukasiewicz have considerably surpassed these limits. He was not only a logician in the most exact meaning of the word, but also a philosopher, methodologist and theoretician of science; he paid a lot of attention to the problem of a relation that philosophy bears to science (with particular regard to the question of the application of modern logic in philosophical researches); he was a theoretician of logic (taking a highly active part in the struggle for overcoming psychologism), and a mathematician; he was also a historian of philosophy and an eminent researcher of the history of logic. Therefore, it is the object of this paper to present and discuss briefly the achievements of Jan Łukasiewicz in the scope of the last of the above mentioned fields of his scientific activities.

How much did Łukasiewicz appreciate the researches on the history of logic, it is clearly testified by the fact that the only two book monographs published by him, namely: *On the law of contradiction in Aristotle* (Cracow, 1910) and *Aristotle's syllogistic from the standpoint of modern formal logic* (1st edition Oxford, 1951, 2nd edition Oxford, 1957) were devoted to nothing else but just the problems of the history of logic. Apart from these two books (and it should be underlined here that the first edition of the latter one is, in principle, a reconstruction of the monograph written in Polish and ready by 1939 in which the author discussed the problems of Aristotle's syllogistic and which was destroyed during the war), Łukasiewicz devoted to the history of logic a number of less comprehensive works, this including, first of all, an

excellent paper *From the history of the logic of propositions* published in the "Przegląd Filozoficzny" XXXVII (1934), pp. 417-437; (some of the theses presented in this illustrious treatise were pointed out in the synopsis of a lecture published under the title *On the logic of the Stoics*, *ibid.*, XXXI (1928), pp. 278-279).¹ Łukasiewicz also paid a lot of attention to the problems of the history of logic in several of his works, otherwise devoted to other subjects (for example: *In defense of logistic*, *The importance of logical analysis for cognition*, a review entitled: *Elements of mathematical logic*, and the like ones).

Most obviously, the full discussion of all the achievements of Łukasiewicz resulting from his researches in the scope of the history of logic greatly surpasses the range of the present article; therefore, only a number of them will focus my attention. This restriction—due to the limited volume of the paper—demands that only the most essential of his ideas and discoveries be chosen, namely those that form elements defining a given theory. These will be, for example, the problems that answer the questions such as: what was the logic of the Stoics, and what was the syllogistic of Aristotle? in what way was an interrelation between these theories understood in the course of the history?, etc. The investigations of Łukasiewicz changed in an important way the picture of the history of logic, especially in the scope of such questions. At the same time, while examining these important questions, Łukasiewicz made a lot of interesting and minute discoveries. Detailed results of his researches on the history of logic include, for example: a solution to the problem of the so-called fourth syllogistic figure of Galenus; an original interpretation of the syllogistic necessity in Aristotle; a formulation of some interesting intuitions resulting from the analysis of the so-called proofs through ectesis in Aristotle; a statement that the so-called De Morgan's laws were known to the Mediaeval logicians, etc.

In his researches devoted to the history of logic, Łukasiewicz was mainly occupied with the following three subjects: 1. The law of contradiction in Aristotle; 2. The history of the logic of propositions; 3. Aristotle's syllogistic. In the course of the article I will try to present briefly some of his opinions on the above mentioned subjects, following the order settled above. According to the suggestions made by Łukasiewicz in his works, the last two subjects will be combined into one whole. Therefore, my further remarks will be divided into two main parts. In the first one I will discuss the opinions of Łukasiewicz present-

¹ A German version of this treatise entitled *Zur Geschichte der Aussagenlogik* was published in the journal "Erkenntnis" 5 (1935-1936), pp. 111-131.

ed in his work *On the law of contradiction in Aristotle*. In the second part I will present the main results of his researches on the history of formal logic, the logic of propositions and syllogistic. This division of subject matters seems to be relevant to the necessary distinction made between the two periods in the activities of Łukasiewicz, namely: the philosophical period and the logical one. In spite of the fact that the two trends observed in the researches of Łukasiewicz had never really been separated and that he was always a logician and a philosopher at the same time, yet at the very beginning of his scientific career the philosophical attitude had been prevailing, while in the second period, that is, starting from the twenties, his philosophical interests receded into the background giving place to purely logical considerations including also the history of logic. The work *On the law of contradiction in Aristotle* belongs to the first period of Łukasiewicz's creative work (the law of contradiction is considered by Łukasiewicz mainly in terms of philosophy), while the works on the history of the logic of propositions and Aristotle's syllogistic represent the second period of his activities.

LAW OF CONTRADICTION IN ARISTOTLE

In his work on the law of contradiction in Aristotle, Łukasiewicz is occupied with three questions: 1. How does Aristotle formulate this principle and what are the types of relations that he can distinguish between the particular formulations? 2. What are, according to Aristotle, the main attributes of this law? 3. What value have the arguments by means of which the Stagirite tries to justify the law of contradiction and its leading role in philosophy? (Of course, here I refer only to the historical aspect of the work of Łukasiewicz.)

1. Answering the first question, Łukasiewicz is of the opinion that Aristotle gives three different formulations of the law of contradiction, namely: ontological, logical and psychological. The ontological formulation is given by the Stagirite, among others, in the text: *Met.*, T3, 1005b, 19–20: "The same cannot be both attributed and not attributed to the same and in the same respect."² After discussing the terminology, Łukasiewicz replaces this verbal translation of Aristotle's text with the following statement: "There is no such object that could possess and not possess at the same time the same characteristic."³

After Łukasiewicz, a logical formulation of the law of contradiction

² Cf. J. Łukasiewicz, *On the law of contradiction in Aristotle*, p. 9.

