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Continuity and Discontinuity. On Method in Leonardo da Vinci' Mechanics

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CONTINUITY AND DISCONTINUITY. ON METHOD IN LEONARDO DA VINCI' MECHANICS*

1. General outline of the problem

The development of science shows its continuity–discontinuity and progressiveness. According to recent studies[1], in the last century the research about the foundations of science seems to have been forwarded increasingly by programs of research, more than through the implementation of a basic theory. Several competitive research programs have characterized the turn of 19th spanning throughout the 20th century[2]. As a matter of fact, discovery seems recast in its scientific value whenever it has not undergone the filter of different approaches and scientific theories, even in conflict to each other, since their foundations; so the evaluation itself of the scientific value of a theory cannot be an absolute one. It is enough to remark that the interesting intellectual effort proposed by Thomas Kuhn (1922–1996) based on the idea of scanning scientific structures in the history of science which can establish themselves as *paradigm* or produce a *replacement* of an old framework. However, today we know that program was not completely adequate[3] to

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[1] A. Drago & P. Cerreta, *Il programma storiografico di Kuhn caratterizzato secondo due programmi di ricerca sui fondamenti della scienza* in: A. Garuccio (ed.), *Atti XXIII Congresso Società Italiana Storia Fisica e Astronomia*, Progedit Editore, Bari 2003, pp. 120–130.

[2] This is evident in mathematics. In physics, too, but with difference between quantum mechanics and relativity. We should note a kind of divergence between Alexandre Koyré (1892–1964) and Kuhn storiographies. See T. S. Kuhn, *The Structure of Scientific Revolutions*, University Chicago Press, Chicago 1962, T. S. Kuhn, *The function of Dogma in Scientific Research* in: *Scientific Change. Historical studies in the intellectual, social and technical conditions for scientific discovery and technical invention, from antiquity to present*, Heinemann, London 1963, pp. 347–369, T. S. Kuhn, *Reflections on my Critics* in: *Criticism and Growth of knowledge. Proceedings of International Colloquium in the Philosophy of Science*, (eds) I. Lakatos & A. Musgrave, Cambridge University Press, Cambridge 1970, pp. 231–278, T. S. Kuhn, *Second Thoughts on Paradigms* in: *The Structure of Scientific Theories*, (ed.) F. Suppe, Illinois University Press, Urbana 1974, pp. 459–482, T. S. Kuhn, *Black-body theory and the quantum discontinuity, 1894–1912*, Oxford University Press, Oxford 1978, A. Koyré (ed.), *From the Closest World to the Infinite Universe*, Johns Hopkins University Press, Baltimore 1957, A. Koyré, *Newtonian studies*, Harvard University Press, Cambridge (Mass.) 1965, A. Koyré, *Du monde de «à-peu-près» à l'univers de la précision*, Armand Colin Librairie, Paris 1961.

[3] M. J. Klein, A. Shimony & T. J. Pinch, *Paradigm Lost? A Review Symposium* in: *Isis* 70, 1979, pp. 429–434, L. Kvasz, *On classification of scientific revolutions* in: *Journal for general Philosophy of Science* 30, 2/1999, pp. 201–232.

understand continuity and discontinuity and/or commensurability in the (historical) development of science. In fact, he used a unique, Newtonian paradigm to analyze the development of theories: in this sense the theories having different foundations as well as Renaissance statics and modern mechanics, Newtonian mechanics and Lagrangean one, chemistry[1] and thermodynamics were scarcely considered in his research[2]. Consequently, the Kuhnian program, which intended to explain all the scientific revolutions through the conceptual scheme of the Newtonian dynamics, found its path obstructed by the history of the evolution of the black body[3]. As a matter of fact, when Kuhn faced this matter to motivate the birth of quanta in *Black Body Theory and the Quantum Discontinuity* he had to give up[4] the Newtonian paradigm that used to be his main inquiry category. The use of other kinds of categories by means of logic and mathematics let the eventual *revolutionary* or *normal* logical character come out in a scientific theory (organization) further than his formal conception of infinite in the use of mathematics. The latter gives us a hint about the choice of formalism and continuous or discontinuous scientific progress[5].

2. Excursus on the scientific and cultural environment

The privileged geographical position of Italy in the Mediterranean caused interesting commercial exchanges with Africa and the Middle East that favoured the free circulation and the widespread of Greek works throughout Italy and Northern Europe[6]. On the other hand, when the Turks captured Constantinople (1453) many Greek scholars moved to Europe (several of them to Italy as well), taking with them important manuscripts and making the knowledge of the classical culture more accessible, compared with the past 12th and 13th centuries. The translation into Latin straight from the Greek language made their contents[7]

[1] R. Pisano, *A history of chemistry* à la *Koyré? Introduction and setting of an epistemological problem* in: *Khimiya Jounal* 17, 2/2007, pp. 143–161.

[2] A. Drago & P. Cerreta, *Il programma storiografico di Kuhn* ... .

[3] A. Drago, *Storiografia del corpo nero: Rivisitazione e nuova impostazione* in: *Electronic Proceedings of XXV SISFA Congress*, http://www.brera.unimi.it/SISFA/atti/atti2005.html, Milano 2008, pp. C08.1–C08.6.

[4] Kuhn merely describes facts without attempting any interpretation. See P. Cerreta, *The birth of quanta: A historiographic confrontation* in: H. Kragh, G. Vanpaemel & P. Marage (eds), *Proceedings of the XXth International Congress of History of Science*, vol. 14: *History of Modern Physics*, Brepols, Turnhout 2002, pp. 249–259.

[5] See R. Pisano & I. Gaudiello, *Continuity and discontinuity. Epistemological inquiry based on the use of categories in history of science* [in press: *Organon* 41, 2009].

[6] See E. Grant, *La Scienza nel Medioevo*, transl. P. Fait, Il Mulino, Milano 1997, pp. 25–54 & pp. 111–122 [original edition: E. Grant, *Physical Science in the Middle Ages*, Cambridge University Press, Cambridge 1971].

[7] It is also necessary to remark that the same Greek writings were originated from as many primary sources in Persian and Arabian language. See G. Ferriello, *L'estrazione delle acque nascoste trattato tecnico–scientifico*

more reliable. Reliability increased thanks to the invention of movable type printing (ca. 1450) by Johann Gutenberg[1] (1400?–1467?). Approximately, since 1474 they started to print works of mathematics, astronomy and astrology in Italy; the edition by Giovanni Campano[2] in 13[th] century might have been one of the first translations of the *Elements*[3] by Euclid (1482) in its Latin version. In such a climate and until Renaissance[4] the image of the new scientist, seen also as a student of natural phenomena, emerged. He was seen as a new type of scientist, re–born and re–qualified, not just an interested and clever astrologer and medieval theologian. Above all he looked now independent from a hypothetical and *general pre–established design.* However, the reconcilement between the divine plan and the new mathematical truths could converge into an outlined project, still divine under many aspects, considering God as the engineer who had planned a cosmological design in mathematical and geometrical terms. Koyré in his Newtonian studies wrote:

> *Once more the book of nature seemed to reveal God, an engineering God this time, who not only had made the world clock, but who continuously had to supervise and tend it in order to mend its mechanism when needed (a rather bad clockmaker, this Newtonian God, objected Leibniz), thus manifesting his active presence and interest in his creation.*[5]

God as an engineer allowed a certain *chance* of studying the divine product that is nature interpreted in mathematical terms, since in this way the object of study was still confined to a religious matter. In fact, this would explain why, among other things, the majority of the Renaissance scientists were theologians as well, who preferred to inquire into nature instead of the Holy Scriptures[6]. Therefore each discovery or mathematical invention was seen as the product of God's engineering work. Though this new way of conceiving it science was limited to the learned and the rich only, since they

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di karaji matematico–ingegnere persiano vissuto nel mille, Kim Williams Books, Torino 2007.

