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Mathematical Thinking and Verifying Erroneous Intuition within the Context of Security. A Case Study

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ABSTRACT

The article presents an example which demonstrates how misconceptions and erroneous intuition of a subject can have an influence on making incorrect decisions. A simple case study demonstrates that being familiar with basic mathematical principles constitutes a tool sufficient to verify the correctness of reasoning by a subject and, as a result, to make decisions.

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Human existence involves constantly making decisions, the consequences of which have a direct impact on broadly-defined *security*. The level of *security* of a given *security subject* is proportional to the dynamics of the subject's development and the level that the subject is able to achieve in the subfield of *security culture*.

The *security culture* of a given subject is the entirety of an established, material and extramaterial human achievement, the purpose of which is military and extramilitary defence[1]. This phenomenon is composed of three intersecting dimensions: the mental-spiritual, organisational-legal and material. Due to the dominating role of the *nation state* as a *security subject*, the culture of national security plays a significant role in the personal and international dimension of security.

In order to be able to develop this *culture*, it is necessary to provide an appropriate environment, and specifically a *security environment*. It is defined as a system dependent on dynamic interactions between numerous factors, particularly opportunities, challenges, threats and risks which arise[2]. Our environment is safe if we surround ourselves with persons who are as familiar as possible with the intricacies of the actions which they perform every day. Security culture serves humans to achieve various goals and meet various needs, which include[3]:

– effective control over emerging dangers, which is aimed at achieving a sufficiently low threat level,
– restoring security if it has been lost,
– optimisation of security in its multitude of sectors,
– awakening within the human consciousness a need of self-improvement and of trichotomous mental, social and material development.

Over the ages, humanity has learned to identify and discern various types of threats. It has realised that it is necessary to alleviate those threats and that there exist measures which allow for avoiding and combating them effectively[4]. Every such action[5] is a result of a particular decision

[1] J. Piwowarski, *Trzy filary kultury bezpieczeństwa*, „Kultura Bezpieczeństwa. Nauka-Praktyka-Refleksje", 2015, no 19, p. 21–33.

[2] *Biała Księga Rzeczpospolitej Polskiej*, BBN, Warszawa 2013, p. 247; J. Piwowarski, *Fenomen bezpieczeństwa. Pomiędzy zagrożeniem a kulturą bezpieczeństwa*, Wyższa Szkoła bezpieczeństwa Publicznego i Indywidualnego „Apeiron" w Krakowie, Kraków 2014, p. 8.

[3] J. Piwowarski, *Fenomen bezpieczeństwa…*, p. 13.

[4] Ibidem.

[5] *Action* as defined by sociology is a type of human activity, with which subjects associate a certain meaning; According to Max Weber, who popularised this definition of action, it is a human behaviour (internal or external act, lack or enduring thereof), with which "the agent [subject] or agents associate a subjective sense" – M. Weber, *Gospodarka i Społeczeństwo*, Wydawnictwo Naukowe PWN, Warszawa 2002, p. 6.

based on, in most cases, our experiences, knowledge or intuition. Therefore, the ability to reason logically and rationally is important – this in turn is associated directly with the ability to think mathematically, which can be described as individual aptitude toward:
– identifying and understanding the role of mathematics in the modern world,
– making judgements based on mathematical reasoning,
– utilising mathematical skills wherever they are needed in everyday situations[6].

These skills should contribute to the mathematical competence of individual security subjects, as well as modifying their way of thinking and, as a result, the way they act. The modern world is changing rapidly and profoundly, which has an impact on the education system and constitutes a challenge for it. Mathematical knowledge and skills are essential in our everyday lives. They prepare us to live in the social and natural reality that surrounds us, seeing as we frequently encounter real problems requiring broadly-defined mathematical skills.

The following part of the article presents an example which demonstrates how our misconceptions and erroneous intuitions may have an impact on us making a decision which may be described as wrong. Knowledge of basic mathematics should constitute a sufficient tool for the purpose of verifying the correctness of our reasoning.

Let us proceed to the example, in which the following situation is analysed:

We have learned that there are hostages imprisoned in one of three buildings (marked as A, B and C). We have no other information, and there is not much time left for a rescue mission. However, there is a special forces unit standing by, which can only enter one building. It is equally likely for the prisoners to be in any of the three buildings.

When selecting the first target, the leader of the task force is aware of the fact that his choice may have an impact on what happens to the hostages, as there may not be enough time for another attack. For the sake of further deliberation, let us assume that the leader decides to storm building A (Fig. 1).

[6] OECD/PISA programme https://www.oecd.org/pisa/.

Fig. 1. The Monty Hall paradox

A few minutes before the planned attack on target (A), the leader unexpectedly receives information that it is certain that one of the other buildings (let us assume that it is building B) is empty. A question arises here: *Should the leader storm building A, his first choice, or should he perhaps switch targets and storm building C?*

At first glance, it may appear as though the question is insignificant and the leader's decision has no impact. However, it turns out that it is not so, as this situation constitutes a paradox, or *a statement which is surprisingly contradictory to commonly held opinions*[7]. Mathematical reasoning is capable of proving that our intuitions are sometimes erroneous.

