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1. Perception of Zeno's paradoxes

It is a widely held opinion that Zeno's arguments hold no water, but are merely ingenious paradoxes that, while may be troubling at first reading, do not stand up to close scrutiny. Among the most critical scholar is surely Calogero (1932: 107), who considers Zeno a zealous follower of Parmenides, unable to further develop the doctrines of his master, seeking only «to build around them a barrier of supporting arguments». Likewise, Zafiropulo (1950: 109) maintains that Zeno's arguments are only linguistic equivocations that «do not raise any difficulties from the logic point of view». For Solmsen (1971: 393) Zeno «delights in intellectual experimentation, in discovery and exploitation of new argumentative methods. Problems, dilemmas, paradoxes, equally defensible alternatives may have fascinated him more – and for their own sake – than a way out of the deadlock, a resolution, and positive "results"». Barnes (1979: 294) holds that «Zeno was no original philosopher [...] negative, destructive, polemical, Zeno was the first of the 'Sophists'. His aims were critical, not constructive; is method subtle not solid». Cordero (1988: 120) concludes that Zeno's contradictory arguments «are based on a solid philosophical position, which, if one should name it, we could not hesitate to name 'nihilist'». On the whole, opinions do not seem to have changed with time: in a recent handbook on pre-Socratic philosophy (Curd, Graham 2008) Zeno does not earn a chapter for himself, but is considered only

for his influence on the Atomists[1]. Rossetti and Pulpito (Barnes et al. 2011: 16) sum up the state of the matter:

> although on the plane of *epistēmē* Zeno does not survive the controls of merit, although the proofs of the validity of his explicit *demonstranda* have utterly collapsed [...] many new questions should be (have been) ready to take shape.

A similar point of view is shown by Fano (2012: 19):

> today, after 2500 years, it is still worthwhile to study Zeno's paradoxes, not because they are convincing arguments in themselves, but because they help to meditate on space, time, continuous, discrete, matter and motion[2].

Undoubtedly Zeno's arguments raise serious problems of interpretation for a number of reasons. The texts of his few remaining fragments show that his writing is extremely synthetic and may lend itself to different interpretations. Besides, the Eleatic doctrine was very early misunderstood in ancient times, owing perhaps to Melissus' coarse formulation, and even Plato and Aristotle seem to have missed the point of Parmenides' poem. Today our position is even more difficult, since Zeno's book is almost completely lost and Aristotle summaries are rather sketchy, as if the arguments were well known in his time and did not require a detailed description.

An idea of the interpretative problems posed by Zeno's prose is given by the remaining fragments. Only two arguments against plurality are fully reported in Zeno's own words. Of them, the most important is undoubtedly in fragment 29 B 1, attested by Simplicius (*Phys.* 140.34–141.8)[3]:

> τὸ δὲ κατὰ μέγετος [ἄπειρον ἔδειξε] πρότερον κατὰ τὴν αὐτὴν ἐπιχείρεσιν. πρδείξας γὰρ ὅτι
> εἰ μὴ ἔχοι μέγετος τὸ ὄν, οὐδ' ἂν εἴη
> ἐπάγει:
> εἰ δὲ ἔστιν, ἀνάγκη ἕκαστον μέγεθός τι ἔχειν καὶ πάχος καὶ ἀπέχειν αὐτοῦ τὸ ἕτερον ἀπὸ τοῦ ἑτέρου. καὶ περὶ τοῦ προύχοντος ὁ αὐτὸς λόγος. καὶ γὰρ ἐκεῖνο ἕξει μέγεθος καὶ προέξει αὐτοῦ τι. ὅμοιον δὴ τοῦτο ἅπαξ τε εἰπεῖν καὶ ἀεὶ λέγειν· οὐδὲν γὰρ αὐτοῦ τοιοῦτον ἔσχατον ἔσται οὔτε ἕτερον πρὸς ἕτερον οὐκ ἔσται. οὕτως εἰ πολλά ἐστιν, ἀνάγκη αὐτὰ μικρά τε εἶναι καὶ μεγάλα· μικρὰ μὲν ὥστε μὴ ἔχειν μέγεθος, μεγάλα δὲ ὥστε ἄπειρα εἶναι.

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[1] An influence that Sedley (2008: 310) tends to discount: "in the course of investigating that question, we will find that Zeno's own role recedes considerably".

[2] Probably, among the forty arguments attributed to Zeno (Procl. *In Parm.* I p. 694, 23; Elias, *In categ.* p. 109), the surviving ones have some aspect that stirs the imagination or leaves the reader dumbfounded. Many other arguments, now lost, might have seemed trivial or senseless if seen outside of the Eleatic context.

[3] The other fragment is 29 B 3, also reported by Simplicius *phys.* 140, 27.

The introductory words suggest that after a preceding argument, which dealt with the reduction of the many to non-existence, a new argument is starting, showing the unlimited magnitude of each of the many. The majority of the authors interpret the argument as a dichotomous partition of a body. Although this traditional interpretation is shared by most scholars, from Zeller onward[4], I think that it needs some reconsideration. In examining this interpretation I will refer to what is, to my knowledge, one of the most recent formulations, which has also the not inconsiderable merit of being perhaps the most detailed and unambiguous description of the argument and of pointing out, unwittingly, some peculiarities that open the way to a crucial alternative: the interpretation given by Barnes (2011) in his lectures held in Ascea (Italy) in year 2009 (published in Barnes *et al.* 2011).

Barnes (2011: 61) effectively paraphrases the argument as follows:

> Take anything which exists – a sausage, for example. It has some bulk and size. So it will have at least two bits to it. Take one of the bits – it too has some size, and so has at least two bits. Take one of those bits – it too has some size … And so on, and on, and on. Therefore the sausage is infinitely large.

Following this interpretation, the argument[5] is clearly absurd: sectioning an object, ideally or actually, does not change the total volume of all its parts. Barnes (2011: 118) himself states that:

> Whereas Zeno's arguments hardly need refutation, it does need to be explained where and how they go wrong.

