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Time Topology for Some Classical and Quantum Non-Relativistic Systems

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1. INTRODUCTION

According to E. Kant time and space are two "forms" of perception. The background of their meaning is mainly intuitional[1]. In physics time is — in principle — a coordinate: in non-relativistic physics, this coordinate may vary independently of the spatial coordinates, whereas in a relativistic case the time coordinate is coupled with its spatial counterparts. This situation holds equally for classic and quantum theories. In a stationary state of a system described by the non-relativistic quantum mechanics, time enters only the phase factor of the wave function which has no influence on the calculated averaged quantities corresponding to the observed data. In effect, in a stationary state no change in time of the observed quantities may occur.

Intuitionally, in classical and quantum mechanics (as well as in special relativistic theory) we assume the time coordinate may vary from a minus infinity called "the past" to a plus infinity called "the future". A time scale of this kind we call linear; see Fig. 1. The purpose of this paper is to indicate that a time scale which is different than linear may be — at least in some cases — of a better use. In our considerations we disregard fully the relativistic theory: we neglect the coupling between time and other (spatial) coordinates into one space-time metrics as well as the problem of time intervals and

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[1] W. S. Fowler, *The Development of Scientific Method*, Pergamon Press, Oxford 1962.

their measurements. The time coordinate is assumed (similarly to Leibniz [2]) solely to be a parameter which allows us to distinguish between "earlier" and „later".

The problem of time topology has its rich bibliography [3]. In this paper we try to approach this problem on the basis of an analysis of simple physical systems, both classical and quantum. For the classical case we limit ourselves to periodic systems. According to Thirring [4] an almost periodic time evolution is a general property of small systems, whereas for large systems this evolution exhibits a chaotic behaviour connected to a mixing of the observables. For the quantum case we examine the time evolution of a system submitted to the action of a small external field called perturbation and denoted by V; we consider only a system having non-degenerate quantum-mechanical states, so the system energies corresponding to different states are different. Beyond the perturbation V, we assume the system is a fully isolated object.

We formulate the problem of time topology for a quantum-mechanical system as follows: In a very distant past, the system was in a unperturbed stationary state (P) in which — in the absence of V — it could remain infinitely. At some moment the system was perturbed by V, and in a very distant future it will be found in another stationary state (F); we

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[2] G. W. Leibniz, *Confessio Philosophi. Ein Dialog*, Klostermann, Frankfurt a/M., 1967.

[3] H. Reichenbach, *The Philosophy of Space and Time*, Dover Publications, New York 1958; J. J. C. Smart (ed.), *Problems of Space and Time*, MacMillan, London 1964; T. Gold (ed.), *The Nature of Time*, Cornell University Press, Ithaca 1967; H. Reichenbach, *The Direction of Time*, University of California Press, Berkeley 1971; A. Grünbaum, *Philosophical Problems of Space and Time, 2nd ed.*, A. Riedel, Dordrecht 1973; J. R. Lucas, *A Treatise on Time and Space*, Methuen, London 1973; L. Sklar, *Space, Time and Space-time*, University of California Press, Berkeley 1974; I. Hinckfuss, *The Existence of Space and Time*, Clarendon Press, Oxford 1975; G. Nerlich, *The Shape of Space*, University Press, Cambridge 1976; W. H. Newton-Smith, *The Structure of Time*, Routledge and Kegan Paul, London 1980; G. J. Whitrow, *The Natural Philosophy of Time*, Clarendon Press, Oxford 1980; D. H. Mellor, *Real Time*, University Pess, Cambridge 1981; R. Swinburne, *Space and Time*, 2nd ed., MacMillan, London 1981; E. Jaques, *The Form of Time*, Russak-Heinemann, New York 1982; M. Friedmann, *Foundations of Space-Time Theories*, Princeton University Press, Princeton 1983; R. Le Poidevin, *Relationism and Temporal Topology: Physics or Metaphysics?*, „The Philosophical Quarterly" 40 (1990), 419—432.