The problem presented above can trace its origins back to the American television game show "Let's make a deal"[8]. The show was broadcast from 1963 to 1976, with Monty Hall as its host. The main prize was

[7] *Słownik Języka Polskiego*, Wydawnictwo Naukowe PWN, Warszawa 2004.

[8] J.S. Rosenthal, *Monty Hall, Monty Fall, Monty Crawl,* „Math Horizons", 2008, p. 5–7.

a car, which was hidden behind one of three doors. Behind the other two doors there were goats. The goal of the player was to select a door behind which they thought there was a car. The host, knowing what is behind each door, would randomly open one of the remaining doors after the player had made their choice, which would always contain the consolation prize in the form of a goat. At that point, Monty Hall would ask the player if they would like to make a different choice. The point of this problem is whether making another choice is actually profitable for the player. The correct answer, though it may be surprising at first, is "yes". If the player chooses the other door, their chances of winning increase twofold! This solution is often considered wrong and contradictory to intuition for most persons who encounter this problem for the first time. Due to this contradiction, mathematics describes this problem as the *Monty Hall paradox*.

A certain interesting fact warrants a mention here – the Monty Hall problem gained widespread attention in 1990 thanks to Marilyn vos Savant, who was an author of a special, very popular Sunday column which was added to 640 various newspapers and magazines across the United States. Marilyn vos Savant gained fame and popularity as the person with the highest recorded IQ – 228. Many readers would submit questions for her to answer in her column "Ask Marilyn", eager to learn the answers such an intelligent person could give. Even though the paradox itself, the way of formulating the problem and its correct solution had been known before, it was the answer published in "Ask Marilyn" by Marilyn vos Savant that made the paradox an object of a public debate, the arena of which was the whole of the United States of America. The publication sparked significant controversy. According to estimates, approximately 10.000 letters were sent to the newspaper, claiming that Marilyn was wrong and that a correction should be published. Interestingly enough, many of the letters came from faculty members of various universities. The controversy reached such a high level that some of the letters were even quite insulting[9].

The key to solving this problem is underestimating the information about the "empty" door. This entire situation actually only involves two decisions. It can therefore be assumed that the problem can be divided into two phases:

[9] M. Savant, *Game Show problem*, http://marilynvossavant.com/game-show-problem/.

– Decision 1 (at the start of the game): door selection (in this case – building selection).
– Decision 2 (after revealing an "empty" door): decision to switch or not.

Let us analyse the entire situation on the grounds of these two decisions. There are three buildings, and the hostages are in one of them.

Phase 1 – target selection. We can select the "correct" building (1/3 chance) or an "empty" building (2/3 chance).

Phase 2 – decision to switch targets. Let us assume that we decide not to switch. This means that the chance of us "winning"[10] are the same as in phase 1, i.e. equal to the chance of us choosing the "correct" building at the start, that is 1/3. The chance of "losing" is the probability of us choosing an "empty" building at the start of the experiment, that is 2/3.

Fig. 2. Mathematical reasoning breaking down erroneous intuitions

However, what happens if we decide to switch targets in phase 2? In that case, **if we originally chose a wrong building (which is the more**

[10] "Winning" (success) is understood to mean selecting the building which contains the hostages, and "losing" (failure) – selecting an empty building.

probable option, seeing as two of the three buildings are empty), we are sure to achieve success if we switch, however, if our original selection was correct, switching targets will make us lose (Fig. 2). Therefore, if we switch targets, our odds of winning are 2/3, and our odds of losing are 1/3.

After listening to the message about the empty building, the leader has the following chances of success: 1/3 if he does not switch, and 2/3 if he does. It follows that if he changes his mind, he will double his chances of success.

Based on our long-time observations, despite thorough analyses of this problem, there often occurs a certain contradictory effect. Though the theory which explains why a solution which contradicts our intuition is correct may be clear for a moment, the false conviction regarding our own infallibility manages to dominate. A type of conflict erupts in the mind of the recipient, which results in the above-mentioned theory appearing as a type of sophism[11]. It is quite challenging to clear up such misconceptions, not to mention eradicate them. Information technology may prove helpful in this case, however. By using commonly available software, such as MS Excel or the freeware programme GeoGebra[12], it is possible to program a simulator of the described situation. Such simulations may quickly present a wide spectrum of cases impossible to arrange using traditional means, presenting them in an accessible form, even for persons unrelated to the sciences (cf. Ratusiński and Szczeblowska, 2016[13]).