Barnes develops his analysis with consummate skill and, at the end of his lectures, he leaves aside any consideration concerning the sums of convergent and divergent series, which he carried out up to this point, and starts the analysis from scratch assuming that the body has originally a finite volume. If this is accepted, then the sum of the volumes of the parts of any partition of the body – and *a fortiori* of a quasi-partition – cannot be infinite. It follows that the series of the infinite elements conceived by Zeno, being a quasi-partition of the body, is not infinite. This conclusion is fatal to Zeno. The statement that a sausage becomes infinitely large if infinitely partitioned leaves no one bewildered, nor does it seem to be a paradox at all, but it is only an obviously false argument, devoid of any subtlety.

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[4] Zeller (1892, vol. I: 592). See also, among many others, Gomperz (1896, vol. I: 304); Lee (1936: 31); Booth (1957); Owen (1958); Vlastos (1971); Abraham (1972); Kirk *et. al.* (1983: 267); McKirahan (1994 and 1999); Hasper (2006) and Palmer (2012).

[5] Barnes calls it the 'Dichotomy'.

That the argument is unsound is generally recognised, so that the main effort of the interpreters is frequently directed to make out what Zeno could have had in mind and why he went wrong. Lee (1936: 31) states: «we can only make sense of Zeno's argument by assuming that the elements in question are supposed to have the characteristics of the geometrical point, besides having magnitude», a somewhat contradictory claim. Owen (1958: 143) thinks that Zeno's aim was to point out that the division can have only two outcomes: «Either the parts have no size, and then there can be no such parts; or they have some size, and then the thing you set out to divide becomes infinitely big», absurd outcomes which he regards as a dilemma equally lethal. Valstos (1971: 131) judgement is severe: «we must [...] reconcile ourselves to the thought that our clever Zeno here walked into a booby trap». Abraham (1972) thinks that the logic of Zeno can be restored if instead of a dichotomous process, in which only the smaller part is further divided at each stage, one reads the fragment as a through and through division of all the infinite generated parts; a difference that obviously does not solve the problem: the sum of the parts remains finite. McKirahan (1999: 139) tries to rescue Zeno by reinterpreting his final conclusion: «He concludes that each of the many things is so large that it has an unlimited number of parts – without committing himself to a view on the question of whether anything with an unlimited number of parts can have a limited size»; a conclusion that may reduce Zeno's argument to the trivial statement that if we divide a quantity into infinite parts, the number of the parts is infinite. Observing that the hypothesis of a through and through division «is difficult to square it with the evidence» (p. 55), Haspers (2006: 83) maintains that the unlimitedness in size of the whole can be accounted for «by ascribing to Zeno an implicit argument according to which the absence of a final part to the series of ever-decreasing parts is sufficient for the conclusion that the whole of such series is of unlimited size, for lack of a limit», which does not absolve Zeno from wording an inconsistent statement.

Since the traditional interpretation of fragment 1 is formulated in terms of a dichotomy, it is sometimes assimilated to two Zeno's arguments against motion, the 'Race'[6] and the 'Achilles', which are apparently similar. I maintain that these similarities are deceptive: the argument of the 'Race' shows that it is not possible to step over all the intervals in which the dichotomy divides the stadium, because they are infinite; it does not say that the stadium has an infinite length. In the *Physics* (VI 2, 233a 21) Aristotle writes: διὸ καὶ ὁ Ζήνονος λόγος ψεῦδος λαμβάνει τὸ μὴ ἐνδέχεσθαι τὰ ἄπειρα διελθεῖν ἅψασθαι ἐν πεπερασμένωι χρόνωι, as if Zeno had stretched to infinity the crossing time, and shows that this conclusion is wrong. However, further on Aristotle (VIII 8, 263a 4) gives a different interpretation: εἰ ἀεὶ τὸ ἥμισυ διιέναι δεῖ, ταῦτα δ' ἄπειρα, τὰ δ' ἄπειρα ἀδύνατον διεξελθεῖν.

Since Aristotle himself also gives this second, more subtle reading, there is no need to saddle Zeno with the absurd statement that crossing the stadium would require an

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[6] Aristotle (*phys.* I 3, 187a 3) calls it the 'Dichotomy', but it is also called the 'Stadium'.

infinite time[7]. Aristotle's second formulation of the argument, however paradoxical, is bewildering: it is obviously impossible to complete an infinite series of tasks, yet these tasks are completed after a finite time[8]. The alleged argument of fragment 1, on the other hand, looks absurd: how can one expect to make a body infinitely large simply by slicing it[9]? It is not easy to imagine that the same Zeno who conceived the clever arguments of the 'Race' and of the 'Achilles', could have concocted such an inferior product.

2. An interpretation of fragments 29 B 1 and 29 B 2

I think still possible to come to the aid of Zeno and at the same time reappraise Plato's evidence in the *Parmenides* stating that Zeno was a faithful follower of his master.

About the initial hypothesis of the argument εἰ δὲ ἔστιν understood as "if many things exist", Barnes (2011: 60) writes:

I myself cannot see how the hypothesis could limit or condition the course of the argument; rather, from a logical and philosophical point of view, the hypothesis is redundant.

About Simplicius' text, he (2011: 59) wonders:

there are one or two curious turns of phrase: why, for example, does Zeno talk about the 'projecting' bits, or say that one bit is 'in front of' another; and why does he change from ἀπέχειν [be distant from] to προέχειν [project]? I do not know the answers to those questions. But, once again, the questions do not touch the philosophical interpretation of the text.

Following the traditional interpretation, the initial hypothesis is actually redundant and the change from ἀπέχειν to προέχειν has no apparent justification. But is this the only possible reading? Indeed, as we have seen, many serious scholars agree with it, but there is disagreement too. Albertelli (1939: 207 n. 4), on whose book many took their first steps with the Eleatics, mentions two interpretations. The first, coinciding with the traditional one, tries to prove the infinity of the body by dissecting it – that is, working inward. The second, on the contrary, proves the infinity

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[7] McKirahan (2002) suggests that the first one is actually Zeno's reading; but it would not be the only case where Aristotle grossly misinterprets his predecessors' opinion just to develop his own ideas.

[8] The paradoxes of the 'Race' and of the 'Achilles', in one form or another, are still discussed as examples of supertasks (see Laraudogoitia 2011).