[4] W. Thirring, *Lehrbuch der Mathematischen Physik. Band 4: Quantenmechanik grosser Systeme*, Springer Verlag, Wien 1980.

assume $P \neq F$. The question we put here is what is the kind of time path which is followed by the system on its way from P to F. Before we approach the problem of a time path for a quantum-mechanical case we examine — in Section 2—4 — the time topology for classical periodic systems.

2. THE OBSERVER; HIS BIRTH, LIFE AND DEATH

As a first step, let us point out that a time toplogy can be a subjective notion depending both on the properties of the examined system as well as the perception ability of the examiner (an observer). We define the observer as a being who can do (and register) measurements. For example, we may imagine an observer who can register solely the coordinate x of the position of a body in a Cartesian coordinate system. Another observer can register, say, the x and y coordinates of this system; the third observer can measure all three kinds of coordinates: x, y and z. Beyond these coordinates we may have observers who can measure — for example — one, or more, velocity components of the body; other observers can register other body properties, for example its mass. Finally, we may imagine an observer whose perception and measuring ability is infinite. We call Him an External Observer.

Any observer who is able to accept only a finite set of observations of a system is an observer connected — in some way — with that sytem. He differs from another observer whose perception ability, i.e. the (finite) set of measuremnts which can be done by him on the system and registered, is different. It is easy to note that the perception ability of an observer depends on the system properties, in particular the system complexity. For example, for a system which is a staright line identical with the x axis, there is no problem of how to measure the space coordinates other than x; similarly in order to measure a position in the system which is a plane there is no problem of how to measure more than two kinds of independent coordinates, say x and y. Let us assume that beyond observables different than time any observer can also measure time. This means that he is able to distinguish between an earlier and a later event; an event for an observer means — as a rule — a measurement. The time measurements enable him to establish the sequence of events. The set of all measurements in instants t_i enclosed within an interval of time done in sequence by an observer we call the life interval of this observer. The earliest of the observations done by the observer (corresponding to the smallest t_i) we call his birth; the latest observation (corresponding to the largest t_i) we call his death. We assume the number of observations in the set of measurements representing the life of an observer is finite, although no limit can be imposed a priori on it.

The instants of time t_i measured by an observer during his life can be represented on the time scale given in Fig. 1. These instants fill the interval BD; see Fig. 1a.

3. TWO KINDS OF THE SCALE OF TIME

Let a simple (linear) harmonic oscillator oscillate along the x axis. With the oscillator an observer is connected who can measure the values of the x coordinate; any measurement of x is accompanied by a corresponding measurement of t. The oscillator moves, say, first in the direction of positive x, next it goes to negative x. We assume the observer starts his measurements immediately after the oscillator moves across the point $x = 0$. This means the life of the observer starts at certain position

$$x_B = \varepsilon \gtrsim 0 \tag{1}$$

for which $t = t_B$. Next x increases until the positive value close to the amplitude of the oscillator is attained; then x begins to decrease, becoming in some instant, a negative number. At the same time the sequence of the measured instants t_i is an increasing set of numbers. After the smallest x is attained the value of x increases. The largest of the increasing but negative x is some

$$x_D = -\varepsilon' \lesssim 0. \tag{2}$$

The next measured value of x is

$$x_{D+1} = \varepsilon \gtrsim 0 \tag{3}$$

so

$$x_{D+1} = x_B \tag{4}$$

because we assume that the rythm of the oscillator as well as that of the measurements of x remains unchanged in the course of the oscillations. This situation repeats infinitely in accordance with the definition of the oscillator as a perfectly periodic system. An important feature of a perfect periodic system is that we have no physical parameter which allows us to distinguish between one oscillaltion period and another. If we assume that there exists a one-to-one correspondence between the measurements of x and those of t, the full periodicity of the sytem implies the same periodic property for the time variable. In effect, the time scale for the oscillator should be not linear but forms a closed line leading to the equality

