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THE FOURTEENTH AND SEVENTEENTH CENTURY PROJECT OF MATHEMATICAL PHYSICS. CONTINUITY OR DISCONTINUITY?*

Introduction

Medieval philosophers while commenting on Aristotle's works on natural philosophy noticed many *apporiai* in his physics. What is more, inspired by William of Ockham (ca. 1280–1349), they rejected Aristotelian prohibition of *metabasis* limiting the use of geometry only to *scientiæ mediæ* (intermediate sciences), such as optics or astronomy, which *deal only with quantified aspects of natural phenomena and not the whole phenomenon*[1]. In the 14th century the members of the famous English School, so–called Oxford Calculators introduced to physics both: mathematics, understood as a proper language of science, and logic understood as the convenient way to pose problems. Therefore, it seems interesting to examine if they have any *project* of mathematical physics and if so whether it made them stand any closer to modern science. Since it is no more doubtful that Galileo, while proving a proper rule of accelerate motion, used the Mean Speed Theorem, formulated by one of the Calculators, William Heytesbury (ca. 1313–1372)[2], and that Newton employed the 14th century theory of compounding ratios[3], it seems attractive to go back to a continuity/discontinuity in the history of natural science.

One of the most distinguished historians of medieval science, Annelise Maier, regarded the history of natural philosophy from the thirteenth to the 18th century as the history of the gradual rejection of Aristotelianism. She claimed that it did not evolve uniformly from century to century, but it

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* The paper is a revised version of an article titled: *Why was Medieval Mechanics Doomed? The Failure to Substitute Mathematical Physics for Aristotelianism* in: *Miscellanea Mediaevalia: Herbst des Mittelalters? Fragen zur Bewertung des 14, und 15, Jahrhunderts*, t. 31, Berlin – New York 2004, pp. 495–511.

[1] For Aristotelian and medieval classification of sciences see e.g. J. A. Weisheipl O. P., *The nature, Scope, and Classification of Sciences* in: D. Lindberg (ed.), *Science in the Middle Ages*, Chicago 1978, pp. 461–482, esp. pp. 477–478.

[2] For the detail analyses of an influence of medieval Mean Speed Theorem on Galileo see an excellent book by P. Damedow, G. Freudenthal, P. McLaughlin, J. Renn, *Exploiting the Limits of Preclassical Mechanics*, New York 1992, pp. 226–236, esp. p. 227.

[3] See E. D. Sylla, *Compounding ratios. Bradwardine, Oresme and the first edition of Newton's "Principia"* in: E. Mendelson (ed.), *Transformation and Tradition in the Sciences. Essays in Honor of I. Bernard Cohen*, Cambridge 1984, pp. 11–43.

occurred in two stages: the first reached its culmination in the 14th century, the second in the seventeenth[1]. However, it was Pierre Duhem, who first clearly expressed the idea that modern mechanics was a product of the Middle Ages, by which he reversed the previous, predominant view, that the period preceding the Scientific Revolution in the 17th century had no importance at all for the development of science[2]. The initial favourable response to Duhem's theory by those historians, who believed that he had succeeded in discovering fourteenth–century precursors of Galileo and Newton, was replaced by the criticism. Marshall Clagett, was convinced that late medieval doctrines reveal *how the very points of criticism of the older system became points of departure for the new*[3]. Annelise Maier saw the greatest weakness of Duhem's theory in his neglect of the different contexts of doing science in the 14th and 17th centuries and in his examinations of medieval theories from the perspective of later scientific beliefs[4].

According to Edward Grant there were contextual and substantive pre–conditions that enabled the Scientific Revolution to develop. The contextual pre–conditions, he understands as the open availability of the translations of Greco–Arabic works on science and natural philosophy into Latin, the formation of the medieval university, and the emergence of the logically trained natural philosophers who created *a social environment.* The substantive pre–condition of the Scientific Revolution, he founds in inventing new methodologies, in using new scientific language, in bringing into light new scientific problems, and in giving new answers to old questions. That resulted in the intensive development of natural philosophy – *the mother of all sciences.* Hence, in Grand's opinion, the new practice in scientific inquiry in the fourteenth–century laid the foundation for the science that came to fruition in the seventeenth–century[5].

Grant's views were strongly criticized by Andrew Cunningham[6], who emphasized the significant difference between natural philosophy, which was practiced in Europe from the early thirteenth to the early nineteenth century,

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[1] See A. Maier, *Die Vorläufer Galileis im 14 Jahrhundert*, Roma 1966, pp. 1–2, A. Maier, *Zwischen Philosophie und Mechanik*, Roma 1958, pp. 373–382.

[2] See P. Duhem, *Etudes sur Leonardo de Vinci*, vol. 1– 3, Paris 1906–1913, P. Duhem, *Le système du monde. Histoire des doctrines cosmologiques de Platon à Copernic*, vol. 1–10, Paris 1913–1959.

[3] M. Clagett, *The Science of Mechanics in the Middle Ages*, Madison 1959, p. XIX.

[4] A. Maier, *Die Funktionsbegriff in der Physik des 14 Jahrhunderts* in: *Divus Thomas* 24, 1946, p. 66.