[9] Antonopoulos (2011) tries to rescue Zeno by pointing to the order followed in eating the pieces of the sausage: you cannot eat a single piece without having first eaten countless others. This, however, doesn't make the sausage infinitely large, it makes it inedible.

working outward (to infinity because there is never a last limit). But what makes the second interpretation more likely are the expressions προύχον and προέξει αὐτοῦ τι of the Zenonian text[10], which by adopting the first interpretation cannot be properly rendered[11].

I hold that the initial assumption, εἰ δὲ ἔστιν, is an essential premise to the argument, but it requires a preliminary reflection on what Zeno means by ἔστιν. To understand it, one must look to the Parmenidean use of ἔστιν, a path that many authors – Barnes *in primis* – rule out by altogether freeing Zeno from Parmenides[12].
Let us look at the core of Parmenides ἀλήθεια in fragment 28 B 4:

λεῦσσε δ᾽ ὅμως ἀπεόντα νόωι παρεόντα βεβαίως
οὐ γὰρ ἀποτμήξει τὸ ἐὸν τοῦ ἐόντος ἔχεσθαι
οὔτε σκιδνάμενον πάντηι πάντως κατὰ κόσμον
οὔτε συνιστάμενον.

and in 28 B 8.22–25:

οὐδὲ διαιρετόν ἐστιν, ἐπεὶ πᾶν ἐστιν ὁμοῖον·
οὐδέ τι τῆι μᾶλλον, τό κεν εἴργοι μιν συνέχεσθαι,
οὐδέ τι χειρότερον, πᾶν δ᾽ ἔμπλεόν ἐστιν ἐόντος.
τῶι ξυνεχὲς πᾶν ἐστιν· ἐὸν γὰρ ἐόντι πελάζει.

Being is a firmly connected whole, closely adhering to itself, since no *non-being* exist (28 B 8.8: οὐ γὰρ φατὸν οὐδὲ νοητόν / ἔστιν ὅπως οὐκ ἔστι) which could divide it into distinct *beings*, either scattered through the whole universe or grouped together. All is full of *being*, and *being* adheres to itself. If we believe that the things we conceive are objective *beings*, representing the "true" partition of *being*, we are introducing interruptions in what exist, thus positing *non-being* together with *being*. So we wander through the inconceivable way of error, which leads nowhere (28 B 2.4):

ἡ δ᾽ ὡς οὐκ ἔστιν τε καὶ ὡς χρεών ἐστι μὴ εἶναι,
τὴν δή τοι φράζω παναπευθέα ἔμμεν ἀταρπόν

But we do not need to enter this impassable way. To avoid it, we have only to admit that the objects we conceive are only names given by us (28 B 8.38: τῶι πάντ᾽ ὄνομ[α]

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[10] On the change from ἀπέχειν to προέχειν see also Calogero (1932 [1977²]: 121 n. 15).

[11] In the same sense translate Pasquinelli (1958: 270); Colli (1964: 93); Dumont (1988: 291).

[12] Barnes (2011: 40): «there is nothing in what survives of Zeno's work which could be construed as a decent defence of Parmenides' monism, and on the other hand there are several elements in it which have as much force against monism as they do against pluralism. (In fact, I do not think that Parmenides was a monist at all [...])». See also Solmsen (1971); Cordero (2004 and 2011); Antonopoulos (2011).

ἔσται, / ὅσσα βροτοὶ κατέθεντο πεποιθότες εἶναι ἀληθῆ) to parts of the whole *being*; parts that we, in our own minds, select as we see fit, tracing arbitrary boundaries: the error lies in believing that these objects are the real articulation of *being*[13].

If Zeno shares this doctrine of Parmenides, in 29 B 1 he does not speak of the act of slicing a sausage, but he shows that a plurality of *beings* does not exist, because, as stated by Parmenides, one cannot find interruptions in *being* (the totality of what exists), since it is a compact *continuum*[14]. When we look for a *being* in space, it must have dimensions (μέγεθος καὶ πάχος) and one bit of it must be distant from another (καὶ ἀπέχειν αὐτοῦ τὸ ἕτερον ἀπὸ τοῦ ἑτέρου), but the same can be said on what is "projecting out" (περὶ τοῦ προύχοντος, *i.e.* what lies immediately beyond the object), and so on, and on, and on... nor is there an extremity (ἔσχατον) to the expansion of the initial entity, because we do not find limits to its being.[15] No division is mentioned in the text and the the iterative process works by progressively expanding the element of the 'many' from which we have started. Thus, each of the 'many' grows up to infinity.

Zeno argues that, if many distinct *beings* existed (εἰ πολλά ἐστι), we could find the boundaries of each one.[16] But beyond the limits that we subjectively allot to each portion of *being* which we at first identify as *an existing object*, *being* continues uninterrupted, and further on it still continues, since there is no limit to what exists. In other words, if on the periphery of what we posit as an initial quantity, which we presume to be a single *being*, we seek its objective boundaries – that is, boundaries set not by us but by a discontinuity in *being* – the object expands to infinity because no such boundary can be found due to the non-interrupted nature of *being*: *being* therefore fills the space and is not divided into a multiplicity of *beings*. In this sense *being* can be said *one*. Thus Zeno is not speaking of a dichotomy – the fragment never mentions the act of dividing or of slicing – but of the infinite expansion of any spatial entity we start from, in the vain effort to reach its objective limits. If this interpretation is accepted, Zeno's argument is sound: he never stated that by slicing a cake thin it would become infinitely large, which would look utterly impossible even to a three year old child, today as well as in Zeno's time.

While the argument of fragment 1 seeks the boundaries of *being* by working outward, fragment 29 B 2 shows, in my opinion, the conclusions of a similar argument which works inward. The fragment is attested by Simplicius (*Phys.* 139.11–15):

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[13] For this interpretation see also Calenda (2011).

[14] Owen (1958: 140): «Zeno certainly held, as a philosophical theory inherited from Parmenides, that there is only one thing in existence».