[1] With Gutenberg we should remark Johann Fust (1400?–1466?) and Peter Shoffer (1425?–1502?). See C. Singer, *Il rinascimento e l'incontro di scienza e tecnica circa 1500–1750* in: *Scienza della Tecnologia*, (eds) C. Singer, E. J. Holmyard, A. R. Hall, T. I. Williams, Boillati Boringhieri, Torino 1966–1968, tomo II, vol. 3, pp. 285–424 [original edition: C. Singer, *From the Renaissance to the industrial revolution, c. 1500–c. 1750* in: *A History of technology*, (ed.) C. Singer, Clarendon Press, Oxford, 1954–1958, vol. 3, pp. 285–424].

[2] Campano di Novara or Giovanni Campano (1220–1296). Details in: I. Grattan–Guinnes, *Companion Encyclopedia of the history and philosophy of the mathematical sciences*, Johns Hopkins University Press, Baltimore 1994, L. Berzolari, *Enciclopedia delle matematiche elementari e complementi*, Hoepli, Milano 1932.

[3] After 1482 was published an edition contended the four books of *Conic Section* by Apollonius from Perge (262?–180 B.C.). See *Opere* Pappus Alexandrini' (290?–350? a.C.) manuscripts and *Aritmetica* by Diophantus (III sec. a.C). The image in the paper is the first page of *Elementa* published posthumously in 1482 by Campano. Credit: http://web.unife.it/altro/tesi/A.Montanari/Euclide.htm.

[4] See E. K. Knobloch, C. Vasoli & N. Siraisi, *Il Rinascimento* in: *Istituto della Enciclopedia Italiana*, vol. 4: *Medioevo, Rinascimento*, Istituto della Enciclopedia italiana, Roma 2001, pp. 605–1044.

[5] A. Koyré, *Newtonian studies*, p. 21.

[6] See E. Grant, *La Scienza nel Medioevo*, pp. 25–54.

had a knowledge of Latin and Greek. The spread of the new culture by print was hampered by two factors. First, a lot of technicians, such as architects and engineers, would have probably welcomed the application of geometry and mathematics as *theoretical science* to arts, navigation and architecture but the precarious diffusion of school education did not give the pioneers of *scienza activa* access to the necessary scientific heritage. In fact:

> *The book* De Architectura *by the Roman architect and engineer Vitruvius (first century AD) became known in the 12th century; nearly all the works of Archimedes (3rd century BC) were translated by William Moeberke in 1269, from Greek into Latin* [...]. *But these writings had no influence on the practice of medieval craftsmen who did not understand Latin*[1].

Therefore, according to some thought currents of history of mathematics[2], the expectation about the spread of the classical culture, instead of encouraging the highest erudition among mathematicians and, in general, of scientific topics[3], paradoxically seemed to exclude just the new–born class of scientists–mechanics who, far more numerous than theoretical scientists, felt a strong interest in the introduction of mechanical devices[4] or of calculating ones within their treatises. Second, the other factor is a more philosophical one. By then theoretical knowledge was the only one to be considered full and definitive, therefore experience was meant to be of secondary use, so the discoveries of technicians were ignored, eventually causing a strange regression toward the medieval culture typical of the Scholastics of 12th century. In particular, due to the lack of mathematical devices, technicians would feed their knowledge through the development of so–called procedures *by comparison*. Models by similitude were typical, after daily practice and based upon *make mistakes and correct*[5], almost to represent a sort of a practical handbook of architecture. Sometimes, as regards some authors, we can guess interesting, though embryonic, references on the study of the strength of materials related (geometrically) to the structure, the *air quantity* or to the height of the columns[6]. The scientific applications will flow into the new technology and will require more and more the integration of local activities and the managing skill of the artisans[7]. This integration and the new

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[1] E. Knobloch, *Mathematical Methods in Preindustrial Technology and Machines* in: *Technological Concepts and Mathematical Models in the Evolution of Modern Engineering Systems*, (eds) M. Lucertini, A. M. Gasca, F. Nicolò, Birkhäuser Verlag, Basel – Berlin 2004, p. 3.

[2] See M. Kline, *Storia del pensiero matematico*, vol. 1, transl. by A. Conte, Einaudi Editore, Torino 1999, pp. 253–322 [original edition: M. Kline, *Mathematical Thought from ancient to modern times*, Oxford University Press, New York 1972].

[3] For economy of space I will not deal with the meaning of science in this historical time. As a matter of fact, the Aristotelian paradigm marked a certain distinction among what science was and what it was not.

[4] On the epistemological role played by scientific instruments one can see a recent book: S. D'Agostino, *Gli strumenti scientifici e la scienza*, Barbieri Editore, Manduria 2005.

[5] See V. Marchis, *Storia delle macchine*, Laterza, Bari 2005, pp. 3–137.

[6] See V. Marchis, *Storia delle macchine*, pp. 46–47.

[7] Charles Singer (1876–1960), Trevor Williams and Thomas Derry, tried to study the role played by

reference to the Euclidean geometry will bring together with other physical–mathematical factors – that will be the casestudy of the present thesis – to the realization of the first projects, after *the aestimatio* model, that is approximated and designed on the spot[1].

Among non–humanists approaching mathematic studies and pure mechanics, Niccolò Fontana, called Tartaglia (1500?–1577)[2] is the most outstanding figure. He could be defined as the scholar who by then best represented the crucial turn from the learned mathematician to the practical one, also endowed with skills in the field of mechanics and architecture. He was able to pick up with great attention from the architects' and engineers' practical knowledge, especially the military ones, theoretical matters which later were to prove crucial for the projections that were then reelaborated by others, even if some never mentioned Tartaglia[3], e.g. see the case of Galilei's work, *Trattato di Fortificazione*[4] and in his theory of projectiles.

3. Leonardo da Vinci' cultural background: continuity or discontinuity?

The figure of Leonardo da Vinci[5] is set in the historical period mentioned

technique in science and in particlar Alistair Crombie (1915–1996) on the birth of modern science as an evolution of artisans' skills. See A. C. Crombie, *Medieval and Early Modern Science*, vols. 1–2, [2nd ed.] Doubleday, Garden City (NY) 1959, A. C. Crombie, *Styles of Scientific Thinking in the European Tradition*, Duckworth, London 1994.