[5] See E. Grant, *The Foundation of Modern Science in the Middle Ages. Their religious, institutional, and intellectual contexts*, Cambridge 1996, pp. 171–203, E. Grant, *Physical Science in the Middle Ages*, New York 1971, E. Grant, *Studies in Medieval Science and Natural Philosophy. Collected papers*, London 1981.

[6] For detailed discussion between Grant and Cunningham see *Open Forum* in: *Early Science and Medicine* 5, 3/2000, pp. 258–300, E. Grant, *God, Science, and Natural Philosophy in the Late Middle Ages* in: L. Nauta & A. Wanderjagt (eds), *Between Demonstration and Imagination. Essays in the History of Science and Philosophy Presented to John D. North*, Leiden – Boston – Köln 1999, pp. 244–266, E. Grant, *God and Natural Philosophy: The Late Middle Ages and Sir Isaac Newton* in: *Early Science and Medicine* 5, 3/2000, pp. 279–298, A. Cunningham, *The Identity of Natural Philosophy. A Response to Edward Grant* in: *Early Science and Medicine* 5, 3/2000, pp. 259–278, A. Cunningham, *A Last Word* in: *Early Science and Medicine* 5, 3/2000, pp. 299–300.

and modern science, which has developed over the last two centuries[1]. In Cunningham's view the essential difference between natural philosophy, which is about *God and His creation*, and modern science, which *does not deal with God or the universe as God's Creation* makes it impossible to describe both periods by the same term *science*, since it offers *a potential source of confusion and mistaken identities*. Therefore, we should not bother ourselves with the question of the continuity or discontinuity in science from the medieval period until the time of Galileo, since science – a product of modern practice – did not exist before the early nineteenth century. Our energy, thus, should be put in examining natural philosophy as natural philosophy that had nothing to do with science. Consequently, Cunningham ideally would wish us:

> *to be referring to a historical sequence of philosophy (ancient period), natural philosophy (medieval and early modern period), and then science (modern period i.e. the 19th and 20th centuries), disciplines fulfilling different roles in their respective eras*[2].

From Cunningham's examination of the history of medieval period one can emerge several lucid and plausible historical explanations of the context of medieval learning. His insistence that natural philosophy had nothing to do with science deprives science of its own history and so, it forces us to examine more clearly its actual history. In every epoch, we find in science irrelevant questions, wrong answers and *ad hoc* solutions. In my opinion, problems do not become or cease to be scientific simply because they are considered by friars, monks or secular clerics, which is Cunningham's main argument against the possibility of doing science in the Middle Ages. I do not believe that I could understand better the medieval, the Galilean or the Newtonian science of motion, explaining particular problems, e.g. of free fall of heavy bodies, just because I would judge that before the 19th century all natural philosophy was about God and His creatures. Nature does not behave differently in different times: stones consistently fall down with accelerated motion in all time period.

Therefore in this paper, I will go back into *the old battle* about the continuity/discontinuity problem of science. In order to refine what the continuity or discontinuity in science and especially mathematical physics means, I will underscore the changes within medieval physics and point up the complete rupture with the medieval past that was executed by Galileo vis-à-vis the Aristotelian world. I will focus on aspects of 14th century science of motion, which historians of medieval science consider responsible for significant departures from the Aristotelian natural philosophy. Since special credit goes for these changes to the Oxford Calculators, that is to William Heytesbury, John Dumbleton (? – ca. 1349), Roger Swineshead (? – ca. 1365), Richard Swineshead (fl. ca. 1340–1354), and Thomas Bradwardine (ca. 1290–

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[1] See A. Cunningham, *Taking Natural Philosophy Seriously* in: *History of Science* 29, 1991, pp. 377–392, A. Cunningham & R. French, *Before Science. The Invention of the Friars' Natural Philosophy*, Aldershot 1996.

[2] See A. Cunningham, *The Identity of Natural Philosophy*, pp. 277–278.

1349), who, along with Richard Kilvington (ca. 1302–1361), constituted the first academic generation of this school, as well as to French masters, like John Buridan (ca. 1300–1362), Albert of Saxony (ca. 1316–1390), and arsilius of Inghen (1330/40–1396), I will briefly discuss their opinions. I will not pay attention to experimental science and the experimental verification of mathematical analyses of motion, since I cannot find any witness of such procedures in medieval mechanics.

The Medieval Tradition in Mechanics

It is beyond any doubt, that notions, such as *velocity*, *gravity*, *impetus*, and *force* have entirely different connotations in the theory of relativity or quantum mechanics and in the classical Newtonian mechanics, as well as in medieval science. I am aware that whereas the term *mechanics* can be applied to the Galilean and the Newtonian theories of motion, its application to medieval science is anachronistic. Although in the Middle Ages we find works on statics and on the science of motion, the notion of *mechanics*[1] did not appear as a description of the science of motion and force, which had a set of principles describing dynamics, kinematics and statics, where, an equilibrium would be treated as a special case of motion, with zero or constant velocity. In this paper, however, I will adopt this anachronistic notion and I will narrow my focus to the study of physics of motion and force.