[15] Fränkel (1942: 119) understands προέχει in the sense of "projects, stretches forward", but he limits the projection to a "layer of skin" and adds: «It goes without saying, however, that each subsequent skin would be thinner and thinner than the preceding one (by a constant ratio, presumably), so that the total extension, as modern mathematicians express it, converges to a certain sum». Surprisingly, the dichotomous scheme always pops out and imposes itself, even though division is never mentioned in the fragment.

[16] Owen (1958: 141): «if you say there are many things in existence how do you distinguish your individuals? [...] no method of dividing anything into spatial or temporal parts can be described without absurdity».

εἰ γὰρ ἄλλωι ὄντι, φησί, προσγένοιτο, οὐδὲν ἂν μεῖζον ποιήσειεν· μεγέθους γὰρ μηδενὸς ὄντος, προσγενομένου δέ, οὐδὲν οἷόν τε εἰς μέγεθος ἐπιδοῦναι. καὶ οὕτως ἂν ἤδη τὸ προσγινόμενον οὐδὲν εἴη. εἰ δὲ ἀπογινομένου τὸ ἕτερον μηδὲν ἔλαττον ἔσται μηδὲ αὖ προσγινομένου αὐξήσεται, δῆλον ὅτι τὸ προσγενόμενον οὐδὲν ἦν οὐδὲ τὸ ἀπογενόμενον.

Unfortunately, we do not have the whole argument in Zeno's words, but we may have traces of it in another passage of Simplicius (*Phys.* 139.26–32):

ἕτερος δὲ ἦν λόγος τῷ Παρμενίδῃ[17] ὁ διὰ τῆς διχοτομίας οἰόμενος δεικνύναι τὸ ὂν ἓν εἶναι μόνον καὶ τοῦτο ἀμερὲς καὶ ἀδιαίρετον. (1) εἰ γὰρ εἴη, διαιρετόν, τετμήσθω δίχα, ὡς ἤτοι ὑπομένει τινὰ ἔσχατα μεγέθη ἐλάχιστα καὶ ἄτομα, πλήθει δὲ ἄπειρα, καὶ τὸ ὅλον ἐξ ἐλαχίστων, πλήθει δὲ ἀπείρων συστήσεται· ἢ φροῦδον ἔσται καὶ εἰς οὐδὲν ἔτι διαλυθήσεται καὶ ἐκ τοῦ μηδενὸς συστήσεται· ἅπερ ἄτοπα. οὐκ ἄρα δειαιρθήσεται, ἀλλὰ μενεῖ ἕν. (2) καὶ γὰρ δὴ ἐπεὶ πάντη ὅμοιόν ἐστιν, εἴπερ διαιρετὸν ὑπάρχει, πάντη ὁμοίως ἔσται διαιρετόν, ἀλλ᾽ οὐ τῇ δὲ οὔ. διῃρήσθω δὴ πάντη· δῆλον οὖν πάλιν ὡς οὐδὲν ὑπομένει, ἀλλ᾽ ἔσται φροῦδον, καὶ εἴπερ συστήσεται, πάλιν ἐκ τοῦ μηδενός συστήσεται. εἰ γὰρ ὑπομενεῖ τι, οὐδέ πω γενήσεται πάντη διῃρημένον. ὥστε ἐκ τούτων φανερόν φησι, ὡς ἀδιαίρετόν τε καὶ ἓν ἔσται τὸ ὄν.

The passage is meant to show that *being* is only one, without parts and indivisible, because (1) if it could be divided, using a dichotomous process of division, only two outcomes are possible: either would remain ultimate magnitudes, minimal and indivisible, infinite in number, or it will vanish and will dissolve into nothing. Part (2) repeats the argument with the explicit assumption that πάντη ὅμοιόν ἐστιν.

The argument is analysed in detail by Makin (1982)[18], who bases his interpretation on the homogeneity of *being*: Zeno wants to show that τὸ ὄν – that is, anything that *is* – is indivisible while his opponents hold that it is divisible. Zeno states that τὸ ὄν is homogeneous, so it cannot be divisible here and not there, but, if divisible, it must be so everywhere. It follows that "what remains" are infinite parts which either have some final indivisible magnitude, or have no magnitude at all[19]. Both alternatives are absurd: the reason for the first one is not stated but, as Makin puts it, the «unstated absurdity is then, presumably, that the whole will itself be an infinite μέγετος»; the second alternative is absurd because, Makin (1982: 227) says, «what remains when the division […]

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[17] Simplicius, in discussing Aristotle's *Physics* (187a 1), quotes the passage from Porphyry, who attributes the argument to Parmenides; but further on (*Phys.* 140.21–26) he remarks that it is more likely Zeno's, as also Alexander thought. A similar argument is explicitly attributed to Zeno by Philoponus (*Phys.* 80.23) in his comment of the same passage of Aristotle's *Physics*.

[18] See also Curd (1998: 173).

[19] Makin (1982: 229–230) explains that «what appears (to us) to be a natural alternative – i.e. that τό ὄν is divisible to a *finite* number of ἄτομα μεγέθε is not considered» «since division cannot be finite» given the homogeneity assumption.

is completed is *not* divisible, since it has been divided; and so it cannot *be*, since what is *is* divisible». Thus *being* is indivisible. The argument, however, is unsound, because the division cannot be completed. Note that if Zeno thought that the division could be completed, he should have admitted that his so called arguments against motion were unsound. Although one is entitled to maintain that Zeno uttered contradictory arguments for purely eristic reasons, I think worthwhile to look for a more appealing solution. Besides, the homogeneity assumption also troubles me: Zeno, as an Eleatic thinker, may accept it, but why should Zeno's opponents do so? Surely the world does not appear homogeneous[20]. It is true that for the purpose of the argument, the homogeneity assumption could be restricted to divisibility; but again, why should the opponent concede the point? We usually identify a plurality of objects and hold them to be divisible, but we do not identify infinite parts of a single object: a desk has legs, a plane, drawers and some other details, but that is all: we make distinctions along preferential lines. What does then διαιρετόν mean? Notice that we never find a reference to division in Zeno's fragments. I think that we can detect here a limit of the interpretations purely based on the logical structure of the argument, without analysing the underlying meaning of the words: they unavoidably end up by depicting Zeno as an eristic controversialist, who seeks rather to confuse the opponent than to persuade him. This is how Plato perceived him, and later Greeks seem to have followed Plato's lead. I believe, however, that we can trace a fuller image of Zeno, showing that he formulates sound arguments to defend an equally sound epistemological doctrine of his master. I hope to show that Zeno can fairly win the argument by giving to his opponents a reason to accept the homogeneity clause, and by doing away with the unsound conclusion that, following an infinite dichotomy, the whole itself vanishes into nothing.