1 E.g. one can consider the golden section–geometry by drawing particular circles in a square.

2 N. Tartaglia, *La noua scientia de Nicolo Tartaglia: con una gionta al terzo libro*, edited by Stephano Da Sabio, Venetia 1537, N. Tartaglia, *Opera archimedis Syracusani philosophi et mathematici ingeniosissimi per Nicolaum Tartaleam Brixianum ... multis erroribus emendata, expurgata, ac in luce posita, ... Appositisque manu propria figuris quae graeco exemplari deformatae, ac deprauatae erant, ad rectissimam symetriam omnia instaurata reducta & reformata elucent*, apud Venturinum Ruffinellum, sumptu & requisizione Nicolai de Tartaleis Brixiani, mense Aprili 1543, N. Tartaglia, *Ragionamenti de Nicolo Tartaglia sopra la sua la trauagliata inuentione: nelle quali se dichiara uolgarmente quel libro di Archimede siracusano intitolato De insidentibus aqueae*, per Nicolo Bascarini, Venetia 1551, N. Tartaglia, *Quesiti et inventioni diverse de Nicolo Tartaglia brisciano* (1st ed. 1546), 1554 [for a modern edition: N. Tartaglia, edited by A. Masotti, Ateneo di Brescia Editore, Brescia, 1959], N. Tartaglia, *Il general trattato di numeri et misure*, per Curtio Troiano Navò, Venetia 1556–1560, N. Tartaglia, *Iordani Opvscvlvm de Ponderositate*, Nicolai Tartaleae Stvdio Correctvm Novisqve Figvrisavctvm. Cvm Privilegio Traiano Cvrtio, Venetiis, Apvd Curtivm Troianvm MDLXV, 1565. For a reading from Tartaglia to Torricelli: R. Pisano & D. Capecchi, *Il principio di Torricelli prima di Torricelli* in: *Proceedings of XXIV Congresso Nazionale di Storia della Fisica e dell'Astronomia*, Napoli – Avellino 2007, pp. 107–112, R. Pisano & D. Capecchi, *La meccanica in Italia nei primi anni del Cinquecento. Il contributo di Niccolò Tartaglia* in: *Electronic Proceedings of XXV SISFA Congress*, http://www.brera.unimi.it/SISFA/atti/atti2005.html, Milano 2008, pp. C08.1–C08.6, R. Pisano & D. Capecchi, *La teoria dei baricentri di Torricelli come fondamento della statica* in: *Physis* 44, 1/2007, pp. 1–29, R. Pisano, *Il ruolo della scienza archimedea nei lavori di meccanica divGalilei e di Torricelli* in: *Da Archimede a Majorana: La fisica nel suo divenire, Proceedings XXVI SISFA Congress*, (eds) E. Giannetto, G. Giannini, D. Capecchi, R. Pisano, Guaraldi Editore, Rimini 2009, pp. 65–74.

3 It is known that at that time it was usually assumed that the learned reader knew certain theories. In this sense the lack of quotation marks sometimes had a meaning different from what it has today.

4 G. Galilei, *Breve instruzione all'architettura militare* in: *Opere Nazionali di Galileo Galilei*, vol. 2, edited by A. Favaro, Barbera Editore, Firenze 1890–1909, pp. 15–75, G. Galilei, *Trattato di fortificazione* in: *Opere Nazionali di Galileo Galilei*, vol. 2, pp. 77–146.

5 R. Pisano & D. Capecchi, *Leonardo da Vinci. Recenti riflessioni storico–epistemologiche sulla deformabilità dei corpi* in: *Proceedings of XLVI Congresso nazionale AIF* 40, 2007 in: *La Fisica nella Scuola*, supp. 3/2008, pp. 120–129, R. Pisano, F. Di Pietrantonio, *Meccanica ed architettura nel Rinascimento. Note su Leonardo da Vinci* in: *Epistemologia e didattica*, vol. 2, Laveglia Editore, Salerno 2007, pp. 197–219. On Leonardo da Vinci: P.–M. Duhem, *Les origines de la Statique*, vols. 1–2, Hermann, Paris 1905–1906, P.–M.

above. Generally, when the so–called *scientia pratica* of the Renaissance is referred to, we are reminded of engineers and, consequently, of Leonardo da Vinci (1452–1519), the great scholar who sums up a multiplicity of competences that nowadays would be considered as different crafts: from the engineer, architect, scientist to the artist. Although some studies, such as from Pierre Duhem (1861–1916), Roberto Marcolongo, (1862–1943), Clifford Truesdell (1919–2000) and Bertrand Gille (1920–1980) suggest a review of Leonardo da Vinci's role as a genius, in favour of a more human figure of a *learned man*, endowed with a quick intelligence, e.g. not all his designs about machines sprang out straight of his fantasy. Thus:

> *Leonardo da Vinci is perhaps overrated for his contributions to science, since his was more the mentality of the engineer; his notebooks are neither systematic nor lucid expositions of physical concepts. Yet he too supplied an important ingredient, wrestling as he did with practical problems of mechanics with great genius and technical ability. He brought alive again the tradition of Jordanus Nemorarius and Albert of Saxony, and his speculations on kinematics and dynamics, if inconclusive, reveal how difficult and elusive were the conceptual foundations of mechanics for its early practitioners*[1].

Further, that is also confirmed by the discovery of a manuscript by the great architect and engineer Francesco di Giorgio Martini from Siena (1439–1501) where in the part on machines several notes in Leonardo da Vinci's hand were re–discovered. Taking into account that modern historiography reached the conviction that Leonardo got his results in part from other sources or that he would have written them previously together with other authors[2], we can

Duhem, *Etudes sur Léonard de Vinci*, Hermann, Paris 1906–1913, R. Marcolongo, *Lo sviluppo della meccanica sino ai discepoli di Galileo* in: *Memorie della R. Accademia dei Lincei*, s. 5°, vol. 13, fasc. II, 1919, pp. 80–138, R. Marcolongo, *La meccanica di Leonardo da Vinci*, Stabilimento Industrie Editoriali Mendionali, Napoli 1932, L. da Vinci, *I libri di meccanica nella ricostruzione ordinata di Arturo Uccelli preceduti da una introduzione critica e da un esame delle fonti*, edited by by A. Uccelli, Hoepli, Milano 1940, C. Pedretti, *Leonardo architetto*, Electa Editore, Milano 1978, C. Pedretti, *Leonardo. Le macchine*, Giunti Editore, Firenze 1999. On the Internet the most important works on Leonardo da Vinci are edited by P. Galluzzi, direttore dell'*Istituto e Museo di Storia delle Scienze di Firenze*: http://www.imss.firenze.it/indice.html), *Museo della Scienza e della Tecnica* "Leonardo da Vinci" di Milano: http://www.museoscienza.org/leonardo/default.asp, *Archivio digitale di storia della tecnica e della scienza, Biblioteca Leonardiana*, http://www.leonardodigitale.com/login.html by Comune di Vinci (Fi), P. Galluzzi, *Leonardo Da Vinci: Engineer and Architect*, Montreal Museum of Fine Arts, Montreal 1988.

[1] W. A. Wallace, *Experimental Science and Mechanics in the Middle Ages* in: Ph. P. Wiener (ed.) *The Dictionary of the History of Ideas: Studies of Selected Pivotal Ideas* vol. 2, Charles Scribner's Sons, New York 1973–1974, pp. 196–205 [available at: University of Virginia Library http://www.historyofideas.org/cgi–local/DHI/dhi.cgi?id=dv2–23, Electronic Text Center].