Medieval thinkers were familiar with two distinct, Greek traditions in mechanics: one deriving from Archimedes and the other from Aristotle. The Archimedean tradition, known to medieval philosophers, was connected with the geometrical and static approach to the science of weight that appeared in the West with the works of Archimedes, Euclid and the Arabic successors of Greek thinkers[2]. The so–called dynamic, Aristotelian tradition began with the work titled *Mechanical Problems*[3] where mechanics is defined as: *a τέχνη, a practical craft that uses natural principles to attain some end contrary to*

[1] The definition of the notion *mechanics* is to be found in Aristotle's *Posterior Analytics*, where he says that τά μεχανικά is related to stereometry (a part of geometry which deals with solids) in the same way as optics is related to geometry and harmonics to arithmetic (see Aristotle, *Posterior Analytics* I, 78b35–39). Therefore, mechanics does not deal with the qualities of a body, but considers only the geometrical attributes of its proper subjects, viz. solids, in the same way as harmonic and optics, which being not interested in sounds and vision as such, consider only lines and numbers (see Aristotle, *Metaphysics*, 1078a).

[2] On the Greek mechanical tradition in the Middle Ages see M. Clagett, *Some General Aspects of Physics in the Middle Ages* in: *Isis* 39, 1948, pp. 24–44, M. Clagett, *The Science of Mechanics in the Middle Ages*, M. Clagett, *Archimedes in the Middle Ages*, vol. 1–5, Madison 1964, M. Clagett, *Archimedes in the Late Middle Ages* in: D. H. D. Roller (ed.), *Perspectives in the History of Science and Technology*, Oklahoma 1971, pp. 239–259, E. A. Moody & M. Clagett (eds), *The Medieval Science of Weight*, Madison 1952, J. E. Brown, *The Science of Weights* in: D. Lindberg (ed.), *Science in the Middle Ages*, pp. 179–205, W. R. Laird, *The Scope of Renaissance Mechanics* in: *Osiris* 2, 2/1986, pp. 43–68, G. Ovitt Jr., *The Status of the Mechanical Arts in Medieval Classifications of Learning* in: *Viator* 14, 1986, pp. 89–105.

[3] *Mechanical Problems* were written by an anonymous Greek author, who most likely belonged to one of the first generations of Aristotelians. G. Owen shares Duhem's opinion that this work is *a systematic development of some suggestions found in his* (sc. Aristotle's) *writings though inconsistent with others*. G. E. L. Owen, *Aristotelian Mechanics* in: A. Gotthelf (ed.), *Aristotle on Nature and Living Things*, Pittsburgh – Bristol, p. 230. On mathematical models in Aristotle see also T. Heath, *Mathematics in Aristotle*, Oxford 1949. On the authorship of *Mechanical Problems* see T. Heath, *Mathematics in Aristotle*, p. 227, J. E. Brown, *The Science of Weight*, p. 179, p. 205.

nature[1]. The main problems, touched here, are focused on some physical and mathematical properties of circular motion, on developed analyses of the lever or balance, and on explanation of equilibrium on a balance of unequal arms.

The record of the transmission of *Mechanical Problems* is obscure. The first complete translation into Latin was not made until 1497. Nevertheless, some 13th and 14th century works include arguments similar in the mode to those presented in the *Mechanical Problems*[2]. The works that show the closest affinity to it are: an anonymous 14th century *Liber de ponderibus*, the *De ratione ponderis*, ascribed to Jordanus Nemorarius (1225–1260), the treatise on kinematics, *De motu* by an early thirteenth–century geometer,Gerard of Brussels, and Emperor Frederick's II (1194–1250) *De arte venandi*[3]. In the opinion of Richard Westfall, those who followed that tradition of the medieval science of mechanics had not recognised any distinction between statics and dynamics, since the simple machines, such as the lever, obviously served not to hold bodies in equilibrium, but to move them; and consequently, were analysed in dynamic terms[4].

It is the work of an Arabic author, Thabit ibn Qurra's (836–901) *On the Kariston* which should be blamed for blending Archimedes' static proofs with the dynamic solutions of the problems considered in *Mechanical Problems.* Undoubtedly, in the Middle Ages the statics with its superior mathematics made its impact through the dynamic Aristotelian tradition. While commenting Aristotle's works on the philosophy of nature, the medieval authors, first and foremost, followed his *Physics*. In accordance with Aristotle there are two kinds of motion: a natural and violent one. The former is described dynamically, since a downward motion of a heavy body is caused by a natural attribute, such as gravity or weight. The latter, on the other hand, is studied statically when a violent motion is considered to be caused by a force acting against a *pondus*, i.e. against the resistant weight of a body. In the 14th century, however, scholars formulated laws describing both types of motion in the same way.

Mathematical Physics in the 14th Century

In order to acknowledge the influence of mathematics upon medieval science of motion, I will now examine the most significant elements of a new approach toward science of motion which resulted in a developing of a project of mathematical physics adopted by the English philosophers. Those elements,

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[1] Pseudo–Aristotle, *Mechanical Problems*, 847a10–11.

[2] On the influence of the *Mechanical Problems* in the Middle Ages see M. Clagett & E. Moody, *The Medieval Science*, p. 147, pp. 150–153, M. Clagett, *The Science of Mechanics in the Middle Ages*, pp. 71–72, J. E. Brown, *The Science of Weight*, p. 203. Heikki Mikkeli states that *Mechanical Problems* was unknown in the Middle Ages and was assimilated into the corpus of mechanical and technological works in the sixteenth century. See H. Mikkeli, *An Aristotelian response to Renaissance humanism: Jacopo Zabarella on the Nature of Arts and Sciences*, Helsinki 1992, pp. 119–120.