Regarding homogeneity, all that Zeno needs to point out is that *existence* is surely homogeneous: something *exists* or not, it cannot exist more or less; but what does not exist is not there. Zeno speaks of *being* – of what *exists* – and in *being* there is no gap. To *exist* is a homogeneous property – at least it is homogeneous for the Eleatics, and we may side with them: it is a statement that can be easily shared.

Since, in what we posit as an initial *being* we do not find objective boundaries which could show it to be actually "one *being*", we can proceed – as if with the help of mental magnifying lens – looking for boundaries of smaller and smaller quantities. Still, owing to the homogeneity of *being*, none of the particles we examine in succession does qualify in its own right as "one *being*". Going on, we can proceed to the infinite, until the particle we vainly try to isolate vanishes into nothing. Thus, the argument shows how is obtained the 'entity' to which μήτε μέγεθος μήτε πάχος μήτε ὄγκος μηθείς ἐστιν, as stated in Simplicius introduction of fragment 2. The conclusion that such entity does not altogether exist may perhaps sound extreme, valid only to the limit, but hardly nonsensical. Accord-

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[20] Makin (1982: 225) gets rid almost casually of the problem, stating that «We may expect Zeno's opponents to agree [...] it was the innovation of the Atomists to deny it».

ing to this interpretation, the arguments of fragments 1 and 2 – usually called "arguments against plurality" – taken together make an effective point against the hypothesis of a plurality of beings, as stated by the conclusion of fragment 1:

οὕτως εἰ πολλά ἐστιν, ἀνάγκη αὐτὰ μικρά τε εἶναι καὶ μεγάλα. μικρὰ μὲν ὥστε μὴ ἔχειν μέγεθος, μεγάλα δὲ ὥστε ἄπειρα εἶναι.

Both fragments, in fact, form a single argument, which shows that, *if we posit a plurality of beings*, each *being* vanishes into nothing (fr. 2) and becomes infinite (fr. 1): hence a plurality of beings does not exist[21]. It can be added that, as Simplicius states (*Phys.* 139.6–7): τῷ πολλὰ εἶναι λεγόντι συμβαίνει τὰ ἐναντία λέγειν; but this antilogy is redundant, since both arguments directly show the thesis: even scanning the whole space we cannot find a plurality of beings. I think, however, we can go a step further. Since *being* exists by definition, only one horn of the dilemma survives: *being* is ἄπειρον, and therefore it is one[22].

3. The arguments against motion

The traditional division of Zeno's arguments (arguments against plurality, against motion and against place) may be useful, but is misleading. Plato in the *Parmenides* (127e 5) makes Socrates ask:

εἰ γὰρ πολλὰ εἴη, πάσχοι ἂν τὰ ἀδύνατα. ἆρα τοῦτό ἐστιν ὃ βούλονταί σου οἱ λόγοι, οὐκ ἄλλο τι ἢ διαμάχεσθαι παρὰ πάντα τὰ λεγόμενα ὡς οὐ πολλά ἐστι; καὶ τούτου αὐτοῦ οἴει σοι τεκμήριον εἶναι ἕκαστον τῶν λόγων, ὥστε καὶ ἡγῇ τοσαῦτα τεκμήρια παρέχεσθαι, ὅσουσπερ λόγους γέγραφας, ὡς οὐκ ἔστι πολλὰ; οὕτω λέγεις, ἢ ἐγὼ οὐκ ὀρθῶς καθαμαντάνω;

and Zeno answers (128a 2):

οὔκ, ἀλλὰ [...] καλῶς συνῆκας ὅλον τὸ γράμμα ὃ βούλεται.

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[21] Fragment 29 B 3 shows an argument against plurality which starts from non-contiguous *beings*: infinite other beings could always be found among them, so the number of *beings* grows to infinity. This is impossible, because they must be as many as they are, neither more nor less: thus they must be finite in number.

[22] Infinite is understood as stretching in every direction, as filling the space. I strongly disagree with those who interpret Parmenides' εὐκύκλου σφαίρης ἐναλίγκιον ὄγκωι (28 B 8.43) as an absurd description of the form of being. I see this interpretation as an example of how easily meaningless doctrines are imputed to pre-Socratic thinkers. Given that the surface of a sphere is the only one whose properties are identical in all its points, the sphere is a very apt simile that shows how the properties of *being* are uniformly identical everywhere, owing to the πεῖρας πύματον, *i.e.* to the strict constriction of logic. Parmenides, though, never stated the infinity of *being*: infinity cannot be rigorously deduced from *being*, but absence of limits might. Today we can conceive an unlimited universe which is neither spatially neither temporally infinite. This may *a posteriori* justify Parmenides' restraint.

Following Socrates' interpretation, confirmed by the Platonic Zeno, all Zeno's arguments aim at proving that "many things do not exist" (οὐ πολλά ἐστι), and show that, once the existence of a plurality of beings is admitted, impossible consequences follow. Further on Socrates declares to Parmenides (128a 8) that Zeno's statement "many things do not exist" is nothing else than Parmenides' own doctrine of the unity of *being*:

σὺ μὲν γὰρ ἐν τοῖς ποιήμασιν ἕν φὴς εἶναι τὸ πᾶν, καὶ τούτων τεκμήρια παρέχῃ καλῶς τε καὶ εὖ· ὅδε δὲ αὖ πολλά φησιν εἶναι, τεκμήρια δὲ καὶ αὐτὸς πάμπολλα καὶ παμμεγέθη παρέχεται.