$$t_{D+1} = t_B \tag{5}$$

similar to (4). This property is represented in Fig. 1b. Topologically the scale is equivalent to a circle; see Fig. 2. The life between the birth (t_B) and the death (t_D) of the observer connected with the oscillator will repeat infinitely and nothing in the system allows for him to detect the repetitions. These repetitions can be discovered only by another observer whose ability to do measurements is larger that that of the observer connected with the oscillator. Me call such an observer an external observer (not capitalized). In order to discover the repetitions of the life of the observer connected with the oscillator, the external observer should have the possibility of counting the oscillations. An example of such an external observer is given in the next section.

4. SIMPLE EXAMPLES OF AN EXTERNAL OBSERVER

Let us consider earth Z which circles about sun S along an ellipse. An observer is connected with the system. We call him an internal observer (i.o.) and assume the observer can measure only the distance r between S and Z and can register the instants of time t corresponding to different r. Let us assume the measuring ability (perception) of the observer begins to work at the smallest possible $r = r_{min}$. The first measurement r_B gives the smallest value of the increasing r, whereas the last measurement, r_D, gives the smallest value of the decreasing r. The set of measurements repeats infinitely without the possibility of detection of this fact by the observer. Because of this periodicity, the perception of the observer goes to zero and begins to work again approximately in the same instant of time, viz. when the distance $r_D \approx r_B$ is attained. The time scale is very much similar to that obtained in Sec. 3 for the harmonic oscilator: it begings at t_B, the time when the distance r_B was measured, and ends at t_D, the time when the distance r_D was attained. The totality of the measurements represents the life of i.o. and — since the system is fully periodic — a circular scale of time should be applied. The i.o. cannot detect that $t_D \approx t_B$.

Now let us introduce an external observer (e.o.) with respect to the internal one; the e.o. can measure more kinds of parameters than i.o. We assume also that e.o. can read the measurements of i.o. but not vice versa. For example, besides the distance $r = |\overrightarrow{SZ}|$ the e.o. can measure the angle of the direction $\overrightarrow{SZ}$ in respect to certain constant direction $\overrightarrow{SA}$ which links sun S with some star A; see Fig. 3. If there is no precession of the ellipse about the axis perpendicular to the ellipse plane and going through the point S, the angles

$$\vartheta = \sphericalangle ASZ \tag{8}$$

measured in each course of Z about S will repeat exactly. Let us assume the perception of e.o. is switched on at the angle

$$\vartheta_m = (\sphericalangle ASZ)_m \tag{7}$$

which is the angle between $\overrightarrow{SA}$ and $\overrightarrow{SZ}$ in the case of

$$|\overrightarrow{SZ}| = r_{min}. \tag{8}$$

If ε'' and ε''' are infinitesimally small numbers, the first measurement gives certain

$$\vartheta_B = \vartheta_m + \varepsilon'' \tag{9}$$

and the last measurement gives certain

$$\vartheta_D = \vartheta_m + 2\pi - \varepsilon''', \tag{10}$$

because the next measurement of ϑ is ϑ_B providing we assume the earth movement and the rythm of the measurements are fully periodic. So the life of e.o., though richer than the life of i.o. by the set of measurements of ϑ bettween ϑ_B and ϑ_D, has the time scale identical to that of i.o.