[2] To support these hypotheses in the 1970's Gille mentioned a *Manuscript* by architect–engineer Francesco di Giorgio Martini from Siena. It is *Ashburhamiano N. 361* and placed in Laurenziana Bibliotheca of Firenze. The manuscript was not complete but included most part of mechanics and notes by Leonardo da Vinci. This aspect brings Gille to conclude that for a long time everyone thought that the author of the manuscript was just him, Leonardo da Vinci. See B. Gille, *Leonardo e gli ingegneri del Rinascimento*, transl. A. Carugo, Feltrinelli, Milano 1972, p. 128, [original edition: B. Gille, *Les ingénieurs de la Renaissance*, Hermann, Paris 1964. On this theme see also L. Russo, *La rivoluzione dimenticata. Il pensiero scientifico greco e la scienza moderna*,

reasonably make the hypothesis that the abundance of materials about his scripts and the lack of it in other cases could also be due to greater care when searching the documents of the brilliant scholar. So it is difficult to make a hypotheses about an artist's inspiration. In fact, without a proper method of historical inquiry it is not so easy to deduce from his manuscripts what one author takes from another and what really represents scientific continuity or discontinuity:

> *After Aristotle and Ptolemy the idea that the earth moves – that strange, ancient, and "entirely ridiculous"*[1] *Pythagorean view – was thrown on the rubbish heap of history, only to be revived by Copernicus and to be forged by him into a weapon for the defeat of its defeaters. The Hermetic writings played an important part in this revival, which is still not sufficiently understood*[2]*, and they were studied with care by the great Newton himself*[3] *such developments are not surprising. No idea is ever examined in all its ramifications and no view is ever given all the chances it deserves. Theories are abandoned and superseded by more fashionable accounts long before they have an opportunity to show their virtues.*[4]

In other words but in the same vein, Mach wrote:

> *1. That branch of physics which is at once the oldest and simplest and which is therefore treated as introductory to other departments of this science, is concerned with the motion and equilibrium of masses. It bears the name mechanics. 2. The history of the development of mechanics, is quite indispensable to a full comprehension of the science in its present condition. It also affords a simple and instructive example of the process by which natural science generally is developed.*[5] [...]. *They that know the entire course of the development of science, will, as a matter of course, judge more freely and more correctly of the significance of any present scientific movement than they, who limited in their views to the age in which their own lives have been spent, contem-*

Feltrinelli, Milano 1996, pp. 385–388.

[1] Ptolemy quoted by Feyerabend: P. Feyerabend, *Against Method*, Humanities Press, London 1996, p. 35, n. 4.

[2] Ptolemy quoted by Feyerabend: P. Feyerabend, *Against Method*, p. 35, note n. 5.

[3] Ptolemy quoted by Feyerabend: P. Feyerabend, *Against Method*, p. 35, note n. 6.

[4] P. Feyerabend, *Against Method*, p. 35.

[5] E. Mach, *The Science Of Mechanics – A Critical And Historical Account Of Its Development*, [4th ed.] Open Court–Merchant Books, La Salle 1996, p. 1.

> *plate merely the momentary trend that the course of intellectual events takes at the present moment.*[1]

Leonardo is an artist but also a technician and a scholar and [...] *it would be a mistake, assuming a position systematically too antithetic to the official thesis, to assimilate his notes to a definitive work of art*[2]. Then, we must say that an indirect continuity in a bend toward science shown by Leonardo emerges when considering that the themes he dealt with had already been studied in early 1400 by Mariano Taccola from Siena who was interested in the scripts of mechanics and military technology of Pneumatica[3] by Philon of Byzantium (280–220 B.C.). As the majority of engineers by that time, Leonardo also studied the engineering works by Heron from Alexandria (I c. B.C.) though considered useless toys. On the other hand, they got enthusiastic before the futuristic technical designs by Leonardo in that when not copying it, they were strongly influenced by them, such as Hero's engine, windwheel, vending machine, force pump, Heron's fountain, et al.[4] Gille ends up his book with a hope:

> *All our engineers were men of war. Such statements of the obvious have the uncomfortable habit of often being true. Yet the sixteenth century had passed beyond warlike preoccupations and had constructed a complete technical system, just as it had built a new scientific system. More than their quest for deadly power, more than the amusements and the love of images, what has attracted us in these men is the difficult apprentice ship they served in a new world. Much remains to do before we understand the processes of their thought, before we appreciate their hesitations and grasp the nature of their ignorance and their failures. We must underline their gradual distortions of accepted truths, their difficult departures from the traditional paths, in order to give them credit for having* [...] *unique advance in the history of thought.* [...]. *But the enquiry remains open: it might bring to light other works still languishing in the dust of libraries, it might also provide a more precise analysis of the notebooks which have never been published and which are full of information.*[5]

Nowadays Leonardo da Vinci's cultural matrix seems clear. Historians agree in considering the Aristotelian physics as the main source of his mechanics.

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[1] E. Mach, *The Science Of Mechanics* ... , p. 7.

[2] L. Russo, *La rivoluzione dimenticata* ... , p. 282 [English transl. is mine – R. P.].

[3] The Italian translation of *Pneumatica* by Filon from Byzantium is reported in the first part of *War and hydraulic machines in 15th century*. See L. Russo, *La rivoluzione dimenticata* ... .

[4] See L. Russo, *La rivoluzione dimenticata* ... , p. 389 [English transl. is mine – R. P.].

[5] B. Gille, *Engineers of the Renaissance*, MIT Press, Cambridge (Mass.) 1966, p. 240.

According to such studies from the analysis of *Codes* by Leonardo, it was possible to deduce some of the titles of the manuscripts, not entirely scientific, used by Leonardo for the researches: Al Kindi, *Libellum sex quantitatum*, Mondino de' Luzzi, *Anathomia*, Giorgio Valla, *De expetendis et fugiendis rebus*, Aristotle, *De phisica* and *De metheoris,* Giovanni Peckam, *De prospective*, Piero de' Crescenzi, *Libro dell'agricoltura*, Francesco di Giorgio Martini, *Trattato di architettura militare e civile*, Paolo dell'Abaco, *Recholuzze del maestro Pagolo astrolacho*, Leon Battista Alberti, *De picture*, Euclide, *De ponderibus, De levi et ponderoso fragmentum* and *De prospective*, Luca Pacioli, *De divina proportione*, Plinio il Vecchio, *Naturalis*, Giovanni di Mandinilla, *Tractato delle più maravigliose cosse e più notabili*, Cristoforo Landino, *Formulario di epistole volgari* ... et al.[1]

Actually, even if Leonardo da Vinci's research works concern almost exclusively the fields he practiced as a technician, a need of a mathematical–geometrical abstraction and of rationalization seems to emerge; apparently neglected until then by technicians, there was an exigency to define technique through observation and the mathematical explanation of phenomena. Nonetheless it is worth remarking that a consequence of this early form of *discontinuity* is the fact that Leonardo da Vinci's method surely did not spring out of nowhere. It is rooted in the scientific tradition of the Aristotelian school, further than in the Archimedean one. More specifically, many are the traces of Aristotle' thought[2] to be found in Leonardo, starting with the concept that the knowledge of *universal things* (*the furthest from our senses, in contrast with the singular things which are the closest to our sensible perception*) is acquired by means of reasoning based on primitive truths that cannot be proved; the latter can be known by induction, that is by means of data of the sensible perception stored in our memory:

> *From sense, therefore, as we say, memory is produced, but from repeated remembrance of the same thing, we get experience, for many remembrances in number constitute one experience. From experience, however, or from every universal being at rest in the soul that one besides the many, which in all of them is one and the same, the principle of art and science arises, if indeed it is conversant with generation, of art, but if with being, of science.*[3]

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[1] See E. Solmi, *Scritti vinciani. Le fonti dei manoscritti di Leonardo da Vinci e altri studi*, La nuova Italia, Firenze 1976. See also R. Pisano & D. Capecchi, *Leonardo da Vinci. Recenti riflessioni storico–epistemologiche sulla deformabilità dei corpi* and R. Pisano, *Il ruolo della scienza meccanica nella progettazione degli architetti e degli ingegneri del Rinascimento*, Ph.D. dissertation from University of Roma "La Sapienza", 2008, vol. 1, pp. 116–134 [available in pdf format via: *International Galilean Bibliography, Istituto e Museo di Storia delle Scienze*, Firenze: http://biblioteca.imss.fi.it/].