If, as I believe, Plato is right, and all Zeno's arguments are meant to show that a plurality of being does not exist, then one is entitled to suppose that each of the arguments against motion, paraphrased by Aristotle, should have started, explicitly or implicitly, with the initial hypothesis εἰ πολλά ἐστι, advanced by Zeno in fragments 1 and 3 and mentioned by Simplicius when he introduces fragment 2. It is easy to surmise why Aristotle omits this assumption: he has no doubts about the existence of the many, as he clearly shows in his critic to Parmenides' monism in the *Physics* (I 3, 186a 22):

ἀδύνατον φαίνεται τὰ ὄντα ἓν εἶναι, καὶ ἐξ ὧν ἐπιδεικνύουσιν, λύειν οὐ χαλεπόν. [...] εἰ μόνα τὰ λευκὰ ληφθείη, σημαίνοντος ὂν τοῦ λευκοῦ, οὐθὲν ἧττον πολλὰ τὰ λευκὰ καὶ οὐχ ἕν· οὔτε γὰρ τῇ συνεχείᾳ ἓν ἔσται τὸ λευκὸν οὔτε τῷ λόγῳ.

For him the hypothesis εἰ πολλά ἐστι is redundant, because, as we have seen, the argument of the 'Race' makes sense even if this hypothesis is disregarded: undoubtedly, it is a paradox, but an ingenious paradox. We have no doubts on the possibility of crossing the stadium, but we don't see how crossing infinite space intervals could be accomplished. Classical mechanics solves the problem assuming the continuum hypothesis and stipulating the convention that the limit, for x that tends to infinite, of the sum S_x of the terms of a dichotomous partition of a given magnitude L is equal to L:

$$\lim_{x \to \infty} S_x = \lim_{x \to \infty} \sum_{n=1}^{x} \frac{L}{2n} = L$$

This convention is justified by the fact that, as large as we chose x, the sum is always less than L and, as small as we chose a magnitude ε, we can always find a value of x large enough to make the difference $L - S_x$ less than ε.

Is Zeno just stating ingenious paradoxes? It looks as if Plato in the *Parmenides* (128d 2) endorses this derogatory interpretation, putting it in the mouth of Zeno himself[23]:

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[23] Plato here is using the same rhetorical device he uses in the *Sophist,* where he puts his own critic of the Eleatic doctrine in the mouth of the "foreigner from Elea". Raven (1948: 73) rightly notes: «it is anyhow very improbable, as has often been remarked, that Plato should have known Zeno's actual motives». About Zeno's

ἀντιλέγει δὴ οὖν τοῦτο τὸ γράμμα πρὸς τοὺς τὰ πολλὰ λέγοντας, καὶ ἀνταποδίδωσι ταὐτὰ καὶ πλείω, τοῦτο βουλόμενον δηλοῦν, ὡς ἔτι γελοιότερα πάσχοι ἂν αὐτῶν ἡ ὑπόθεσις, εἰ πολλά ἐστιν, ἢ ἡ τοῦ ἓν εἶναι, εἴ τις ἱκανῶς ἐπεξίοι. διὰ τοιαύτην δὴ φιλονικίαν ὑπὸ νέου ὄντος ἐμοῦ ἐγράφη.

Following this statement, Zeno, when still young and passionate, was only formulating clever paradoxes in order to pay back Parmenides' detractors in the same coin. Plato, for his part, suggests an answer to the problems raised by Zeno, by making Socrates ask (*Parm.* 129c 3): εἰ δ' ἐμὲ ἕν τις ἀποδείξει ὄντα καὶ πολλά, τί θαυμαστόν, λέγων, ὅταν μὲν βούληται πολλὰ ἀποφῆναι, [...] ὅταν δὲ ἕν. For both Plato and Zeno the solution of the paradox could follow from the words βούληται ἀποφῆναι: as one "wants to show". Plato and Zeno, however, do not speak the same language. For Plato (129c 8) human knowledge holds an underlying ontological base: πλήθους γὰρ οἶμαι μετέχω [...] μετέχων καὶ τοῦ ἑνός. One can choose the aspect he wants to stress, because Socrates partakes both in the (existing) idea of *plurality* and in that (also existing) of *oneness*. On the contrary, if Zeno is defending Parmenides, he not only aims to show that *being* is one, but also that all the objects and entities we name are just names, that is, mental dissections of *being*. So space can be divided at will, in a dichotomous process as in infinite other less perverse ways, but the way we choose (what we "want to show") does not change a bit of *what is*. Zeno maintains that no ontological status attaches to human knowledge and he finds his solution in epistemology: there is an unfathomable gulf between human knowledge and *being*. The knowledge of *being*, that is, *being* itself (28 B 3: τὸ γὰρ αὐτὸ νοεῖν ἐστίν τε καὶ εἶναι, and 28 B 8.34: ταὐτὸν δ' ἐστὶ νοεῖν τε καὶ οὕνεκεν ἔστι νόημα), is the only true knowledge, but men cannot adequately describe it: men can only give names to arbitrarily chosen parts or aspects of *being*.

Thus, the paradox of the 'Race' ceases to be simply a paradox, ingenious as it may be, and becomes a sound argument. It is sound because in the conditional εἰ πολλά ἐστι Zeno uses ἐστι in the Eleatic sense, meaning the absolute existence of the space partition that he conceives. If the space was in itself divided in the infinite intervals generated by the dichotomous process, the athlete, in order to complete the race, would have to cross all these intervals, and this is clearly impossible. But we know that the athlete crosses the stadium. So now? Are we back to the ingenious paradox? No, because the conclusion is different: once shown that εἰ πολλὰ εἴη, πάσχοι ἂν τὰ ἀδύνατα, one must admit that πολλὰ οὖκ ἐστι. It follows that the partition of the race devised by Zeno, and all other partitions we may choose to devise, are "not existing" in the Eleatic sense – that is, in themselves, objectively, out of our minds.