³ *Ibid.*, p. 10.

is contained in the text: Met., T6, 1011b, 13–14: “And it is the most undeniable of all the rules that the contradictory propositions cannot be true at the same time.”⁴ After discussing the terminology, Łukasiewicz replaces this formulation of Aristotle with the following definition: “Two propositions of which one attributes to an object the very characteristic that the other refuses to attribute to it cannot be true at the same time.”⁵

A psychological formulation of the law of contradiction is—according to Łukasiewicz—contained in the text: Met., T3, 1005b, 23–26: “Nobody can believe that the same be present and not present at the same time, as some claim was said by Heraclitus, because he who is speaking needs not believe in what he says.”⁶ Having determined that the original Greek word *ὑπολαμβάνειν* does not mean (in this particular case) ‘to accept’, that is ‘to suppose’, but ‘to believe’ or ‘to be convinced,’ Łukasiewicz replaces this formulation of the psychological law of contradiction with the following statement: “Two judgements corresponding to two contradictory propositions cannot exist in the same mind at the same time.”⁷

Aristotle did not give a definition of the semantic differences which occur between these three formulations of the law of contradiction, and yet Łukasiewicz is of the opinion that he was fully aware of these differences. This is confirmed by the remarks of the Stagirite concerning the interrelations by means of which different formulations of the law of contradiction are combined together. Łukasiewicz thinks that all the enumerated formulations are semantically different from each other, that is, they express different rules. Most important among them is the ontological law of contradiction which for the Stagirite is the law of contradiction *κατ'ἐξοχήν*.⁸ This principle is equipollent to the logical law of contradiction as both these rules result from one another due to the very definition of truth,⁹ but still the equipollency is only of a logical character, ad it is not a real *equipollency* (underlined by Łukasiewicz—TK). Says Aristotle: (Met., T10, 1051b, 6–9): “You are not white because we speak the truth calling you white, but because you are white we—by saying this—speak the truth.” Basing, among others, on this text, Łukasiewicz gives the following interpretation of the Aristo-

⁴ Ibid.

⁵ Ibid., p. 11.

⁶ Ibid.

⁷ Ibid., p. 12.

⁸ Ibid., p. 13.

⁹ In the verbal translation this definition reads as follows: “To speak about the existing that it exists, and about the non-existing that it does not exist, this is the truth.” Cf. Met., T 7, 1011b, 26–26.

telian justification for distinguishing the ontological law of contradiction: "... being is both a sufficient logical condition for the truth of propositions as well as a real cause of making these propositions, while the truth of the propositions is only a sufficient logical condition and not the real cause of being." ¹⁰

Łukasiewicz is of the opinion that Aristotle was not discussing the psychological law of contradiction on a level equal with the ontological and logical rules as he thought that the latter ones are absolutely primary and uncontestable. On the other hand, he tried to prove the psychological law of contradiction basing on the logical or ontological rules. This argumentation was interpreted by Łukasiewicz in the following way: "If two judgements corresponding to contradictory propositions existed in the same mind and at the same time, then this mind would possess at the same time contradictory characteristics. It follows, however, from the logical law of contradiction that there is no such object which could possess at the same time contradictory characteristics, and therefore, two judgements corresponding to contradictory propositions cannot exist in the same mind at the same time." ¹¹ Łukasiewicz criticizes this argumentation very severely. It is not possible now to go into this subject at some length, and therefore I wish to underline only some of the objections, namely the statement that says that in his proof of the psychological law of contradiction Aristotle confuses the logical and psychological problems. According to Łukasiewicz, this proof is the first symptom of psychologism in the history of logic. ¹² The psychological law of contradiction is—as claimed by Łukasiewicz—an empirical theorem, and as such it cannot be proved by way of deduction. It is possible to look for the justification a posteriori, and this means that it can be regarded only as a probable theorem. Łukasiewicz doubts, however, whether it is possible to ascribe to this law even that quality. He would rather consider it to be false. He thinks that it is possible to imagine some situations in which the contradictory propositions might be accepted. For example, some acts of the religious faith may be of this nature.

As I am not able here to discuss this opinion of Łukasiewicz in a more detailed way, or at a greater length, I only wish to remind the readers that the problem of a relation between psychology and logic, and therefore that of the psychological law of contradiction, remains open for discussion all the time. It has also been taken into consideration in Poland. In his book published in 1971 and devoted to the

¹⁰ Łukasiewicz, *op. cit.*, p. 19.

¹¹ Łukasiewicz, *op. cit.*, pp. 24–25.

¹² *Ibid.*, pp. 38 and 156.

problem of a relationship between the semantics and pragmatics, Dr. L. Koj accepts the assumptions describing graphic relations of denotation such that they result in a psychological version of the law of contradiction.¹³ It should be underlined that Łukasiewicz has never really abandoned his idea of the psychological law of contradiction. He spoke about it very explicitly once again during his Underground University lectures delivered in Warsaw in July 1942.¹⁴

2. The problem of attributes ascribed by Aristotle to the law of contradiction was restricted by Łukasiewicz to an ontological and logical formulation of this rule. Łukasiewicz paid a lot of attention to this problem but most of his analyses seem to have lost their value by now. Therefore, I think that while discussing this subject it will be sufficient to give only a few relevant remarks.