[2] Aristotle is one of the most important sources for Leonardo, especially for mechanics. See M. E. Bottecchia Dehò, *Per una edizione dei Mechanica di Aristotele* in: *Annali della Facoltà di Lettere e Filosofia dell'Università di Padova* 2, 1977, pp. 43–53, Aristotle, *Mechanical Problems* in: Aristotle, *Minor Works*, transl. W. S. Hett, Harvard University Press, Cambridge (Mass.) 1936.

[3] Aristotle, *The Posterior Analytics*, 354a17, transl. O. F. Owen in: Aristotle, *The Organon or Logical Treatises of Aristotle*, Bohn H. G., London 1853. See also L. da Vinci, *I libri di meccanica nella ricostruzione*

At the same time, Leonardo draws[1] on Archimedes' *scientia*, in particular he shares the methodology based on the study of the equilibrium of bodies, that is he follows the rational criteria that the mathematician from Syracuse had set to determine the centres of gravity[2]. Leonardo introduces the concept of *pratica* as the basis of any of his studies, declining it either as observation, a study of buildings, of human anatomy and natural phenomena, or as an experiment aimed at checking up the calculations derived from his observation. On the other hand, he defines himself *discepolo della sperienza*[3]. To him, from *experience* we can derive, beyond good building practices, also rules that are not only the expression of aesthetic research but principally requirements for the proper performance of the *building organism*, considered at the same time as a living organism or a *macchina–ingegno*[4]. With Leonardo, it very often recurs, perhaps for the first time, the idea of an absolutely efficient building–machine. Within it daily activities are made rational and mechanic: e.g. a fireplace automatically operated, a laundry, the model of a stable[5]. The building is conceived as a *living organism* but, at the same time, in a sense, taking Vitruvio' concepts to the extreme, he suggests also the way round. In other words, living organisms too – men and animals – are turned into *macchine*. In this sense, he detects in any organism, living or not, a unity of process and function based on movement and considers animals as a human body and buildings as a whole of mechanical devices, that he calls *elementi macchinali*[6]. *Bird is a device performing after a mathematical law*[7] [...] *and nature cannot make animals move without 'mechanical devices'.*[8] Leonardo da Vinci's considerations around such *mechanical elements* and his studies of anatomy are really interesting, proving study and performance methods very similar:

ordinata ... , p. 38, Aristotle, *Aristotle's Prior and Posterior Analytics. A Revised Text with Introduction and Commentary* by W. D. Ross, Clarendon Press, Oxford 1949.

[1] See L. da Vinci, *I libri di meccanica nella ricostruzione ordinata* ... p. 70.

[2] See R. Pisano, *Brief historical notes on the theory of centres of gravity* in: *The Global and the Local: The History of Science and the Cultural Integration of Europe – Proceedings of the 2nd International Conference of the European Society for the History of Science*, (ed.) M. Kokowski, Polish Academy of Sciences, Warszawa 2007, pp. 934–941, J. Renn, P. Damerow & P. McLaughlin, *Aristotle, Archimedes, Euclid, and the Origin of Mechanics: The Perspective of Historical Epistemology*, Max Planck Institute for the history of science of Berlin print n. 239, Berlin 2003, pp. 43–59.

[3] C. Pedretti, *Leonardo. Le macchine*, p. 36.

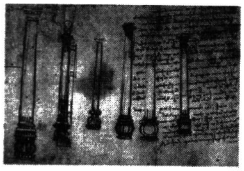
[4] In the Renaissance, as in the Old Age, the term *ingegno* pointed out the mechanism of a machine, and by extension, the machine itself. For Vitruvius, e.g. the *ingegno* is a machine that requires only one person to be put at work. The term *building* could apply to the concept of machine (e.g for war or water) and vice versa to machine (e.g. Milano Dom machine). See C. Pedretti. *Leonardo architetto*, p. 309.

[5] The model of the stall is described in *Manoscritto B* (f. 38v.–39r.). See also L. da Vinci, *Frammenti sull'architettura* in: L. da Vinci, *Scritti rinascimentali di architettura*, edited by A. Bruschi, C. Maltese, M. Tafuri, R. Bonelli, Il Polificio Editore, Milano 1978, pp. 308–309.

[6] P. Galluzzi, *Gli Ingegneri del Rinascimento*, Giunti Editore, Firenze 1996, p. 192.

[7] C. Pedretti, *Leonardo. Le macchine*, p. 44: *L'uccello è strumento oprante per legge matematica*. [English transl. is mine – R. P.]

[8] P. Galluzzi, *Gli Ingegneri del Rinascimento*, p. 192: *E la natura non può dar moto alli animali sanza strumenti macchinali*.



All such instruments will generally be presented without their armatures or other structures that might hinder the view of those who will study them.[1]

This *uniformity of treatment* emerges in his drawings as well, either anatomic, where bones and muscles are handled as geometrical schemes of *ingegni*[2], or of machines and tools, in which relevant specific elements insist, such as the cannons–columns[3] that seem to claim the universality of the planning project.

4. What kind of tradition until Leonardo?

Generally, the physical Aristotelianism and its mechanization[4] can essentially be viewed as the whole of the knowledge based upon experience, concerning all outer phenomena of the sensible universe. Leonardo da Vinci's mechanics, after the tradition of the Aristotelian physics, focuses on the motions that can occur in a mechanism as its study object. Under the aspect of logic and organization of the theory, Leonardo studies problems of engineering concerning his profession, e.g. strength of masonry in buildings, arch–thrust, wall cracking. From his studies it is evident[5] that his reasoning, at first intuitive, later proceeds by analogy and approximation, by means of a series of experiences, in search of absolute rigour. In order to do this, approximation useful also in practical applications is sufficient. As a matter of fact, it might have been the early use of such approaches to kind of works previously approached by traditional means and hardly rational evaluation methods[6]. Leonardo also studies the typical problems of mechanics that are nowadays solved by the building science, axial and flexional analysis of a wood beam: materials submit to strength by a column axially and flexionally loaded and strength of horizontal beams. Through a rough calculation and after *experience*, he is able to set the laws upon deformability of a wood beam, strength of beams with square or circular section, simply supported or clamped at one or two extremities, strength of composite beams. Leon Battista Alberti (1404–1472) and Francesco di Giorgio Martini also had dealt with the same problem, but Leonardo was among the first who tried to elaborate it by mathematical and geometrical approach. Leonardo aims to establish the *universal* laws ruling the attitude of the materials, of the building elements, of *ingegni*, in an attempt at reconciling what Paolo Galluzzi defines as *two*

[1] L. da Vinci, *Codice di Madrid I*, c. 82r.: *E tali strumenti si figureranno in gran parte sanza le loro armadure o altra cosa che avessi a impedire l'occhio di quello che le studia.*

[2] The design by Leonardo upon the wings of the birds as part of machines hypothesized to fly. *Codice Windsor* RL 12656, ca. 1513–1514, *Codice sul volo degli uccelli*, f. 17r, ca. 1505.