Now let us assume the ellipse performs a slow precession in its plane. The life of i.o. does not change, but e.o. may now see and label different lives of i.o. This is so because the angle ϑ_m, therefore the whole set of the observed angles between ϑ_B and ϑ_D, is now different for any course of Z about S. The distinction between two different courses allows e.o. to discover that the next course of Z about S begins immediately after the end of the former course. This means a property of the system (precession) accompanied by the corresponding increase of the ability to do measurements (angles ϑ), make the time scale of e.o. different than the time scale of i.o. We may assume, for example, that precession of the ellipse covers a full angle 2π exactly during v courses of Z about S. Then e.o. will discover that his life is v times longer than the life of i.o. But qualitatively, the time scale of e.o. remains the same as that of i.o., which means that the scale of e.o. is also of a circular shape. This is so because after the precession covered a full angle 2π the set of the measurements obtained by e.o. repeats exactly[5]. The e.o. cannot discover this repetition by himself, but this can be done by another observer, let us call him e.e.o., and having a larger ability for doing measurements than e.o.; simultaneously, the change of a new physical parameter is necessary. For example, the system Z, S and A is a part of a galaxy. We assume the plane ZSA on which Z, S and A are placed, rotates about the axis SA. We assume e.e.o. is connected with the rotating system and — beyond r and ϑ — he can measure the rotation angle φ of the plane ZSA about SA. Then e.e.o. will discover that different full precessions of the ellipse end at different positions of the plane ZSA. So e.e.o. has his own time scale, larger than that of e.o. The e.e.o. may count different full precessions and discover that the end of one life of e.o. is immediately followed by his — e.o. — next life.

We summarize this section by concluding that the time scale of an observer connected with a mechanical sytem may depend both on his measuring ability and the physical properties of the system. If the notion of the observer is dropped out, we may speak about a convenient time scale for a system. A set of measurements can be performed on a system and the time scale for any system can be dependent on this set.

5. PERTURBATION OF A QUANTUM-MECHANICAL SYSTEM AND TOPOLOGY OF TIME

In non-relativistic mechanics, both classic and quantum, a straight-linear time scale is usually assumed; Fig. 1. In preceding sections we considered some mechanical systems and gave arguments for a different than linear, namely circular, scale of time; Fig. 2. In fact, the topology of the time scale is, to a large extent, a matter of convenience. A well-

[5] L. D. Landau, E. M. Lifszits, *Mechanics*, Vol. 1 of Course of *Theoretical Physics, Nauka*, Moscow 1973 (in Russian).

-known problem of similar nature concerned spatial coordinates and was given in astronomy: the planets movement can be described equally in the Ptolemaic, or geocentric, system as well as in the Copernican, or heliocentric, system, but the second system is more convenient than the first. The purpose of the remainder of this paper is to examine the topology of the time scale for a non-relativistic quantum-mechanical system. To this purpose we choose the problem of a perturbation of a non-degenerate stationary state of a quantum-mechanical system.

If the system is in its stationary state, no measurement done for it can provide a distinction between a later and an earlier instant of time, see Sec. 1. In fact, for such a system the notion of time looses its sense. In reality the stationary states on only a few quantum-mechanical systems are exactly known. To such systems belong, for example, a particle in the field of a constant potential, the harmonic oscillator, and the states of the non-relativistic hydrogen atom (athough in description of the last two cases some special functions are necessary). We often seek for states which differ only slightly from the exactly known states, for example states of the hydrogen atom in the weak electric or magnetic external field. The states which are well-known are called unperturbed, whereas states obtained from the action of a supplementary field are called perturbed states. Schrödinger gave a mathematical procedure which leads from unperturbed to perturbed states [6]. This is a complicated iterative process which represents the perturbed energies and perturbed wave functions as a combination of series based on the unperturbed quantities. Iteration means that contributions of the next step (higher order) can be expressed successively by those obtained in a former step (lower order). This makes the whole calculation extremely complicated and in practice limited to only a few steps, on condition that we assume that results calculated in these few steps adequately approximate the exact solutions.

A more systematic approach to the perturbation method can be done on the basis of field theory [7]. Although we look for a perturbed stationary state, the time parameter is intro-

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[6] L. I. Schiff, *Quantum Mechanics*, 3rd ed., McGraw Hill, New York 1968.