Aristotle considered the law of contradiction the most valid ($\beta\epsilon\beta\alpha\iota\omicron\delta\acute{\alpha}\theta\eta$) and most obvious (the most cognizable — $\gamma\nu\omega\rho\iota\mu\omega\delta\acute{\alpha}\theta\eta$) of all the rules, as well as uncontestable and absolutely primary which out of its very nature ($\phi\acute{\upsilon}\sigma\epsilon\iota\ \acute{\alpha}\rho\chi\acute{\eta}$) is a basis for all other axioms.¹⁵ Łukasiewicz argues these attributes. He states, for example, that the law of identity is simpler, more obvious and more basic, i.e. logically anterior, than the law of contradiction.¹⁶ Of course, this is what Łukasiewicz said in his work published in 1910. He would certainly not make this statement several years later. It would be quite impossible for such an eminent logician not to know that both these laws are simply equipollent on the basis of the classical calculus of propositions, the equipollency resulting from a thesis: $(p \supset q) \equiv \sim(p \sim q)$, substituting q/p .

In his attempts to impair this opinion of Aristotle according to which he ascribed the above mentioned attributes to the law of contradiction, Łukasiewicz made references to some idealistic philosophical theories which—according to him—allow for an inconsistency, namely Hegel's philosophy and Meinong's theory of objects. Most certainly, Łukasiewicz gave up these arguments later on.

The main argument of Łukasiewicz in this question is a rather strange and apparently false opinion concerning the nature of the definition, and in particular the definition of a true proposition. Łukasiewicz states that although the law of identity is more basic than the law of contradiction, yet even this rule is not final because it is not

¹³ Cf. L. Koj, *Semantics vs. pragmatics*, Warszawa 1971, pp. 36–39.

¹⁴ Notes of those lectures were made and published by M. Bizoń under the title *Genesis of logic*, Katowice, 1947. The problem in question is discussed on pages 69–70.

¹⁵ *Met.*, T 6, 1011b, 13–14; T 3, 1005b, 12–23; T 3, 1005b, 32.

¹⁶ Łukasiewicz, *op. cit.*, pp. 43–49.

the statement true by itself. Only definitions are true by themselves because in the definitions we express—either openly or in a concealed manner—a singular fact corresponding to the words such as: “I define,” “I determine,” “I call,” “I understand.” As such, every definition is a singular judgement as it expresses an individual act of its creation. And because it refers to “the fact which is created together with it and is contained in it,” therefore it is always true.¹⁷ It does not mean, however, says Łukasiewicz, that every definition is a rule or final premiss. Only one definition is of this character, namely the definition of a true proposition. All other definitions are based on it because they bear the notion of truth. Apparently ultimately, the law of contradiction seems also to be based on this definition (which does not mean that after Łukasiewicz the law of contradiction results from the definition of truth).¹⁸ This extraordinary point of view requires no detailed critical analysis. I think that it is enough to ask a question: is it possible to have only one definition of the true proposition? How could we justify an affirmative answer to this question? And I am sure that with the theory of definition explained here only a negative reply to this question is possible which, in turn, creates a lot of possibilities for the cognitive relativism. It should be stressed here that Łukasiewicz has never come back to this opinion.

3. I can say only a few words about the very penetrating and detailed criticism of the arguments by means of which Aristotle tried to prove the law of contradiction. First of all, let me stress the fact that in his critical observations Łukasiewicz seems to pay little attention to the problem of inconsistency which can be formulated in the following way: Aristotle considers the law of contradiction to be absolutely uncontestable and primary, while at the same time he tries to prove this very law. And it is of no importance to Łukasiewicz whether this law be really primary or not; what is important to him is only if it is true and firm, and therefore he would be glad to see any relevant proof of this rule. The critical analysis has shown, however, that Aristotle's arguments are not these proofs as they contain some very essential logical misconceptions, such as: *petitio principii*, *ignoratio elenchi* and the formal error, that is, *non sequitur*.¹⁹

The objections of Łukasiewicz concerning Aristotle's arguments in

¹⁷ Ibid., p. 52.

¹⁸ Ibid., pp. 53–55.

¹⁹ It is believed that in his “proofs” of the law of contradiction Aristotle applies the following false logical thesis: $\sim SaP \rightarrow SeP$. Cf. I. M. Bocheński, *Ancient Formal Logic*, Amsterdam, 1951, p. 36.

question are right but only with an assumption that for the Stagirite this argumentation was really and truly a demonstration in the full meaning of this word. Such an assumption seems, however, to be quite false. Aristotle is not proving anything in the sense of his scientific ἀπόδειξις, he merely performs some verbal-logical actions by means of which he wants to obtain an approval of the auditory (interlocutors, readers, listeners) when he is referring his point of view or when he is compelled to give up an idea which he thinks to be false. And to realize an objective of this type it is not absolutely necessary to ensure full consistency of reasoning. The Stagirite knows that in his case the fulfilment of such a condition would not be possible; as we know his principle is βεβαιοδάθη and γνωριμοδάθη in an objective sense: τῇ φύσει of these words which does not necessarily mean that it is so in the subjective meaning of these words (πρὸς ἡμᾶς). It may always happen so that somebody does not understand not only this principle but all that he confirms, sometimes against his personal convictions; hence the necessity for mental and verbal actions which force reflections and provoke a mental effort. This is—I think—the very aim that Aristotle wanted to achieve when he was arguing in favour of the law of contradiction. He quoted no proofs in the full sense of the word, i.e. the proofs which—similarly as a definition of truth—would attack the domain of semantics, but he gave some arguments which are to function in the pragmatic domain of a graphic relationship.

To complete the task of presenting the ideas of Łukasiewicz expressed in his work *On the law of contradiction in Aristotle*, I wish to make a few critical remarks. It seems that in an evaluation of this type two main aspects should be covered, namely the substantial aspect and the historical one. The substantial value of this book mainly consists in that the profound and versatile analyses presented in it were helpful in overcoming the false conviction which claimed that the law of contradiction is an axiom of Aristotle's logic in the same sense as the law of parallels is an axiom in the geometry of Euclides. By the way, Łukasiewicz himself was of the same opinion at the time when he was writing his book and therefore he did not attack this standpoint expressis verbis.