Strictly speaking, the so-called arguments "against motion" are not against motion at all: they use our perception of motion to prove that *being* is one, a compact whole. These

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book Fränkel (1942: 125) pointedly comments: «have we not just heard that, far from being ashamed of it, he is still reciting it to an eager audience?». On Plato reception of Parmenides and Zeno see Palmer (1999 and 2009).

arguments can be understood as counterfactual thought experiments: they show that, if the objects conceived by us were the objective reality, we could conceive a partition of space in such a way as to make the movement impossible[24]. In fact, how could the athlete perform the infinite acts in which the dichotomy divides the race through the stadium? How could Achilles cross the infinite space intervals in which his pursuit of the tortoise is divided? How could the arrow exit from the place, equal to itself, in which it is now, if the space is, in itself, stiffly divided in only two parts: the part in which now the arrow is and the part in which now it is not?[25] We see, however, that the athlete crosses the stadium, that Achilles reaches the tortoise, and that the arrow darts through the air, and we are forced to conclude that the partitions of space we have devised are not real beings, but are only the way we mentally divide what exists.

One could say that, even accepting this interpretation, Zeno's argument of the 'Race' is unsound, because we cannot imagine an infinite dichotomous partition. In a sense we cannot: we can conceive it, but we cannot exhaustively visualise it, not even in our imagination. So what? Do we have a criterion to decide which of our conceptions is an actual partition of what exist, and which is not? Some may think they have, but Zeno cannot be blamed for not believing it. He believed just the opposite: that, as stated by Parmenides, all partitions we conceive are mental facts, not the actual composition of the existent whole.

4. Conclusions

What is the meaning of all this? The objective truth, Parmenides' ἀλήθεια cannot be described in words: the words the Goddess uses in Parmenides' poem are only signs (σήματα) which attempt to give, in human words, an idea of the compact core of tautological Truth (28 B 1.29: Ἀληθείης εὐκυκλέος ἀτρεμὲς ἦτωρ). The distinctions we perceive and the entities we conceive are all made and named by us, and we can select them as we think it useful or interesting for us. However, Parmenides says, not every way of making a selection has the same value, but some ways are better (for us) than others (28 B 1.31:

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[24] I believe that a similar meaning has also the argument of the 'Stadium' reported by Aristotle (*Phys.* VI 9, 239b 33): ὁ περὶ τῶν ἐν σταδίωι κινουμένων ἐξ ἐναντίας ἴσων ὄγκων παρ' ἴσους. Furley (1967: 73) writes: «it is generally agreed that this argument of Zeno's has no force at all *unless it is directed against a theory of indivisible magnitudes*. Once this condition is granted, it is clever and to the point», but he adds: «there is no sign whatever in our text that these units were supposed to be indivisible; the mere use of the word *onkos*, "body," certainly does not prove it». Barnes (1979: 291) states: «There is no evidence that anyone prior to Zeno had entertained the atomistic theory he is imagined to be attacking; and there is no reason why he should himself have invented such a theory simply to knock it down». Zeno, however, had a strong reason to do it, if he wanted to show that, if our partitions of the world were actual reality, absurd conclusions would follow. Actually, we do not know how faithful Aristotle was in reporting all Zeno's arguments against motion, and I find especially appealing the interpretation of the 'Stadium' given by Mansfeld (1982: 326), who does away with the fixed masses «*introduced by Aristotle* in order to *refute* Zeno».

[25] The same argument is less strikingly voiced in fragment 29 B 4.

καὶ ταῦτα μαθήσεαι, ὡς τὰ δοκοῦντα / χρῆν δοκίμως εἶναι; and 28 B 8.61: τόν σοι ἐγὼ διάκοσμον ἐοικότα πάντα φατίζω, / ὡς οὐ μή ποτέ τίς σε βροτῶν γνώμη παρελάσσηι). We do not know if Parmenides or Zeno ever hinted at what makes one διάκοσμον better than another. I think they did not. However this is a problem that epistemology has yet to solve conclusively, and which, likely, will never find a final solution.

Have we reasons to believe that the doctrine just described could be the Eleatic doctrine of knowledge, and not an anachronistic projection of our own epistemological stance? I think we have. The status of truth seems to be a serious concern in late archaic and classical Greece, in science, in historiography, in legal proceedings, in political decisions. Doubts on the fact that truth could be proved were widespread. Xenophanes was explicit, even though we cannot evaluate the depth of his views. Heraclitus pointed to the gulf existing between God's knowledge and human opinions (22 B 78: ἦθος γὰρ ἀνθρώπειον μὲν οὐκ ἔχει γνώμας, θεῖον δὲ ἔχει; 22 B 28: δοκέοντα γὰρ ὁ δοκιμώτατος γινώσκει, φυλάσσει): men do not capture reality as God sees it, but look to the world through their needs and interests[26]. In the second half of the fifth century Protagoras showed that each man has his own truth and that each statement can be reversed – a 'dangerous' doctrine against which Plato struggled tirelessly his whole life. Only one step divides the ineffable *being* of Parmenides from Gorgias' negation of the existence of *being*. This, however, is a step that Parmenides could not cross, because his whole argument is based on the tautologically true statement that only the whole existent existed.

If this interpretation of his so-called "paradoxes" is accepted, one could not call Zeno a philosopher without philosophy[27], except in the sense, highlighted by Rossetti, that "philosophy" is an anachronistic word before Plato's days. To Zeno we must recognize a *de facto* philosophical dignity[28]: he is to all intents and purposes an Eleatic thinker, sharing Parmenides' views. His arguments allow us to imagine Parmenides and a few selected friends – Zeno among them – reasoning about knowledge while strolling through the streets of Elea or resting in the shade of a tree, arguing about a new but elusive point of view – the compact and ineffable nature of *being*, entailing the anthropic, subjective character of what we conceive – and looking for arguments to prove their insight.

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[26] Osborne (2009: 435): «we perceive with an interested gaze: [...] the world is not the same for all because we come with distinct preferences and interests».

[27] See Rossetti (2011).

[28] As stated by Rossetti (2011: 172).

REFERENCES

ABRAHAM, W. E., 1972, "The Nature of Zeno's Argument against Plurality in DK 29 B 1", *Phronesis*, 17(1), pp. 40–52.

ALBERTELLI, P., 1939, *Gli Eleati. Testimonianze e frammenti*, Bari.

ALLEN, R. E., FURLEY, D. J., 1975, *Studies in Presocratic Philosophy. Vol. II. Eleatics and Pluralists*, New York.

ANTON, J. P., KUSTAS, G. L., 1972, *Essays in Ancient Greek Philosophy. Vol. I*, Albany.