[7] S. Raimes, *Many-electron Theory*, North-Holland Publ. Comp., Amsterdam 1972.

duced into the calculation. This parameter is taken along the linear scale of time. Let us consider the state of a system which has the lowest energy, so it is called the ground state. It is assumed that at time t being very far in the past ($t = -\infty$) the state was unperturbed whereas now ($t = 0$) it is perturbed. There is an operator (the time-development operator) which leads from the situation at $t = -\infty$ to that at $t = 0$. The interaction operators are the perturbation operators represented in the so-called interaction picture and ordered with the help of the chronological operator. The sense of this last operator is that it arranges the sequence of the interaction operators from the earlier to the later times. So, for example, the product $H_A(t_1)\,H_B(t_2)$ of two interaction operators does not change on condition $t_1 > t_2$, but changes upon the action of the chronological operator into $H_B(t_2)\,H_A(t_1)$ if $t_2 > t_1$. The intervals of the integration performed over the time variables extend from $-\infty$ to zero; therefore, they correspond to a half of the scale given in Fig. 1. The time-development operator obtained in the above way can be averaged over the unperturbed wave function of the ground state. The imaginary part of the time derivative of the logarithm of this average represents exactly the correction to the energy of the unperturbed state, so when added to this energy, the correction gives the energy of the perturbed state. Such a treatment, although elegant, seems to make the perturbation calculation still more laborious than the Schrödinger iterative procedure. Especially, the treatment does not give a clear insight into how different components terms (sums), entering the Schrödinger perturbation series, can be obtained. This situation changes, however, when a circular scale of time — similar to that given in Fig. 2 — is taken into account instead of the linear scale. This kind of perturbation theory has been presented in some detail elsewhere[8]. In the next two sections we outline those of its features which seem to be important from the point of view of the topology of time.

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[8] S. Olszewski, *Time Scale and its Application in Perturbation Theory*, „Zeitschrift für Naturforschung" 46a (1991), 313—320.

6. PERTURBATION ENERGY OBTAINED FROM A CIRCULAR SCALE OF TIME

The time evolution of a quantum-mechanical system which goes on according to the circular scale of time can be represented by cycles of collisions of this system done with some perturbation potential V.

We assume that a system which is originally in a stationary state n is transfered under the action of the first collision in a cycle with the potential V into some other state p. After a sufficiently long time, state p can be considered as stationary similarly to n. It is convenient to assume that in state n the system is characterized by its own time variable t_n, in state p the system has its own time variable t_p, etc. Consequently, the collision with V changes t_n into t_p. Another collision transfers the system from a stationary state p to a stationary state q; then the time variable t_p is changed into t_q. Further collisions with V can transfer the system to state r, state s, etc.; accordingly, the time parameter will assume the time variables t_r, t_s, etc. The last collision in a cycle is necessarily that which transfers the system back into state n. The next collision with V begins a new cycle. The number of cycles is unlimited and tends to infinity. An example of a cycle composed of four collisions (three intermediate collisions) is represented in Fig. 4.

Any collision gives its contribution to the energy change of the system. This change is proportional to the integral of the product of (a) the complex conjugate of the wave function of the system before collision, (b) the perturbation potential, (c) the wave function after collision. The integration is performed over the position-dependent, or spatial, as well as the time-dependent coordinates. Any collision defines the space and time variables over which the integration has to be done. In general, the result of the succesive integrations, corresponding to succesive collisions, tends to zero. The exception is the situation when a cycle ends with some collision which transforms the system back to its initial state n. We say then that a cycle is closed and then we obtain a non-zero contribution to energy known from the Schrödinger perturbation theory.

The integrals over space and time can be separated. The result of the integration over time for a (closed) cycle of collisions we call a kinetic part, whereas a similar integral done over space variables in a cycle is called a static part.