This paper refers to sheets based on GeoGebra scripts, which can be saved as HTML files. Such applets simulate situations in which users can make their own decisions, and then decide to change them or not. It is also possible to order the computer to conduct a series of experiments and present the results in a synthesised form. This way, observers can

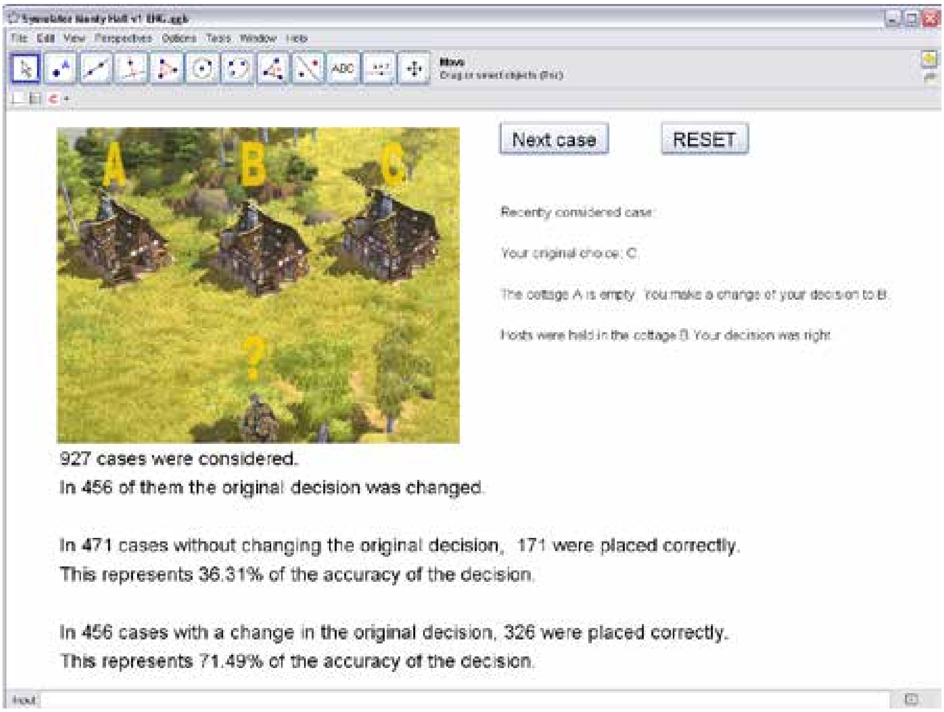
[11] In this case, sophisms are understood to mean misleading mathematical "proofs", which appear to be correct, but are actually wrong, containing deliberate logical errors which are difficult to identify at first glance.

[12] GeoGebra (www.geogebra.org) is a multiplatform, free DGS (Dynamic Geometry System) programme combining CAS (Computer Algebra System) elements, aimed primarily at the sciences. It enjoys widespread use in many areas of mathematics. The programme's advantages are its intuitiveness and simplicity of use. This makes GeoGebra extremely popular with researchers and teachers.

[13] T. Ratusiński, D. Szczeblowska, *Przykład wykorzystania nowoczesnych technologii w procesie szkolenia personelu medycznego celem poprawy bezpieczeństwa pracy*, „Security, Economy & Law", 2016, No 2 (XI), p. 54–66.

follow every single case, as well as observing the results of numerous, e.g. a thousand, repetitions of a single experiment. Such a simulation would be difficult and time-consuming without a computer. Fig. 3 constitutes an example of how such a simulator operates. Observing the practical frequency of certain results confirms the correctness of their theoretical equivalent, i.e. their probability of occurrence. For some users, such direct observation of hundreds (and sometimes even thousands) of examples enables them to deconstruct misconceptions which, if left uncorrected, may negatively influence the decision-making process, impacting the safety of e.g. subordinates.

Fig. 3. A computer simulator of the Monty Hall problem programmed in GeoGebra



In conclusion, as demonstrated in the example above, intuition-based actions of security subjects can be far from optimal, so much so that they may be considered wrong. Even though at first glance, mathematical reasoning may appear as something entirely specific, seeing as apparently it can neither be classed as part of the experimental sciences nor the arts, its practical uses can be found in all areas of life. Mathematics itself can be perceived as a special field as within its scope, more so than in the scope of any other scientific discipline, lies the search for the answer to the question of what is true and what is false. Therefore, reasoning becomes a tool for determining the correctness of certain formulations, demonstrating the implementational aspect of mathematics which is useful in practice. In the demonstrated case, the point was to verify the correctness of the thesis claiming that it was worth it to change a decision that has already been made. Rudimentary knowledge of probability theory and simple mathematical reasoning confirms the correctness of this, apparently unnatural, solution.

In the age of rapid changes, how our society is educated should also undergo transformations. It is a worthy effort to strive for all levels of education to rely more on mathematical reasoning, as it allows to verify the correctness of decisions made in the future.

The problem indicated in the article is significant from the point of view of the broadly-defined security culture, as well as for the life of the hostages who participated in it. The authors of this paper hope that this translation sparks a wider discussion of the topic.

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