The most valuable historical discovery described in this book seems to be a differentiation made between the three formulations of Aristotle's law of contradiction, notwithstanding the fact that it has quite often been criticized. For example, I. M. Bocheński says that the texts of the Stagirite include only two formulations of the law of contradiction, namely the logical formulation and the metalogical one. The logical formulation corresponds to the ontological one in Łukasiewicz, while the

metalogical formulation includes what Łukasiewicz considers to be logical and psychological formulations. In his polemics with Łukasiewicz, Bocheński asserts that it is hardly probable that Aristotle might have formulated the psychological law of contradiction.²⁰

I think that it is Łukasiewicz who is right. The ontological law of contradiction in Aristotle is not equipollent to any of the modern logical theses just labelled with this name. In the context in which the ontological law of contradiction is discussed, the Stagirite clearly speaks about being and its important characteristics (cf. the beginning of the 1st Book of *Metaphysics*). Inconsistency is this very important characteristic of being, and I would rather think that Aristotle regards this characteristic first of all as a law, and only in the second place he takes into consideration its verbal embodiment. Whether it be this way or the other, this verbal formulation is for the Stagirite an ontological theorem (and even a metaphysical one, since inconsistency is, first of all, an attribute of the real being).

Łukasiewicz is also right if he detects in Aristotle a formulation of the psychological law of contradiction. In the text which according to Łukasiewicz contains this formulation, the Stagirite clearly differentiates between the word *ὑπολαμβάνειν* 'to believe' and the word *λέγειν* 'to speak.' He is fully aware of the difference that exists between the mental act of conviction and the truth of a proposition. The arguments used by Bocheński to question the possibility of formulating by Aristotle the psychological law of contradiction are not very convincing.

MOST IMPORTANT IDEAS OF ŁUKASIEWICZ IN THE FIELD OF FORMAL LOGIC

As I have already mentioned, in the field of formal logic the researches of Łukasiewicz were mainly concerned with two subjects, that is: the logic of the Stoics and Aristotle's syllogistic. Of course, both these themes were strictly combined with each other and his main achievement consisted in determining the differences that occur between these logical theories. Therefore, further discussion will assume the following course: 1. I shall compare—after Łukasiewicz—the Stoics' logic and Aristotle's syllogistic; 2. I shall present some of the researches made by Łukasiewicz and directly related to the logic of the Stoics; 3. I shall discuss some of his ideas resulting from the investigations into Aristotle's syllogistic, excluding modal logic.

²⁰ Ibid., pp. 38–39.

1. The theorems in which Łukasiewicz determined the difference between the Stoics' logic and Aristotle's syllogistic meant, at the time when the author published his work for the first time, a real revolution and a sensational discovery. It is enough just to think that the historians of logic, who had been writing their works before Łukasiewicz, were entirely unaware of the difference that occurs between the logic of propositions and the logic of predicates.²¹ And it was Łukasiewicz who discovered that the variables which occur in the syllogisms of the Stoics are proposition variables, while the variables encountered in Aristotle's syllogisms are predicate variables. Therefore, the logic of the Stoics is a primary form of the logic of propositions, while Aristotle's syllogistic is a fragment of the logic of predicates. This is the first and principal difference between these two theories.²²

The second difference consists in that Aristotle's syllogisms are logical theses in the form of implications, while the syllogisms of the Stoics are inferential schemes or inferential rules. At the same time, Łukasiewicz confirms that the Stoics were familiar with the formula which enables a transition from the scheme to a thesis. The formula is based on a difference between the so-called valid and invalid conclusions. The conclusion about premisses α and β and inference γ is valid if the implication, the antecedent of which is a conjunction of premisses and the consequent is an inference, is true. If such an implication is false, the conclusion is not valid.²³ So, the Stoics were fully aware of the difference between a logical thesis and a scheme of reasoning.

The third difference between Aristotle's syllogistic and the Stoics' logic consists, according to Łukasiewicz, in that the first of these theories is only formal while the second one is formalistic. In the application of his syllogisms, Aristotle was referring to a meaning of the terms contained in these syllogisms, while the rules of reasoning applied by the Stoics allowed for abstractions from the meaning and were of a structural and descriptive character.²⁴

Concluding this comparison of the Stoics' logic and Aristotle's syllogistic, Łukasiewicz asks one question: did the Stoics know anything about the importance of their logic of propositions, and—in particular—were they aware of the fact that they were creating a logical system

²¹ Cf. J. Łukasiewicz, *From the problems of logic and philosophy*, Warsaw, 1961, p. 179.

²² *Ibid.*, p. 180.

²³ *Ibid.*, p. 181.