[3] The other influential dynamical tradition of ancient time was enriched by translations of the work *On the Heavy and the Light and the Comparison of Bodies to Each Other*, which scattered correlations between force, volume, and speed (see J. E. Brown, *The Science of Weight*, p. 186).

[4] See R. Westfall, *Force in Newton's Physics*, London – New York 1971, pp. X–XI.

in my opinion, are: the accommodation of the Archimedean influence in mechanics, the New Rule of Motion invented by Richard Kilvington and developed by Thomas Bradwardine, the quantification of qualities leading to kinematics and, finally, the mental experiments based on the *secundum imaginationem* procedure that was frequently accompanied by the *ceteris paribus* method. The role of John Buridan, famous for the *impetus* theory, will also be invoked here, even though he did not accept mathematical methods in physics.

Various scholars stress the predominant role of one or the other of these aspects as the essential reason for the changing views in science in the fourteenth–century. Alistair Crombie finds the sources for medieval mathematical physics in Robert Grosseteste (ca. 1175–1253)[1]. According to Crombie:

> [...] *mathematics could often provide the reason for occurrences in the world of experience, Grosseteste held, because although the subject that mathematics studied was abstract quantity, mathematical entities actually existed as quantitative aspects of physical things. In fact 'quantitative dispositions are common to all mathematical sciences* [...] *and to natural science'*[2]. *Therefore, as Aristotle*[3], *had said the different branches of mathematics logically subordinated to themselves different physical sciences concerned with physical things. The superior mathematical science then provided knowledge of the reasons for facts provided by the lower physical science.* [...] *essential to this use of mathematics was measurement, which meant performing operations which resulted in a number*[4].

While saying this, Crombie is only partly right. When he talks about subordination of physical sciences to mathematics, Crombie is wrong. We cannot forget that Grosseteste's physics was based on his metaphysics of immaterial light, strongly connected with optics which found use for such science as geometry. Still, in Aristotle's opinion, optics, one of the *scientiæ mediæ*, is subordinated to mathematics, but because it deals with immaterial bodies it is not a legitimate part of physics, that studies material bodies in motion. We cannot forget that a measurement found its use only in intermediate sciences and not in mechanics. We should not treat *measurement* in optics to be the first step in experimental science of motion, since none of the medieval philosophers of nature was interested in a measuring of observed objects. There were no systematic observations of motion, which would result

[1] See A. Crombie, *Augustine to Galileo. The History of Science A. D. 400–1650*, London 195, A. Crombie, *Robert Grosseteste and the Origin of Experimental Science 1100–1700*, Oxford 1953.

[2] Robertus Grosseteste, *Commentarius in Posteriorum Analyticorum libros*, edited by P. Rosii, Firenze 1981, 170,19–171,28. See, Aristotle, *Posterior Analytics* I, 75b14, Aristotle *Physica* II, 193b23 sqq.

[3] See Aristotle, *Posterior Analytics* I, 75b14, 76a32 sqq., 87a31 sqq.

[4] A. Crombie, *Robert Grosseteste*, p. 91.

in developing the techniques of measurement, that would provoke questions posed from the point of view of experimental science.

Crombie is right, however, when stressing that geometry, as a *propter quid*, i.e. a demonstrative science *could provide the 'reason for the fact', in the sense that it could be used to describe what happened, could correlate the concomitant variations in the observed effects*[1]. Therefore, in accord with Grosseteste, mathematicians were at liberty to think for their own purposes, e.g., of space as empty or of infinite extent, because it was not the same thing as real space. The geometer could not be accused of being false because he used abstract concepts not to be observed in physical world. Mathematics, then, was a convenient tool for describing both physical and mental experiments[2]. It seems that fourteenth–century English philosophers closely followed Grosseteste, even thought they did not accept his metaphysics of light. The authority of the *Lyncolniensis* was invoked in many works of the Oxford Calculators in the 14th century.

Marshall Clagett, who states that the later development of mathematical physics was associated with the impact of Archimedes, not only on the elementary geometrical treatises, such as Thomas Bradwardine's *Speculative geometry* but also on the questions concerning motion, is also partially right. The very best example that confirms Clagett's opinion, comes from Richard Kilvington's commentary on Aristotle's *Physics*. Kilvington explains here a hypothetical motion of an elementary body in a void by means of the following Jordanus Nemorarius's postulates coming from his famous treatise *De ponderibus*: *1) That which is heavier descends more rapidly; 2) This is heavier in descending, whose motion toward the center is more direct; 3) This is heavier positionally, whose path at a given position of descent is less oblique*[3]. Following a medieval tradition, Kilvington blended Euclid–Archimedean views, as represented by Nemorarius, with Aristotle's theory.

Already before Bradwardine, Kivington[4] used the first theorem of Jordanus which runs as follows: *The proportion between the speeds of descent of any given heavy bodies is the same as that between their respective weights.* He did so in order to corroborate his original rule of motion relating speeds, forces, and resistances in an exponential function[5]. The New Rule of Motion generally is in accord with Aristotelian principle of motion, which holds that a speed of motion is proportional to a proportion of an acting element, which we call a force (F) to a suffering element, which we call a resistance (R). With regard to a rule formulated in Book VII of *Physics* a body is in motion only

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[1] A. Crombie, *Robert Grosseteste*, p. 96.