ANTONOPULOS, C., 2011, An Infinity of Priorities. *Eleatica 2008: Zenone e l'infinito*, in: Barnes *et al.* (2011), pp. 121–134.

BARNES, J., 1979, *The Presocratic Philosophers,* (1982²), London.

BARNES, J., 2011, "Zenone e l'infinito", in: Barnes *et al.* (2011), pp. 37–118.

BARNES ET AL., 2011, *Eleatica 2008: Zenone e l'infinito*, (L. Rossetti, M. Pulpito ed.), Sankt Augustin.

BOOTH, N. B., 1957, "Were Zeno's Arguments a Reply to Attacks upon Parmenides?", *Phronesis*, 2 (1), pp. 1–9.

CALENDA, G., 2011, *Epistemologia greca del VI e V secolo a.C.. Eraclito e gli Eleati*, Roma.

CALOGERO, G., 1932, *Studi sull'eleatismo*, (1977²), Firenze.

COLLI, G., 1964, *Zenone di Elea*, (E. Berti ed.), Milano.

CORDERO, N. L., 1988, « Zénon d'Élée, moniste ou nihiliste? » *La Parola del Passato*, 43, pp. 100–126.

CORDERO, N. L., 2004, *By being, It Is: the Thesis of Parmenides*, Las Vegas.

CORDERO, N. L., 2011, «Commentaire à propos du Zénon de Jonathan Barnes», in: Barnes *et al.* (2011), pp. 141–144.

CURD, P., 1998, *The legacy of Parmenides*, (2004²), Princeton.

CURD, P., GRAHAM, D. W., 2008, *The Oxford Handbook of Presocratic Philosophy*, Oxford.

DUMONT, J.-P., 1988, *Les Présocratiques*, Paris.

FANO, V., 2012, *I paradossi di Zenone*, Roma.

FRÄNKEL, H., 1942, "Zeno of Elea's Attacks on Plurality", *American Journal of Philosophy*, 63, pp. 193–206.

FURLEY, D. J., 1967, *Two studies in the Greek Atomists*, Princeton.

GOMPERZ, T., 1896, *Griechische Denker*, Leipzig.

HASPER, P. S., 2006, "Zeno Unlimited", *Oxford Studies in Ancient Philosophy* 30, pp. 49–85.

HÜLSZ PICCONE, E., 2009, *Nuevos Ensayos Sobre Heráclito*, Mexico.

LAKS, A., LOUGUET, C., 2002, *Qu'est-ce que la Philosophie Présocratique?* Lille.

LARAUDOGOITIA, J. P., 2011, "Supertasks", *The Stanford Encyclopedia of Philosophy,* http://plato.stanford.edu/archives/spr2011/entries/spacetime-supertasks/.

LEE, H. D. P., 1936, *Zeno of Elea*, Cambridge.

LONG, A. A., 1999, *The Cambridge Companion to Early Greek Philosophy*, Cambridge.

MAKIN, S., 1982, "Zeno on Plurality", *Phronesis* 27, pp. 223–238.

MANSFELD, J., 1982, "Digging up a Paradox: A Philological Note on Zeno's Stadium", *Rheinsches Museum* 125, pp. 1–24 (repr. in: Mansfeld 1990: 319–342).

MANSFELD, J., 1990, *Studies in the Historiography of Greek Philosophy*, Assen.

MCKIRAHAN, R., 1994, *Philosophy Before Socrates*, Indianapolis.

MCKIRAHAN, R., 1999, "Zeno", in: Long (1999), pp. 134–158.

MCKIRAHAN, R., 2002, « La dichotomie de Zénon chez Aristote », in: Laks, Loguet (2002), pp. 465–496.

OSBORNE, C., 2009, "If all things were to turn to smoke, it'd be the nostrils would tell them apart" or Heraclitus on the pleasures of smoking, in: Hülz Piccone (2009), pp. 415–441.

OWEN, G. E. L., 1958, "Zeno and the Matematicians", *Proceedings f the Aristotelian Socety*. LVIII, pp. 199–222 (repr. in: Salmon 1970: 139–163).

PALMER, J., 1999, *Plato's Reception of Parmenides*, Oxford.

PALMER, J., 2009, *Parmenides & Presocratic Philosophy*, Oxford.

PALMER, J., 2012, "Zeno of Elea", *Stanford Encyclopedia of Philosophy*, http://plato.stanford.edu/entries/zeno-elea/

PASQUINELLI, A., 1958, *I Presocratici. Frammenti e testimonianze*, *Vol. 1*. Torino.

RAVEN, J. E., 1948, *Pythagoreans and Eleatics*, Cambridge.

ROSSETTI, L., 2011, "Un filosofo senza filosofia", in: Barnes *et al.* (2011), pp. 171–183.

SALMON, W.C., 1970, *Zeno's Paradoxes*, (2001²), Indianapolis.

SEDLEY, D., 2008, "Atomism's Eleatic Roots", in: Curd, Graham (2008), pp. 305–332.

SOLMSEN, F., 1971, "The Tradition about Zeno of Elea Re-Examined", *Phronesis* 16, pp. 116–41.

VLASTOS, G., 1971, "A Zenonian Argument Against Plurality", in: Anton, Kustas (1972), pp. 119–144.

ZAFIROPULO, J., 1950, *L'école éléate*, Paris.

ZELLER, E., 1892, *Die Philosophie der Griechen in ihrer geschichtlichen Entwicklung*, Leipzig.

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Are Zeno's Arguments Unsound Paradoxes?

Zeno's arguments are generally regarded as ingenious but downright unsound paradoxes, worth of attention mainly to disclose why they go wrong or, alternatively, to recognise them as clever, even if crude, anticipations of modern views on the space, the infinite or the quantum view of matter. In either case, the arguments lose any connection with the scientific and philosophical problems of Zeno's own time and environment. In the present paper, I argue that it is possible to make sense of Zeno's arguments if we recognise that Zeno was indeed a close follower of Parmenides, who wanted to show that, if the plurality of beings existed, then various absurd consequences would follow. He intended to highlight the compact and inarticulate nature of the *being*, and the human character of the system of world partitions producing the entities and the objects on which our knowledge is based.