²⁴ Cf. J. Łukasiewicz, *Aristotle's syllogistic from the standpoint of modern formal logic*, Oxford, 1957, pp. 18–19.

quite different from the Aristotelian one? Basing on some texts of Alexander and Galenus, Łukasiewicz answers the question affirmatively.²⁵

2. The dialectic of the Stoics is a two-way logic. It uses the following functions of truth: negation (which according to the Stoics should definitely be placed before the negative proposition) and implication which they adopted from Philonus. They also used disjunction which at first they (Chrysippus) most probably understood as the one which excludes a combination of two propositions (Chrysippus), and then—following the definition: “if not p , then q ”—as an alternative. Apart from the material implication of Philonus, the Stoics also knew the implication of Diodorus which Łukasiewicz considers to be an ancient correspondent of C. J. Lewis’ close implication.²⁶

The ideas of Łukasiewicz concerning the way of understanding particular functions of truth and of their number were completed by his successors and some of their aspects have been revised. B. Mates, who has examined this problem with great care, assures that the Stoics were familiar with not two but four different implications, namely: the implications of Philonus, Diodorus, Chrysippus and the fourth one, the validity of which was defined in the following way: “...the conditional proposition is true if its consequent is to some extent contained in its antecedent.”²⁷ According to this definition all the implications of identity were thought to be false.²⁸

Contrary to Łukasiewicz, Mates states that Diodorus’ implication is different from Lewis’ close implication which corresponds to Chrysippus’ implication. The criterion of truth for Diodorus’ implication is determined in the following way: the true conditional proposition is the one which cannot and could never have a true antecedent and a false consequent. On the other hand, the criterion of truth in Chrysippus’ implication reads as follows: a conditional proposition is true when the negation of its consequent is inconsistent with its antecedent.²⁹ Basing on thus defined criteria of truth, I. M. Bocheński gives the following definitions for the implications of Diodorus and Chrysippus:

$$(1) p \rightarrow q = \text{Df. (t). } p(t) \supset q(t)$$

$$(2) p \Rightarrow q = \text{Df. } \sim (p \sim q).$$

²⁵ Id., *From the problems of logic and philosophy*, pp. 187–188.

²⁶ Ibid., pp. 182–183.

²⁷ Cf. B. Mates, *Logic of the Stoics*. Polish version translated by A. Kruk, ATK, Warsaw, 1971, p. 73.

²⁸ Cf. Sextus Emp., *Hyp. Pyrrh.*, II, 110.

²⁹ Mates, *op. cit.*, p. 85.

The second one is evidently identical with Lewis' definition of close implication.³⁰

B. Mates also questions the idea of Łukasiewicz according to which the Stoics understood disjunction as an inseparable alternative. In his opinion it is not justified to quote the definition "if not p , then q " because the text upon which it is based should rather be understood as a starting point for another definition, namely: " $\sim p \equiv q$."³¹ Mates is convinced that the Stoics knew the notion of alternative but in the reference materials no explicit definition of this conjunction can be found.

Formulating their logic in the form of a deductive system, the Stoics adopted five inference schemes without proofs and they reduced the other schemes to these five. The distinguished schemes are as follows:

- (I) If p , then q ; so p , and therefore q .
- (II) If p , then q ; so not- q , and therefore not- p .
- (III) Not both p and q ; so p , and therefore not- q .
- (IV) Either p or q ; so p , and therefore not- q .
- (V) Either p or q ; so not- q , and therefore p .³²

Łukasiewicz is of the opinion that the logic of the Stoics was further developed by the Mediaeval logicians, among others, by Peter the Spaniard and Duns Scotus in their researches on the science of consequences. An example of the Mediaeval discoveries in this respect is the so-called law of Duns Scotus and the theorems which some centuries later were called De Morgan's laws.³³

3. Aristotle's logic, and in particular his syllogistic of assertive propositions, was one of the main subjects of the research works carried out by Łukasiewicz. The investigations which he made on this subject resulted in the fact that he considered false all the so-called traditional interpretations of syllogistic represented, among others, by the works of Prandtl, Zeller and Maier. According to Łukasiewicz, the main mistake of these authors consisted in that they did not understand the Stoics' logic and—consequently—they could not understand the main difference between this logic and Aristotle's logic. Neither could they conceive of Aristotle's logic although they thought themselves to be its advocates and experts. This last statement is confirmed by the fact that these authors could not differentiate between the original Aristotelian form of syllogism, which is an implication, and the so-called traditional syllogism which is a scheme of inference.³⁴ The most comprehensive presentation

³⁰ Bocheński, *op. cit.*, p. 90.

³¹ Mates, *op. cit.*, p. 85.

³² Łukasiewicz, *From the problems of logic and philosophy*, p. 184.

³³ *Ibid.*, pp. 188-190.

³⁴ *Ibid.*, pp. 179 and next.

of the results of his researches on Aristotle's syllogistic is contained in Łukasiewicz's book entitled *Aristotle's syllogistic...* and the concise summary of his ideas which I wish to present in course of this paper is based precisely on this book.

The Aristotelian theory of syllogism of assertive propositions is—according to Łukasiewicz—a system of true theorems referring to constants, A, E, I, O, in the meaning ascribed to these symbols by the Mediaeval logic. These symbols represent the relations in which the elements are only general terms such as “man” or “animal.” The theory does not account for unit and empty terms. Constants A, E, I, O, are the only specific constants in the theory while its variables are symbols representing general terms. Apart from the variables and the specific constants, the theorems of syllogistic include two constants which belong to the language of the logic of propositions, and the logic of propositions, being obviously a more fundamental theory, forms implicitly a basis for the theory of syllogism which, in turn, being a logic of predicates, is logically a later theory. These non-specific constants of syllogistic, which belong to the language of the logic of propositions, are functors of the implication “if ..., then ...” and of the conjunction “and.”³⁵

The above mentioned functors of the logic of propositions play a very important role in the language of syllogistic due to the fact that the Aristotelian syllogisms are—after Łukasiewicz—logical theses in the form of an implication having for their antecedent a conjunction of premisses, and for their consequent—a conclusion. Due to the fact that they are theses in the form of an implication, Aristotle's syllogisms differ from the traditional syllogisms which are schemes of inference.³⁶ All the explicitly formulated theorems of syllogistic are implications. An exception might only be two laws of identity: “A belongs to every A” and “A belongs to certain A” which were not formulated expressis verbis by the Stagirite but were known to peripatetics.³⁷

The theses of syllogistic, which were explicitly formulated by Aristotle and were particularly carefully discussed by him, are, first of all, three laws of conversion and fourteen syllogisms mentioned in the initial chapters of the *Primary Analytics*. The syllogisms were divided by the Stagirite into three figures. The syllogisms of the fourth figure were not mentioned in that place but Aristotle knew of their existence and later on Theophrast included them into the first figure.