[2] A. Crombie, *Robert Grosseteste*, p. 99.

[3] *Elementa Jordani super Demonstrationem ponderum* in: E. A. Moody & M. Clagett (eds), *The Medieval Science of Weight Science*, pp. 128–129. See also J. E. Brown, *The Science of Weight*, pp. 190–196.

[4] On the influence of Richard Kilvington upon Thomas Bradwardine, who primarily achieved his fame as much in the Middle Ages as in modern time among historians of medieval science for *his* rule of motion see E. Jung, *Works by Richard Kilvington* in: *Archive d'Histoire Doctrinale et Littéraire du Moyen Age* 67, 2000, pp. 181–223.

[5] For Latin quotes see E. Jung, *Why was Medieval Mechanics Doomed?*, p. 503, n. 38.

when a force is greater than resistance, i.e. when a proportion of F:R is greater than 1. A speed of motion is proportional to F:R proportion[1]. Consequently, in the world of Aristotle, there is no possibility for motion to occur with a speed less than 1. To avoid this apparent weakness of Aristotle's theory Kilvington, and later Bradwardine, applies Euclid's definition of a double proportion. They state that a speed is doubled only if the ratio F:R is multiplied by itself and a speed is halved when we determine the *middle* ratio, i.e. when we find a ratio that equals a square root of F:R ratio[2]. Such a procedure has the undeniable benefit of mathematical as well as physical consistency. It guarantees that in the description of any motion continuously diminishing by continuous *halving* an initial ratio of a speed of motion will always be greater than 0. Aristotle explains that a speed of motion is doubled either when a force is doubled or when a resistance is halved, so then a proportion of F:R is simply multiplied by two[3]. Kilvington is aware that the proper understanding of Euclid's definition of operations on ratios necessitates a new interpretation of Aristotle's rules of motion.

This explanation, however, had nothing to do, like many of the historians of medieval science claim, with a new rule for free fall of a heavy body, but it was only a clarification of Aristotle's rules, which were correct only in one case, when the ratio of force to resistance was 2:1, since then both in multiplying the proportion by two and by itself (that is squaring it) one arrived at the same result.

Since Kilvington's commentary on the *Physics* was written ca. 1326, it is obvious that the situation in medieval physics did not *change dramatically in 1328, when Thomas Bradwardine wrote his "Treatise on the Proportions of Velocities in Motion"*, and that Bradwardine did not *remove the whole problem of relating velocities, forces and resistances from the context of an exposition of Aristotle's words* and did not *investigate it in its own right* as Edith D. Sylla and John Murdoch assert[4]. First of all, this was because Bradwardine followed Kilvington, who had shared Grosseteste's opinion that mathematics was a proper method in physics, useful in all branches of scientific inquiry dealing with *measurable* objects. This belief, which had encouraged Kilvington first to argue against Aristotle, Archimedes, and Jordanus, and then to explain their statements with mathematical precision enforced by Euclid's definition of a double proportion, was later taken over by Bradwardine[5].

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[1] Aristotle, *Physics* VII, 250a–b.

[2] If, anachronistically, force–resistance proportion is generalised symbolically by exponential function, then if $F_1:R_1$ gives rise to a speed v_1, the increasing of this proportion to $F_2:R_2$ will yield v_2, in such way that the following relations are hold: $F_2:R_2 = (F_1:R_1)^{v2/v1}$. Hence, if $v_2=1/2v_1$ $F_2:R_2=(F_1:R_1)^{1/2}$; if $v_2=2v_1$, $F_2:R_2=(F_1:R_1)^{2/1}$. Thanks to this we always have a proportion of F:R greater than 1 even if the speed of motion decreases.

[3] See Aristotle, *Physics* VII, 250a–b.

[4] See J. Murdoch & E. Sylla, *Science of Motion* in: *Science in the Middle Ages*, D. Lindberg (ed.), pp. 224–225.

[5] It seems no more useful to seek the origin of Bradwardine's law in medieval pharmacological treatises. See Clagett, *The Science of Mechanics in the Middle Ages*, p. 439, n. 35, S. Drake, *Medieval Ratio Theory vs. Compound Medicines in the Origins of Bradwardine's Rule* in: *Osiris* 64, 1973, pp. 67–77, M. McVaugh, *Arnald*

Oxford Calculators were just as much influenced by Grosseteste's *Commentary on the Posterior Analytics* in their logic and methodology as their older colleague, William of Ockham was. Accordingly to his minimalist ontology, Ockham accepts substance and quality as the only two distinct realities and – as André Goddu points out – he denies *the existence of quantity as a thing distinguishable from a thing that is quantified and places quantity and mathematical terms under connotative terms and concepts*[1]. In Goddu's opinion, Ockham's theory of connotation *developed as an interpretation of mathematical entities that subordinates logical considerations to what we might call a more pragmatic conception of mathematics. It is a conception that makes mathematics into language or into another tool of analysis*[2]. I strongly agree with Goddu, who maintains that Ockham's theory have influenced many authors of his time and especially the Oxford Calculators, liberating them *to use mathematics in science that does not involve measurement, where mathematics is a kind of theoretical formalism that enables us to resolve thorny questions about qualitative contraries, time, place and the like*[3].