In order to get different contributions to the Schrödinger perturbation energy, the system must be submitted to different (closed) cycles of collisions which begin and end at state n. The number of collisions in a cycle corresponds with the order of the Schrödinger perturbation term. If the time evolution in the collision cycle is represented by a non--branched path, then we obtain only one Schrödinger term for each perturbation order. This is not a satisfactory state of affairs, since for any perturbation order larger than 2 the number of the Schrödinger terms (sums) is larger than unity; for a high perturbation order this excess in the number of terms becomes very high. In order to get a correct number of the Schrödinger perturbation terms, branched paths of the time evolution of a system during its collision cycle should be assumed. Graphically the non-branched time path for a cycle can be represented by a single loop (circle), called a main loop, whereas the branched time paths are composed of several loops, called side loops, which spring out of the time loop having the beginning-end state n. The branched time paths can be obtained with the aid of the elimination principle discussed in Sec. 7.

Let us note that in order to get a correct representation of Schrödinger's result for perturbation energy, all collision cycles have to be different, either in their shape or in the indices which label the states met in the collisions. A repetition of a cycle gives no contribution to energy.

7. ELIMINATION PRINCIPLE FOR EQUAL TIMES

In quantum mechanics of many-electron systems we have a principle given by Pauli which makes reference to symmetry properties of the wave function of the system. If the wave function of a many-electron system is approximated by a combination of products of the one-electron wave functions the Pauli principle becomes an exclusion principle which states that any one-electron wave function cannot be occupied by more than one electron. The exclusion principle is of a fundamental importance for describing physical and chemical properties of matter. Historically, the explanation of the periodic system of elements, as well as the electric and thermal properties of metals, represented its first success. The assumption that electrons in a system behave like identical particles was also at the basis of the principle.

Although physically it has a completely different sense, the formulation of the elimination principle in this paper has some similarities to that of the Pauli principle. The elimination principle concerns the time instants of collisions with V and states that the energy contribution given by any cycle in which the system has two or more simultaneous collisions with V should be subtracted from the perturbation energy. At the same time, we postulate that the static part of two cycles having the same pattern of collisions with V are identical independently of that whether a cycle is represented by a main loop, or a side loop of time.

In applying the elimination principle, as well as the postulate of identity for the static parts, the beginning-end state n is considered in a different way than the intermediate states in a cycle. First, the own time of state n can never be equal to the own time of an intermediate state; second, the beginning-end state for any static part in a cycle represented by a side loop should be put equal to state n. This equality makes the beginning-end state for a static part the same for all cycles.

The elimination principle needs to launch a combinatorial analysis for any collision cycle. This analysis has, as its purpose, to select all cases for which the intermediate collision times can be equal. Two or more equal collision times divide the original (main) loop of time into two or more loops. For example, for a cycle which has three intermediate collisions (represented by a loop on Fig. 4) we can have the following cases of equal times: the collision instant 1 is simultaneous with collision instant 2; collision in 1 is simultaneous with that in 3; collision 2 is simultaneous with 3; finally we can have simultaneously all three intermediate collisions (instants 1, 2, 3). The colon on Fig. 5—5c represents a symbol of equal times. An important point is that any two times, when put equal on a given path, should give loops which may touch; but no crossing of the time path is allowed. For example, a path is not allowed on which simultaneous time are selected in the way represented in Fig. 6. The allowed kind of selection of the time paths implies that the sequence of collision times with V should be preserved. If we put collision time 1 equal to time 3, then time 2 (which is intermediate between 1 and 3) cannot be equal to time 4 which is later than 3.

The elimination principle can lead to time paths composed of many loops. For example, for a cycle of four intermediate collisions, where a collision time 1 is put equal to collision time 4, we obtain a path represented by two loops. However, the elimination principle requires also to take into account the case of time 2 equal to time 3 on the path given on the right side of Fig. 7. This leads to the path represented in Fig. 7a and a separate Schrödinger term corresponding to it is attained.