Łukasiewicz is of the opinion that the division of syllogisms into

³⁵ Cf. Łukasiewicz, *Aristotle's syllogistic ...*, pp. 77-78.

³⁶ *Ibid.*, pp. 1-3; 20-23.

³⁷ *Ibid.*, p. 20.

figures is not, nor was it for Aristotle, of any substantial importance. It is only a technical operation which makes easier full enumeration of the correct syllogisms.³⁸

Much more important seems to be a division of syllogism into perfect and imperfect; the division in which Łukasiewicz can see a formulation of the theory of syllogism in the form of an axiomatic system. Perfect syllogisms are the syllogisms of the first figure. The syllogisms of the remaining figures are imperfect. Łukasiewicz thinks that Aristotle considered perfect syllogisms to be axioms of the systems, while the reduction of the syllogisms of the remaining figures to the syllogism of the first figure proves the syllogisms which are not axioms. At first, Aristotle accepted as axioms all the four systems in the first figure. Next, he reduced this list of the axioms included into his theory to two syllogisms, accepting as assumptions only syllogisms *barbara* and *celarent*. According to Łukasiewicz, this fact requires special attention as it is the first case in the history of logic when the postulate of accepting the least possible number of assumptions was realized. Carrying into effect (*de facto*) this postulate, Aristotle was a precursor of the similar trend in modern logic.³⁹

Aristotle applies three methods of argumentation (reduction) of syllogisms basing on the accepted axioms, i.e. argumentation through conversion, argumentation through reduction to absurdity and argumentation through *ectesis*. The simplest are the proofs of the first type. In these proofs Aristotle uses the laws of conversion and he intuitively applies some rules of the logic of propositions.

Applying reduction to absurdity, Aristotle proves the syllogisms *baroco* and *bocardo*. After a critical analysis of these proofs, Łukasiewicz interpreted them as proofs based on the laws of extended transposition.⁴⁰

Proofs through *ectesis* (ἐκτεσις 'distinction') play no important role in this system but, according to Łukasiewicz, they are interesting by themselves, irrespective of their importance in the syllogistic system.

The formulation of syllogistic into an axiomatic-deductive system is considered by Łukasiewicz one of the greatest achievements of Aristotle in the field of logic. The system is, however, not free from some imperfections and drawbacks. One of them is its incompleteness. The axioms (perfect syllogisms) accepted by Aristotle are not sufficient for deducing all the laws included into his theory of relation: A, E, I, O. It is not possible to derive even these laws that the Stagirite formulated

³⁸ Ibid., p. 23.

³⁹ Ibid., p. 46.

⁴⁰ Ibid., pp. 55-58.

himself such as, for example, the laws of the conversion of propositions A and E and some laws of the so-called 'logic square'. Next, it was not stated clearly what are the theses of the logic of propositions which were used by Aristotle in his proofs *implicite*. The Stagirite did not make a division of the specific terms of logic (that is the terms denoting relations, A, E, I, O) into the primary and secondary ones. Łukasiewicz is, however, of the opinion that the system in question can be corrected on the basis of modern logic without any deviations from what was intended by its creator.

Applying the intuitions expressed in the respective texts of the Stagirite, Łukasiewicz creates a syllogistic system satisfying the requirements of modern logic. While recording the theorems and proofs, he uses his brackets-free symbols. As specific constants of the syllogistic language he accepts the above mentioned symbols, i.e. "A", "E", "I", "O", while the variables are denoted with the first small letters of alphabet "a", "b", "c", "d", etc. Using these symbols, he marks the elementary propositions of the theory in the following way: "Aab" = "each a is b", "Eab" = "no a is b", "Iab" = "certain a is b", "Oab" = "certain a is not b". Apart from this, he accepts the following definitions: "Eab" = "NIab", "Oab" = "NAab". In the formulae of the theorems he uses three functors of the logic of propositions, namely: "C" (implication), "N" (negation) and "K" (conjunction).

The set of all the assumptions in this system is divided by Łukasiewicz into two systems, namely: A. assumptions of assertion, and B. assumptions of rejection. To include into the system both these mental operations is in agreement with Aristotle's point of view; he not only proved the theses of his syllogistic but also justified the rejection of incorrect formulae (propositional functions).

A. Assumptions of assertion:

a) Axioms

- | | |
|-----------------|---|
| (1) Aaa | } laws of identity which were not
explicitly formulated by Aristotle |
| (2) Iaa | |
| (3) CKAbcAabAac | (<i>barbara</i>) |
| (4) CKAbcIbaIac | (<i>datisi</i>) |

The syllogistic system, constructed on the basis of axioms (1), (2), (3), (4), assumes—as a basis—the implicative-negative calculus of propositions in which the symbols of implication "C" and negation "N" are primary terms. The third functor of the logic of propositions, used in the denotation of the syllogistic theses, is introduced through the definition: "Kpq" = "NCpNq." ⁴¹

⁴¹ Ibid., p. 88.

b) Primary rules of assertion:

(1) two rules of substitution based on the above mentioned definitions of expressions "E" and "O".

(2) rule of substitution: If α is an expression accepted in the system, then every expression obtained from α by its correct substitution is also an expression accepted in the system. A correct substitution consists in substituting for predicate variables, such as for example a, b, c , other variables included into the same category.