It seems also that Ockham's theory of connotation, which treats motion as a connotative term describing the change that a body undergoes, was the first step in the separation of the two different aspects of motion: i.e. the dynamics and kinematics later developed by the Oxford Calculators when they pondered the problem of qualitative changes explained in terms of local motion with regard to its effects, i.e. velocity, time and path. *Thus the growth of interest in kinematics* – as Goddu claims – *was caused by the tendency to replace the view of motion as a qualitative accident with entirely quantitative and relation considerations based not on empirical evidence or measurement but on mathematical consistency or coherence*[4]. These tendencies matured in the works of William Heytesbury, who was able to develop metalinguistic and mathematical analyses that grew out of the discussion of the philosophical problem of the intension and remission of forms. It eventually led to the distinction of quality or intensity of speed from quantity of speed and to the concept of latitude of forms or range of degrees. Medieval thinkers dealt with the *measuring* of the speed of the various ways of distribution of different qualities over or through given subjects. Qualities could be uniformly or difformly distributed in a subject either *over distance* or *over time*.

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of Villanova and Bradwardine's Law in: *Isis* 58, 1967, pp. 56–64. What is more, we should not forget that the idea of measuring motion in the propagation of light by means of double and triple ratios first appeared in Groseseteste's treatise *De luce seu de inchoatione formarum*, written between 1220–1235.

[1] See A. Goddu, *The impact of Ockham's readings of the Physics on the Mertonians and Parisian terminists* in: *Early Science and Medicine* 6, 3/2001, p. 214.

[2] See A. Goddu, *The impact of Ockham's readings* ... .

[3] See A. Goddu, *The impact of Ockham's readings* ... . See also E. Jung, *Natura more geometrico. Średniowiecze jako pośrednik w recepcji matematyki greckiej* in: *Studia Warmińskie* 26, 2000, pp. 129–179, E. Jung, *Procedura secundum imaginationem w czternastowiecznej filozofii przyrody* in: E. Jung (ed.), *Księga pamiątkowa ku czci profesora Zdzisława Kuksewicza*, Łódź 2000, pp. 57–79.

[4] A. Goddu, *The impact of Ockham's readings* ..., p. 223.

From the considerations of accelerated motion the Oxford Calculators derived the so–called Mean Speed Theorem. It states that a uniformly accelerated motion corresponds to its mean degree of speed, which means that a given latitude of motion uniformly gained in a given time always makes a mobile traverse a space equal to that which would be traversed if the body moved with the middle degree of the latitude for the whole time. Numerous arithmetic and geometric proofs of this theorem were formulated in the 14th century. The first arithmetical proof was presented by William Hetesbury. The best known geometric proof, which was the most original and comprehensive extant treatment of the intension and remission of qualities and the most elaborate application of the mean–degree measure of speed in motion was formulated by Nicole Oresme[1]. The Mean Speed Theorem accepted and applied by numerous English and French philosophers of the late Middle Ages was also known and used by Galileo[2].

In my opinion, however, one of the most interesting achievements of the Calculators' *mathematical physics* lays in their deep awareness of the various degrees of abstraction. Their theories never renounced empirical verification, and they frequently used *secundum imaginationem* procedure. The *secundum imaginationem* analyses were usually accompanied by the *ceteris paribus* method, when all circumstances in a study case being considered are the same, and that only one factor, which changes during the process, causes the changes of the results. The followers of Kilvington raised questions, which could never have emerged from sense experience, since the structure of nature could be discovered only in highly abstractive analyses. This abstraction, however, was drawn from genuine realities and did not contradict them. Therefore, both ways of describing natural phenomena, i.e. physics and mathematics, were complementary. Realities give a starting point for more complicated mental constructions, which in turn make realities comprehensible. While mathematics was a proper instrument to solve problems, logic was the most convenient method to pose them. They both guaranteed the objective and demonstrative character of natural science[3].

The Oxford Calculator's tradition continued by the next generation of English scholars such as Roger Swineshead, John Dumbleton, and Richard Swineshead was adopted also by many French thinkers. There were two ways that continued the Calculators's ideas. While Nicholas of Oresme fully accepted the *secundum imaginationem* procedure leading to mathematical

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[1] Guillelmus Heytesberus, *Regule solvendi sophismata*, Venetii 1494, VI, ff. 37r–39v. See also E. Sylla, *The Oxford Calculators and the Mathematics of Motion 1320–1350. Physics and Measurement by Latitudes*, New York 1991. On Oresme proofs of Mean Speed Theorem see Nicole Oresm, *Tractatus de configurationibus qualitatum and motuum*, (edited and translated by M. Clagett in: *Nicole Oresme and the Medieval Geometry of Qualities and Motion*, Madison 1968.

[2] See n. 2, p. 151.

[3] On Kilvington's methodology see E. Jung & R. Podkoński, *Richard Kilvington on proportions* in: J. Biard & S. Rommevaux (eds), *Mathématiques et théorie du mouvement XIV^{e}–XVI^{e} siècles*, Tour 2008, pp. 84–86. On William Heyetesbury *speculative physics* see J. Longeway, *William Heytesbury on Maxima and Minima. Chapter 5 of "Rules for solving sophismata" with an anonymous fourteenth–century discussion*, Dordrecht 1984.

physics and developed his own system for measuring motions[1], the others, like John Buridan, eliminated mathematics from natural philosophy because of the Aristotelian *metabasis* prohibition. The hypothetical cases and mental experiments, discussed by Buridan and his followers, were connected with the metalinguistic, logical and theological analysis related to God's absolute and ordained powers[2].