[3] The picture in the text: *Macchine come elementi architettonici: studi di artiglieria* in: L. da Vinci, *Codice Atlantico*, f. 28v.–a, ca. 1495–1497.

[4] E. J. Dijksterhuis, *The mechanization of the world picture*, transl. C. Dikshoorn, Clarendon Press, New York 1961.

[5] See B. Gille, *Leonardo e gli ingegneri del Rinascimento*, pp. 212–213.

[6] See B. Gille, *Leonardo e gli ingegneri del Rinascimento*, p. 213.

mechanical traditions[1]. First, mechanics, until the Middle Age, had generally neglected the practical aplication. Second, mechanics practiced and transmitted within the renaissance *botteghe*. The historical contextualization of tradition suggested by Gille[2] is interesting, either concerning his interpretation on setting the authors until the Renaissance or thereafter by different traditions of mechanics[3], engineering and theoretical. I adapted and re–interpreted *ad hoc* it in the following scheme, basing it on a distinction between Aristotelian and Archimedean modes[4], as thought currents until Leonardo, too.

Theoretical and Archimedean tradition:	**Engineering and Aristotelian tradition:**
J. Fontana (1393?–1455?)	K. Kyeser (1366–?)
L. B. Alberti (1404–1472)	Brunelleschi (1377–1446)
F. di Giorgio Martini (1439–1502)	Mariano J. detto il Taccola (1382–1458?)
N. Tartaglia (1500?–1557)	Valturio (1413–?)
Leonardo da Vinci (1452–1519)	

Starting from the *method of comparison*, Leonardo compares the strength of beams and columns composed by same composition, different height and the same line section and vice versa. A beam is more rigid than another one if, under the same strength, it deforms less, or, in order to get an equal deformation, the introduction of a greater strength is required. In this case, the concept of strength is presented as stress (imagined–)concentrated in a point. This way it is possible to neglect what today we define such as a modulus of elasticity of bodies and, for the beams of the same section line, the momentum of inertia belongs to the section. By associating mathematical calculations and experiences, Leonardo da Vinci obtained that (1) in the case of supports with square section uniformly loaded at the top the strength to axial force is proportional to the surface of the section and inversely proportional to the ratio between height and the side of the square (or the radius of circular cross section)[5], (2) for a square section beams simply supported and loaded in the middle (*mezzeria*) by a weight, the strength is inversely proportional to the weight, inversely proportional to the length and directly proportional to the square of the side of the section.

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[1] P. Galluzzi, *Gli Ingegneri del Rinascimento*, p. 78. It is necessary to remark that upon this subject Drake already envisaged a distinction between two Italian schools of mechanics, based on the different geographical–cultural position and on the different affairs that every component of a school usually had in comparison to the others. See S. Drake & I. E. Drabkin, *Mechanics in Sixteenth–Century Italy: Selections from Tartaglia, Benedetti, Guido Ubaldo, and Galileo*, University of Wisconsin Press, Madison 1996, pp. 13–16.

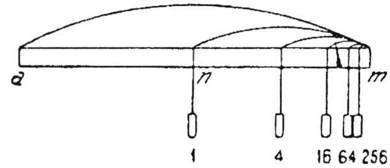
[2] See B. Gille, *Leonardo e gli ingegneri del Rinascimento*.

[3] J. E. Brown, *The "Scientia de ponderibus" in the later middle ages*, University of Wisconsin Press, Madison 1967–1968, E. Grant & J. E. Murdoch, *Mathematics and its applications to science and natural philosophy in the Middle Ages: essays in honour of Marshall Clagett*, Cambridge University Press, Cambridge 1987, M. Clagett & E. A. Moody, *Medieval science of weights*, University of Wisconsin Press, Madison 1952.

[4] R. Pisano, *Brief historical notes on the theory of centres of gravity*, pp. 934–941.

[5] That's the problem of buckling. Tension (in theory of elasticity) is directly proportional to fourth grade of length of section and inversely proportional to square of length of column. D. Capecchi, *Scienza delle costruzioni*, CISU, Roma 1997, pp. 293 sq. For a historical reading on foundations: C. A. Truesdell, *Essay in the history of mechanics*, Springer Verlag, Berlin – Heidelberg – New York 1968, C. A. Truesdell, *The rational mechanics of flexible elastic bodies* (1638–1788) in: *Introduction* to *Leonhardi Euleri Opera Omnia*, Venditioni Exponunt Orell Fussli Turici, II s., vol. 11, Losanna – Zurich 1960.

Se 'l diamitro del quadrato composto dalla fronte del trave fia la ventesima parte di tutta la sua lunghezza, e 'l mezzo d'esso trave posto per linia equidiacente resista a mille, sappi che la metà della lunghezza d'esso trave ha il diamitro della sua fronte che sarà di doppia proporzione alla lunghezza sua a comparazione di quella di prima, e crescerà per modo che la sua potenzia fia quadrupla alla prima, e dupla potenzia a quel di prima acquista per essere la metà più corta; che direno adunque che la resistenzia del mezzo mn fia secupla e am.



Pict. 1 *Codice Forster*, II f. 96r.

As regards his studies of strength of buildings, Leonardo is interested in the causes of sliding, collapse and crack:

> *When the crevice in the wall is wider at the top than at the bottom, it is manifest sign, that the cause of the fissure in the wall is remote from the perpendicular line through the crevice.*[1]

Furthermore, he spots the relation between the solidity of walls and the composition of the ground:

> *Parallel fissures constantly occur in buildings which are erected on a hill side, when the hill is composed of stratified rocks with an oblique stratification, because water and other moisture often penetrates these oblique seams carrying in greasy and slippery soil, and as the strata are not continuous down to the bottom of the valley, the rocks slide in the direction of the slope, and the motion does not cease till they have reached the bottom of the valley, carrying with them, as trough in a boat, that portion of the building which is separated by them from the rest.*[2]

Later, he adds suggestions and precautions to be taken for construction, e.g. raising the walls by successive layers using a mortar perfectly uniform for

[1] Transl. J.-P. Richter in: *The notebooks of Leonardo da Vinci*, compiled and edited from the original manuscripts by J.-P. Richter, Sampson, Low, Marston, Searl and Rivington, London 1888, p. 771 [reprinted: Dover Publication, New York 1970]. L. da Vinci, *Codice Arundel*, f. 157v: *Quando la fessura del muro è più larga di sopra che di sotto, elli è manifesto segno che la muraglia ha la causa della ruina remota dal perpendicolare d'essa fessura*. See also L. da Vinci, *Frammenti sull'architettura*, ... p. 77.