(3) rule of detachment: If $C\alpha B$ and α are expressions accepted in the system, then B is also a theorem accepted in the system.

B. Assumptions of rejection:

Aristotle was applying two procedures for the rejection of false syllogistic moods. The first one consists in selecting such variables the substitution of which gives a syllogism with true premisses and a false conclusion. The second way consists in proving that if some syllogistic moods are rejected, then the other syllogistic moods must be rejected, too.⁴² To formulate his own assumptions determining the procedure of rejecting false syllogistic moods, Łukasiewicz utilizes the ideas included in the second method applied by Aristotle to reject false syllogistic moods. The first arrangement of these assumptions is as follows:

a) Axioms of rejection (theorems rejected axiomatically):

(1*) $CKAc bAabIac$

(2*) $CKEcbEabIac$.⁴³

b) Rules for rejection:

(a*) Rule of detachment for rejection:

If implication "if α , then B " is accepted but its consequent B is rejected, then its antecedent α must be rejected, too.

(b*) Rule of substitution for rejection:

If α is substituted for B and α is rejected, then B must be rejected, too.⁴⁴

The aforesaid assumptions of assertion are sufficient (as proved by Prof. J. Śłupecki⁴⁵) to prove all the true theorems that can be formulated in the language of syllogistic. The assumptions of rejection, are not, however, sufficient. The system will become fully determinable when we accept one more rule of rejection which Łukasiewicz calls Śłupecki's

⁴² Ibid., pp. 68-71.

⁴³ Ibid., p. 96.

⁴⁴ Ibid., pp. 71-72.

⁴⁵ Cf. J. Śłupecki, *From the researches on Aristotle's syllogistic*, Wrocław, 1948 (mainly Chapter V); Łukasiewicz, *Aristotle's syllogistic ...*, pp. 100-101.

principle. A formulation of this principle should be preceded with the following conventional agreement concerning the way of understanding the term "elementary expression of the syllogistic language": elementary expressions of the syllogistic are all the simple expressions such as Aab , Eab , Iab and Oab and all the expressions such as Ca_1 , Ca_2 , Ca_3 , ... $Ca_{n-1}a_n$, where all a 's are simple expressions.

Applying this agreement, we formulate Słupecki's principle in the following way:

If a and B are simple negative terms and γ is an elementary term, then if terms $Ca\gamma$ and $CB\gamma$ are rejected, term $CaCB\gamma$ must be rejected, too. This rule refers to but is not identical with the common metalogical principle: *utraque si pressa neget nil inde sequetur*.⁴⁶

An inclusion of Słupecki's principle into the above accepted assumptions of rejection not only makes the syllogistic system determinable but also enables a rejection through argumentation of formula (2*). In this way, the list of the syllogistic formulae rejected axiomatically finally contains only one item, viz. formula (1*).

To finish this very incomplete presentation of the ideas of J. Łukasiewicz as a historian of logic, I wish to stress once again that these opinions have played a very important role in modern researches on the history of logic. Of course, the greatest interest of the experts investigating into this subject was aroused by the book entitled *Aristotle's syllogistic ...* which is a most comprehensive lecture stating the results of the researches that Łukasiewicz carried out in the history of logic. This utmost interest found its expression, among others, in numerous publications devoted either directly to this first modern monograph on Aristotle's syllogistic (several tens of critical reviews and papers), or indirectly inspired by it. Now, attention should be drawn to some questions which were raised in these publications and which have still remained open to discussion.

ARISTOTLE'S SYLLOGISTIC

One of the controversial opinions of Łukasiewicz is his statement that Aristotle was the first one in the history of science to apply variable symbols in the formulation of the respective theorems and that this was one of his greatest achievements. This idea is contradicted, among others, by J. L. Austin who thinks it to be an evident exaggeration. Austin opts rather for Whitehead's formulation who is of the opinion that Aristotle and his successors were approaching the concept of logic-

⁴⁶ Cf. Łukasiewicz, *Aristotle's syllogistic ...*, p. 103.

al variables without really discovering it.⁴⁷ Personally, I think that this point of view is not justified and the statement of Łukasiewicz is not an exaggeration at all. Łukasiewicz asserts (and he is right) that Aristotle was the first one to introduce variables but this does not mean that Aristotle determined all their functions. On the contrary, Łukasiewicz underlines that Aristotle was not aware of all the possibilities resulting from the derivation of the variables; the Stagirite understood, however, their application so well that he could make a clear distinction between the formal thesis of syllogistic and its definite substitution.

Another point quite often argued is the statement that Aristotle formulated his syllogisms only as theorems assuming the form of an implication. The main objections and doubts concerning this idea can be summarized in the following way: 1° Aristotle is not fully aware of the difference that exists between the logical thesis and the rule of inference.⁴⁸ 2° In Aristotle's texts on logic it is possible to find out some examples of the syllogisms which are conclusions and not implication theses.⁴⁹

Ad 1°. I think that the first of these statements may be considered right. This does not mean, however, that the fact that the Stagirite was really formulating syllogisms in the form of implication theses and not in the form of inference schemes is not of a vital importance for the interpretation of Aristotle's syllogistic. I am convinced that this fact fully justifies the preference given to the interpretation of Łukasiewicz. This interpretation is additionally supported by the fact that the Stoics were already aware of a difference existing between the logical thesis and the inference scheme (this being also acknowledged by the opponents of Łukasiewicz⁵⁰), and that they chose the latter one as standing in distinct opposition to peripatetics.