Medieval mechanics, however, owes John Buridan the *impetus* theory[3]. He was the first to use this term in order to describe the projectile motion, the free fall of a heavy body and the motion of the heavenly spheres. The discussion started with Aristotelian theory of projectile motion presented in his *Physics*. Aristotle claims that each violent motion, such as projectile one and freely falling body need a constant presence of a mover and therefore when a mobile is no more in touch with its mover, a medium takes its role[4]. The theory of permanent impetus elaborated by Buridan was accepted by other Parisian masters like Albert of Saxony (ca. 1316–1390) and Marislius of Inghen (ca. 1340–1396). In their opinion *impetus* is a not self–expending quality and it can be only diminished by resistance. *Impetus* is a variable quality whose force is determined by the speed and quantity of the matter in the subject, so that the acceleration of a falling body can be understood in terms of its gradual accumulation of units of *impetus*. In the case of projectile motion *impetus* is the same as the force impressed by the mover, and its quantity depends on a primary matter. If there is more matter, a greater force can be impressed. In a case of free fall *impetus* is a cause of acceleration of a heavy body. In the case of motion of celestial bodies, *impetus* is the force impressed by God, which causes their everlasting circular motion. This concept of *impetus* helps to describe all types of motion: terrestrial, violent and natural motion of heavy bodies as well as motion of celestial, non–material bodies. Buridan's statement is often presented as the first law of inertia.

The Novelty of Medieval Mechanics vis–à–vis Aristotelian and Galilean Theories

For my own purpose, I now summarize and review what appear, in the

[1] On the French *secundum imaginationem* analysis see H. Hugonnard–Roche, *Analyse sémantique et analyse secundum imaginationem dans la physique Parisienne au XIV siècle* in: S. Caroti (ed.), *Studies in Medieval Natural Philosophy*, Firenze 1989, pp. 133–153, E. Grant, *Medieval Departures from Aristotelian Natural Philosophy* in: S. Caroti (ed.), *Studies in Medieval Natural Philosophy*, pp. 247–252, H. Hugonnard–Roche, *L'hypothetique et la nature dans la physique parisienne du XIV^e siècle* in: P. Suffrin & S. Caroti (eds), *La nouvelle physique du XIV^e siècle*, Firenze 1997, pp. 161–177, J. Sarnowsky, *God's absolute power, thought experiments, and the concept of nature in the "new physics" of XIVth century* in: P. Suffrin & S. Caroti (eds), *La nouvelle physique du XIV^e siècle*, pp. 179–203.

[2] On Buridan see e.g. J. Zupko, *John Buridan*, http://plato.stanford.edu/entries/buridan/.

[3] There is an extended secondary literature on *impetus* theory. See for example A. Maier, *Die Vorläufer Galileis im 14 Jahrhundert*, Roma 1966, pp. 133 sq., A. Maier, *Zwischen Philosophie und Mechanik*, pp. 343–373, A. Maier, *An der Grenze von Scholastik und Naturwissenschaft: die Struktur der materishen Substanz, das Problem der intensive Grösse, die Impetustheorie*, Roma 1952, pp. 199 sq., M. Clagett, *The Science of Mechanics*, pp. 505–540, H. Butterfield, *The Origins of Modern Science 1300–1800*, New–York - London 1965, pp. 13–29.

[4] See Aristotle, *Physics* VI, 232b.

opinion of some historians of medieval science, to be the most important departures of the 14th century mechanics from Aristotle's physics. First of all, there is a blend of the Aristotelian dynamic tradition and Archimedean static and mathematical tradition. Secondly, there is a refutation of Aristotle's prohibition of *metabasis* and use of mathematics as the proper method in physics. As I emphasized, it was for the first time in the medieval period that mathematical strictness forced natural philosophers to invent a new rule describing motion. Thirdly, there is the separation of dynamics and kinematics, which led to the formulation of the Mean Speed Theorem formulated in order to compare speed in accelerated motion. Fourthly, there is the promotion of mental experiment. Fifthly, there is the *impetus* theory, which allowed to explain heavenly circular motion, the free fall of a heavy body and the projectile motion of bodies by means of the same concept of *impetus*.

Deeper insight into medieval mechanics, however, reveals the constant presence of the Aristotelian background. Even though Kilvington and Bradwardine had broken Aristotelian prohibition of *metabasis*, they still remained within the framework of his physics, in which motion takes time because of two factors: force and resistance, acting as its causes. The speed of motion is determined by the ratio of force to resistance and the New Rule of Motion does not break this principle. Like Aristotle, Kilvington, Bradwardine, and their followers, had considered a force responsible for a speed and not for a constant acceleration, which was later properly recognized by Galileo and formulated as the second law of motion by Newton in the 17th century.