[2] Transl. J.-P. Richter in: *The notebooks of Leonardo da Vinci*, p. 770. L. da Vinci, *Codice Arundel*, r. 15: *Li fessi paralleli sono universalmente generati in quelli edifizi che si edificano i' lochi montuosi, li quali sien composti di pietre faldate con obliquo faldamento. E perchè in tale obliquità spesso penetra acqua e altra umidità portatrice di certa terra untuosa e sdrucciolente, e perchè tali falde non sono continuate insino al fondo delle valli, tali pietre si movan portando con seco quella parte de lo edifizio che per loro si separa dal suo detto rimanente..*

the regularity and the compactness of the whole[1]. Leonardo da Vinci's observations on beams concern either the axial and flexional behaviour. For this last issue he focused more attention on its buckling[2]. These considerations are interesting though not always formal and precise experimentally. More in detail for the decline Leonardo is more concerned with deformability than strength[3]. The reason could be that he refers mainly to the timber used in buildingwar machines and ships. These beams are very thick and resistant to failure, so they are essentially dimensioned for deformation. In the following comments by Leonardo where he dealt with clamped and truss beam[4]:

> *(P_1) One beam of 6 braccia is stiffer the double in its middle, than four equal sized beams of 12 braccia joind together*[5].

Based on recent research[6], the previous observation of Leonardo is in accordance with modern theory of elasticity of beams: a supported beam of constant section, highlighted *l* by means of a concentred force *f* applied to *mezzeria*. The arrow *v* is mathematically interpreted[7] by the following formula:

$$v = \frac{1}{48}\frac{fl^3}{EI} \quad (1),$$

where E is the longitudinal modulus and I the moment of inertia of the section. From the previous track and considering (1), from *6* to *12* arms, that is, doubling the light, the same section and force *f* by formula above the arrow increased 8 times or *rigidezza* (rigidity) decreases 8 times. But *4* of *12* auctions arms absorb each *1/4* of the force *f* to which the arrow of four auctions together is equal to that of an individual charged with *1/4 f*. The fall of each beam of 12 arms worth *1/4* to *8* times so it is only 2 times that of an arm of *6*. It is thus the result of Leonardo in (P_1).

The (P_1), however, does not appear as a general law, like: a beam composed of *n* arms (*braccia*) is *8* times more rigid than a beam of *n/2* arms. It seems rather the reporting of test results, idealized as reported to integer

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[1] L. da Vinci, *Codice Arundel*, f. 157v., L. da Vinci, *Frammenti sull'architettura*, p. 297.

[2] They often were interested in the problem of force applied to the top of a beam supported at the side and clamped other one. See D. Capecchi, *Storia della scienza delle costruzioni*, Progedit Editore, Bari 2003.

[3] Galilei was interested in resistance. See G. Galilei, *Discorsi e dimostrazioni matematiche sopra due nuove scienze* in: *Opere Nazionali di Galileo Galilei*, vol. 8, pp. 49–362, G. Galilei, *Le Mecaniche* in: *Opere di Galileo Galilei*, vol. 2, pp. 46–188. *Le Mecaniche* by Galilei appears in two versions: *versione breve* (4 manuscripts) and *versione lunga* (14 manuscripts). *Le Mecaniche* in the Italian national edition by A. Favaro (1891) is *versione lunga*. See G. Galilei, *Galileo Galilei. Le Mecaniche*, edited by R. Gatto, *Le Mecaniche*, Olschki Editore, Firenze, 2002. See also G. Galilei, *Le Mecaniche* in: *Opere di Galileo Galilei*, vol. 2, pp. 155–193.

[4] The quotations are not clear and it looks like when they refer to a square section. See D. Capecchi, *Storia della scienza delle costruzioni*.

[5] L. da Vinci, *I libri di meccanica nella ricostruzione ordinata* ... , p. 230: (P_1) *L'aste di 6 braccia è più forte il doppio nel suo mezzo, che non quattro aste di 12 braccia di simili grossezze, legate insieme.* in: L. da Vinci, *Codice Atlantico* 211, recto b, new edition page: 562 r.

[6] D. Capecchi, *Storia della scienza delle costruzioni*, R. Pisano, *The role played by mechanical science in the architects and engineers design in the Renaissance*, vol. 1, pp. 24–148.

[7] See D. Capecchi, *Storia della scienza delle costruzioni*.

numerical values. Moreover, the experimental capabilities of the times, and perhaps the approach to these studies did not allow to design and verify when the law is different from that expressed by integers. It should not be forgotten that the language of mathematics based on the proportions made it difficult to determine the structure for each physical law that applied sizes varying from each others. In the case of the cantilever Leonardo still makes a quantitative rule:

> *Let the support nb to be as much stiff in n as the support cd in c. The reason is that, since the support cd has a diameter double than the above quoted support ab, it results, thanks to fifth proposition, 4 times greater than support ab; but by considering equal distance from their fixed parts, it carries* 4 times the weight. [...] *Now one half* [in length] *of the support ab, that is nb, is of equal stiffness in n than in c, the extreme point of the support cd, because, if the dimension of one is 20* [or: *2?*] *times the dimension of the other,* the other one is *in the same proportion, then if subjected to equal weight, they will have equal resistance.*[1]

If the following sentence *sia di tanta resistenza* (a lot of strength), in the first part of the passage, and *sia di pari sostentamento* (equal sustenance) in the second part could by interpreted by *have the same rigid situation* and making the hypothesis that the reference to the *less rigid* 20 (*snellezza*) has only a limited value, then the P_2 provides the following law:

$$v \propto f \frac{l^2}{h^2} \quad (2)$$

Following analyses by Danilo Capecchi[2], I agree that expressing an opinion on the historical foundations of inconsistency from (P_1) to (P_2) is still complicated. One can assume that Leonardo considered that cantilevers could be subject to different laws than inflexional beams[3]. Or, more simply, that the report (2) could refer to two separate studies in time and of Leonardo da Vinci's thought. In another proposition Leonardo seems to contradict Leonardo da Vinci's rule (P_2) because he presents a case where the bodies vary according to an inverse proportion to l and not to l^2:

[1] See D. Capecchi, *Storia della scienza delle costruzioni*, p. 235. See also L. da Vinci, *Codice Atlantico* f. 86 verso a, new edition page: 234 v.: (P_2) *Il sostentaculo* n b *sia di tanta resistenzia in* n, *quanto il sostentaculo* cd *si sia in* c. *La ragion si è, che 'l sostentaculo* cd, *per l'essere di duplicato diamitro nella sua grossezza al sostentaculo di sopra, ab, viene, per la quinta proposizione, a essere 4 tanti più grosso che esso sostentaculo* ab*; e però, in pari distanzia dai loro, immobili fermamenti, 4 tanti più peso sostiene.* [...] *Ora il mezzo del sostentaculo ab, cioè nb, sia in n di pari sostentamento a c, stremità del sostentaculo c d, perchè, se 'l diamitro della grossezza dell'uno entra nella sua medesima grossezza 20 volte, l'altro si trova in sé della medesima proporzione, e però a uno medesimo peso saranno di pari resistenza..*

[2] See D. Capecchi, *Storia della scienza delle costruzioni*.