Ad 2°. This statement should also be considered right. It is true that Aristotle sometimes formulates syllogisms in the form of inferences or, at least, he quotes the data which enable the construction of such syllogisms. These examples do not belong, however, to the theory of syllogism itself but they either represent certain form of its application,⁵¹ or are not included into it at all. The latter case refers, for ex-

⁴⁷ Cf. J. L. Austin, a review of the book: *Aristotle's Syllogistic from the Standpoint of Modern Formal Logic* by Jan Łukasiewicz, "Mind" 61 (1952), p. 397; A. W. Whitehead, *The Organisation of Thought*, "Proceedings of the Aristotelian Society," New Series, XVII (1916-1917), pp. 72-73.

⁴⁸ Cf. Austin, *op. cit.*, p. 398; O. Becker, a review of *Aristotle's syllogistic ...*, by Łukasiewicz, "Gnomon" 24 (1952), p. 508; W. Kneale, a review of *Aristotle's syllogistic ...*, by Łukasiewicz, "Philosophy," London, (1952), p. 291.

⁴⁹ E. Tielsch, *The genuine Aristotelian Syllogisme* von Łukasiewicz, "Philosophia Naturalis," Meisenheim/Glan, 8 (1964), p. 265.

⁵⁰ Cf. Austin, *op. cit.*, p. 397.

⁵¹ Cf. G. Patzig, *Aristotle's Theory of the Syllogism*, Dordrecht, 1968, p. 4.

ample, to the syllogisms comprising unit terms (which can also be encountered in Aristotle's texts but they are not included into the system of syllogistic terms).⁵²

Therefore Łukasiewicz is right when he asserts that the system of Aristotle's syllogistic is inadequate in relation to the structures of reasoning applied by the Stagirite in practice. This fact—says Łukasiewicz—negates the principal assumptions of the Aristotelian theory of argumentation according to which every problem can be formulated using four syllogistic propositions (A, E, I, O) which, in turn, makes the syllogism the only tool of argumentation. Łukasiewicz points out that the latter consequence is the main error in the discussed theory of argumentation.⁵³

Agreeing with this critic, I still sustain that it should be completed with an additional explanation not given by Łukasiewicz. Aristotle formulated his syllogistic into a system which may be considered a logical theory independent of these or other philosophical ideas of the Stagirite, in other words, a "purely" logical theory. But if the syllogistic is understood as a component of the Aristotelian theory of argumentation, then it is no longer treated as a theory independent of the Stagirite's philosophical system. It should be remembered that Aristotle was distinguishing especially one notion of the scientific argumentation, namely this one which he accepted in his concept of the perfect science. And it seems to me that this very concept assumes a syllogistic structure of the argumentation.

Usually, the discussions on the ideas of Łukasiewicz disclosed in his monograph on Aristotle's syllogistic are nothing else but a general and quite often enthusiastic acceptance of his opinions. This is, however, not always the case. Some serious polemical publications have also appeared in which the truth of the interpretation accepted by Łukasiewicz was seriously questioned, and some totally different solutions were proposed. The most radical opponent of Łukasiewicz is J. Corcoran. Below I present some of his crucial ideas: the system of Aristotle's logic is basic and therefore it does not allow for another system, e.g. that of the logic of propositions; Aristotle's logic is not an axiomatic system but a system of the natural deduction; the system of Aristotle's logic is coherent and complete.⁵⁴ Let me stop at that. To present and analyze Corcoran's opinions on Aristotle's logic would require a separate treatise. These are highly controversial views and the polemic itself contains a number

⁵² Cf. *ibid.*, pp. 4f.

⁵³ Cf. Łukasiewicz, *Aristotle's syllogistic ...*, p. 44.

⁵⁴ Cf. J. Corcoran, *Aristotle's Natural Deduction System*, in: *Ancient Logic and its Modern Interpretations*, Dordrecht, 1974, pp. 85f, and 122–123.

of misunderstandings. Some of these misconceptions have been discussed by M. Mulhern in his very interesting paper entitled *Corcoran on Aristotle's logical theory*.⁵⁵

In the recapitulation of the first five chapters of his monograph on Aristotle's syllogistic, Łukasiewicz points out that the introduction of Śłupecki's principle, which helped to solve the problem of the determinability of syllogistic, was a crucial step meaning the real end of the investigations into this system.⁵⁶ Ph. Boehner in his critical review on the work of Łukasiewicz is also of the opinion that this book definitely brings to an end the researches on the syllogistic of assertive propositions.⁵⁷

And yet this opinion was not fully confirmed. In the course of almost twenty-five years that have elapsed since the book of Łukasiewicz made its appearance for the first time, the investigations into Aristotle's syllogistic have been undertaken many times and, as a consequence of this, new and valuable publications were issued. It can easily be imagined, however, that these new researches and new publications devoted to Aristotle's logic are, in prevailing part, genetically dependent on the work of Łukasiewicz. Usually, one of the two cases occurs: either 1° the authors accept in principle the ideas of Łukasiewicz and their own contribution only makes these ideas more profound, more concise and complete (G. Patzig), or 2° they carry out their own investigations to prove that Łukasiewicz was wrong in his interpretation and to develop their own interpretation (J. Corcoran). In both these cases, however, the very source of inspiration for the new researches on Aristotle's syllogistic is nothing else but the work of Jan Łukasiewicz.

⁵⁵ The paper has been included into the collection *Ancient Logic and its Modern Interpretations* (cf. pp. 133-148).

⁵⁶ Cf. Łukasiewicz, *Aristotle's syllogistic ...*, p. 131.

⁵⁷ Cf. Ph. Boehner, a review of *Aristotle's syllogistic ...*, by Łukasiewicz, "Journal of Symbolic Logic" 17 (1952), p. 210.