Secondly, the notions *uniform*, *uniformly difform* and *difformly difform* motion were used not only to describe the distribution of changes in uniform, accelerate and decelerate motions. For when medieval natural philosophers considered the difformly difform speed, they had in mind not only difform changing of speed, but also uniform changes of acceleration, i.e. a motion with equal increments of acceleration. Such motion does not exist in nature. Furthermore, such terms as *uniformly difform* motion and *uniform increasing of velocity* were used in both contexts of free fall, i.e. downward motion and of uniformly accelerated upward motion. This is a part of medieval mechanics to which we do not pay enough attention, since we look only for properly recognized problems[1].

Thirdly, the *impetus* theory remains solidly within the Aristotelian matrix of mover and mobile and, as a matter of fact, did not reject the principles of his physics. Furthermore, claiming that it is possible for a body to move eternally with circular motion when there is no acting force, Buridan was unable to formulate the proper law of inertia, since in fact there must be an acting force changing the path in motion if circular motion is to occur. On the

[1] The very best example here is Kilvington's the third question on motion from his commentary on Aristotle's *Physics*. For details see E. Jung, *Motion in a Vacuum and in a Plenum in Richard Kilvington's Question: Utrum aliquod corpus simplex posset moveri aeque velociter in vacuo et in pleno from the 'Commentary on the Physics'* in: *Miscellanea Medievalia* 25, 1997, pp. 179–193. Although Kilvington invented New Rule of Motion, he did not apply it to described free fall only. Since Aristotelian world was *symmetric* with regard to gravity and levity, there was any inconsistency in imagination and *proper* description of accelerated upward motion.

other hand, the medieval concepts of *impetus* finally lead to formulate proper definitions of violent and natural motions and to replace the whole Aristotelian system. Although ontologically different, *impetus* is analogous to Galileo's early use of *impeto* and Newton's *quantity of motion*. But despite its revolutionary implications, Buridan did not use the concept of *impetus* to transform the science of mechanics.

Finally, popular as they might be, thought experiments were rationalistic, only mental and not empirical experiments, and they did not stimulate the development of the experimental science of motion.

Still, I agree with Murdoch and Sylla, who point out that [i]*t would be an error to regard these new and distinctive 14th century efforts as moving very directly toward early modern science*[1]. Galileo's familiarity with late medieval physics' departures from Aristotle, which even made him repeat some of their wrong solutions, did not affect his general idea, since he used fragments of medieval mechanics for completely different aims. Galileo, whom we want to make responsible for the beginnings of the Newtonian dynamics, rejected or rather did not take into account the New Rule of Motion and went back to theory expressed by Avempace; he also changed the theory of *impetus*. Likewise he read Archimedes's works in a different way, what allowed him to create mathematical physics while recognizing the distinction between statics and dynamics. It also permitted him to consider mechanics as a contemplative and mathematical science under geometry that could provide the mechanical arts with their principles and causes. With the two major achievement of Galileo's mechanics, namely the conception whereby horizontal motion is held to be a state in which it remains until some external force causes it to leave, and the identification of free fall as a uniformly accelerated motion and the exposition of its role in nature, the new concepts in mechanics began a career that culminated in Newton's theory. In spite of this, Galileo was able to profit from the *secundum imaginationem* and *ceteris paribus* procedures, making a broad use of thought experiments to convince his readers to accept Copernicus's heliocentric theory. Galileo's approach to the problem of a possibility of applying mathematical principles to physical events was to view these principles not as pure mathematical abstractions but as laws that governed experimental science of motion.

Conclusions

To answer the main question of this paper, I would like to stress that each step taken by a new generations of 14th century natural philosophers was a step forward, even though it was a step taken on the dead–end road of science of motion. In my opinion, medieval mathematical physics was doomed, since even if it succeeded in refuting the restrictive prohibition of *metabasis* associated with Aristotelian philosophy and accepted mathematics as its method, it did not develop empirical mathematics and experimental physics. This was because, ironically, liberation of mathematics from the limitations of

[1] J. Murdoch & E. Sylla, *The Science of Motion*, p. 249.

actual experience created a tool of theoretical analysis that would make it impossible to cross over the threshold of exact science. Even though a tradition in *mathematical physics* continually developed in England from Grosseteste to the middle of the 14th century and then was continued by French, Italian, and Spanish thinkers such as Alvarus Thomas or Domonik de Soto until the end of the 16th century, never made a step forward to abandon Aristotle. Paradoxically, Aristotelian physics appeared to be able to accommodate all medieval attempts at providing it with mathematical precision. The 14th century revolution in mechanics was a revolutionary movement against the background of previous medieval theories, but not against the 17th century–mechanics. The revolution was held in the details. In its history medieval science, while taking an Aristotelian course thoroughly explored that framework exposing its paradoxes and weakness and reached the point where it was unable to overcome the lingering doubts. The decisive break was left to the successors of medieval philosophers of nature.

After a deliberate study of medieval science of motion and secondary literature I am forced to formulate the final conclusion: fourteenth–century revolution in science should not be regarded as a first step towards the Scientific Revolution. In my opinion medieval science should be treated as a specific phenomenon of medieval culture. Its story is over, yet it still waits for someone to tell it: what it was and what it is not with respect to the modern science. Nevertheless, I am still not ready to accept Andrew Cunningham's point of view, discussed in the introduction of this paper, since those were not friar's cassocks, that unable them to move freely, but it was Aristotle.