[3] See R. Marcolongo, *La meccanica di Leonardo da Vinci*, p. 100.

> *If you join together 9 beams having equal properties* [sections and materials], *you will find as much force and resistance, as in the ninth part* [in length] *of one of them. ab supports 27 and if they are 9 beams, so cd, that is the ninth part of them, can support 3; in this way, e f, that is the ninth part in length of cd will support 27 because is 9 time shorter than it.*[1]

The proposition is consistent and valid if it is assumed that – in this case – Leonardo, for the first time, takes into account the strength at break and not the deformability. Hence, on the basis of the laws of statics[2], the strength of the beams varies in inverse proportion to the length. In fact, the flexional momentum in case of power concentrated in the *mezzeria* varies linearly with the light of the beam and the momentum of strength is constant.

5. Concluding reflections

Aristotelian physics and the same approach to Archimedean mechanical method gives Leonardo da Vinci's method a certain *continuity* with the ancient foundations of ancient science, but we should also remark that the contribution of Leonardo appears to offer a certain early vein of *discontinuity* with the tradition of the late Middle Ages as concerning the applications to architecture, especially from the methodological point of view. In fact:

> *Most historians are agreed that some break with Aristotle was necessary before the transition could be made from natural philosophy to science in the classical sense.* [...] *The fourteenth century marked the high point in optical experimentation and in the conceptual development of mechanics during the late Middle Ages.*[3]

Indeed, compared to the other *artisan–engineers* who were his contemporaries, Leonardo had in fact the merit of trying a more thorough analysis of the causes and the general principles underlying the sensible phenomena. In short, Leonardo's studies in mechanics can be summed up as follows:

a) search for a law based on the proportions and a thesis derived inductively from experience,
b) Leonardo does not submit a general law,
c) difficulty in conceiving the type of relationship between size and law expressed using integer numbers,
d) extension of the proposition to geometrically different beams: a method of

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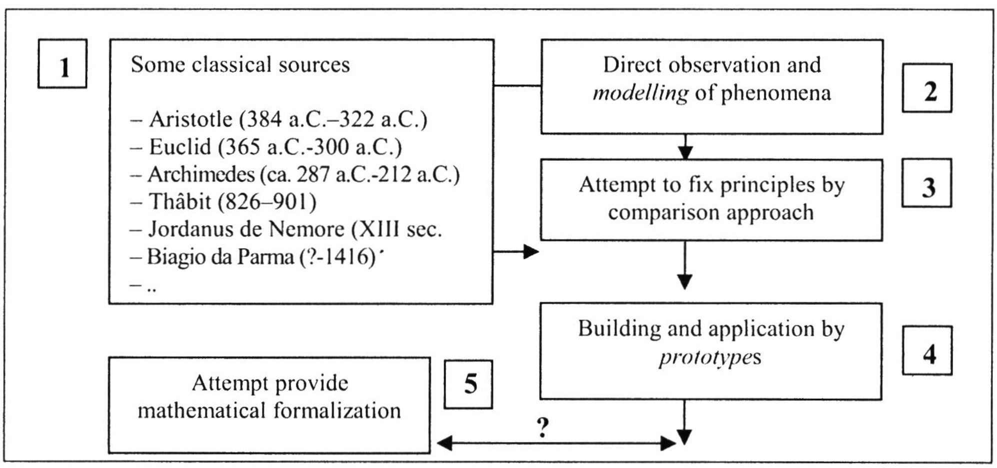
[1] L. da Vinci, *Codice Atlantico* Af. 152 recto b, new edition page: 410 r.: (P_3) *Tu troverai tal forza e resistentia nella collegatione di* 9 *travi di pari qualità, quanto nella nona parte d'una di quelle.* a b *sostiene* 27 *e son 9 travi, adunque e d, ch'è la nona parte d'essi, sostiene* 3*; essendo così, e* f*, che è la nona parte della lunghezza di* d*, sosterrà* 27 *perchè è* 9 *volte più corto di lui*. See also: L. da Vinci, *I libri di meccanica nella ricostruzione ordinata* ... , p. 236.

[2] It is independent in composition but assuming a flexional physical system and ignoring the case of cut–torsion.

[3] W. A. Wallace, *Experimental science and mechanics in the Middle Ages*.

analogy and comparison conserving the ratios.

To summarize the details of the process of knowledge:
a) moves from qualitative (less deformability) to quantitative concepts (two beams have numerically equal rigidity),
b) replacement of new numerical values for new geometric configurations, conserving ratios,
c) he was not interested in absolute rigour, but used approximations as techniques usable in life,
d) by the technique of comparing (proportionality).
From the epistemological point of view Leonardo da Vinci's contribution can be summarized as follows:
a) an attempt to unify the research on the *sensible world*, by applying the same criteria of representation and of a geometric–mathematical analysis to different fields: anatomy, architecture, industrial technique, painting.
b) centrality of sensory observation and experience, to be translated into design, which becomes an instrument of inquiry, explanation and graphic modelling,
c) a universal point of view; interpretation of the biological universe and of the mechanical one, considering systems governed by the same rules,
d) development of a process of theoretical and experimental research that starts from tasks and requirements of a practical nature and then develops theoretical considerations, compared with the classical and medieval primary sources of scientific knowledge, to be verified experimentally, in order to build up general mathematical rules applicable to specific cases,
e) pragmatic and realistic approach to the problems: Leonardo does not seek absolute rigour in the results of his research, but an approximation recognized as useful, clearly an attempt to rationalize all human activities, including his own.



Sources and methodology in Leonardo[1]

[1] It is a fact that by that time the translation of Archimedes by W. Moerbeke (1269) was not so largely

Leonardo seems to establish a closer relation between natural phenomena and theoretical science[1], trying to elaborate also mathematical reasoning for the interpretation of the observed phenomena: *No human investigation can be called real science, if it cannot be mathematically demonstrated*[2].

In conclusion, even in this brief essay, it is evident that the studies by Leonardo represent an important and partly correct attempt to formulate a general theoretical organization involving greater formalization – than his predecessors –which can clear up and preview, e.g. the deformability of bodies in mechanics and architecture. One of his aims was to avoid further planning mistakes to ensure the proper functioning of the *building–human organism* and of the *building–machine*. In this sense he is remote from his contemporaries. Later, toward the end of the Renaissance this new way to decide the theory that assumed a particular cultural value mainly proceeding towards an *analytical* perspective of conceiving mechanics that seemed to be coming to a crossroads: *physical or mathematical science*? That way another historiographic problem emerges: a crucial continuity–discontinuity problem appears when a theory is included in another theory, e.g. mathematics in mechanics (*rational mechanics*), astronomy in mechanics (*celestial mechanics*) mathematics in thermodynamics (analytical theory of heat), mechanics in engineering (*structural mechanics*).

spread. The same with some manuscripts by Jordanus de Nemore at the end of 13th cent. – ca. 1260 upon *scientia de ponderibus*: e.g. *Elementa Jordani super demostrationem ponderum*.

[1] See C. Pedretti, *Leonardo. Le macchine*, p. 12.

[2] Transl. C. Pedretti. C. Pedretti, *Leonardo. Le macchine*, p. 34: *Nessuna umana investigazione si può dimandare vera scienza, s'essa non passa per le matematiche dimostrazioni*.