A consequent application of the elimination principle, together with the postulate of identity of the static parts, gives a full set of terms of the Schrödinger perturbation series for energy. The sign with which the terms enter the series is also given by the elimination principle. Two collision times assumed equal on a given non-branched path means that the resulted term has to be subtracted from the term representing the cycle having a non-branched path. For example, three collision times assumed equal together, can be considered as coming from a path of two equal collision times, so the result for three equal collision times should be subtracted from that obtained for two equal collision times, etc.

8. SURVEY

In this paper we examined a problem of the topology of the time scale. As an alternative to an open, or linear, scale there are presented arguments for a closed, or circular, scale of time.

As the first step, we pointed out that the time scale can be a subjective notion dependent on the physical properties of a given system and the perception ability of an observer connected with that system. In a fully periodic classical system, there is no parameter which allows for an observer to distinguish between one cycle of events (observations) and another cycle. In this case the circular scale time is naturally fitted to the periodic properties of the system. The length of the scale may depend on: (i) the number and kind of parameters characterizing the system, (ii) the perception ability of an observer. In the case of a larger scale both (i) and (ii) are larger than in the case of a shorter scale. Only an observer in a periodic system having a larger scale can detect the periodicity of a system having a shorter scale.

As a second step, in order to have an idea about the topology of the time scale for microphysical systems, this topology was examined for a non-degenerate quantum-mechanical system perturbed by a small potential. It is pointed out that the Schrödinger perturbation energy of this system can be obtained in a rather simple way on condition that a circular scale of time is assumed. Beginning with some unperturbed quantum state, the system is submitted to a cycle of collisions with the perturbation potential; after the last collision in a cycle the system return to its beginnig state and a new cycle of collisions begins. An elimination principle is taken into account in order to subtract the contributions given by the cycles in which two or more simultaneous collisions with the potential occur. With the aid of this principle there exists a strict correspondence between the colllision cycles and the terms of the Schrödinger perturbation series. On the other hand, the application of a linear scale of time to a non-degenerate ground state can reproduce Schrödinger's theory of perturbation energy of a system in a much more complicated way.

I dedicate this paper to the memory of my father, Paweł Olszewski, who stimulated my interest in the problem of time. I am grateful to Tadeusz Kwiatkowski for his assistance in checking the components of the perturbation series and to Joseph A. Dziver for his collaboration in preparing the English version of the manuscript.

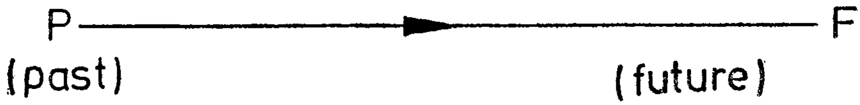

Fig. 1. Linear scale of time.

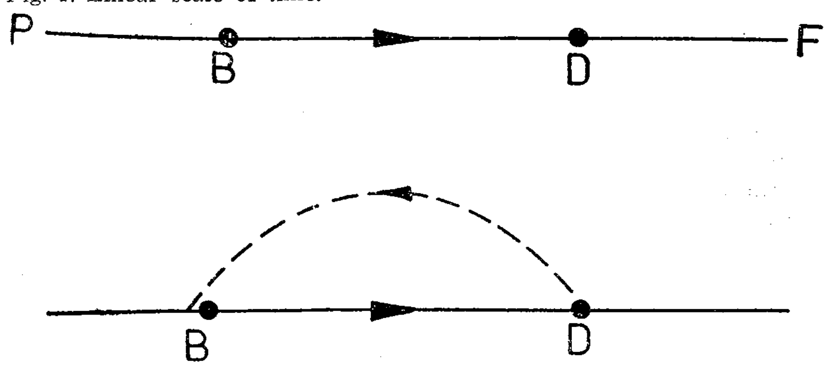

Fig. 1a, 1b. Linear scale of time for a classical periodic system. B — the first observed point; D — the last observed point. For a strictly periodic system the observation of D takes place immediately before the observation B.

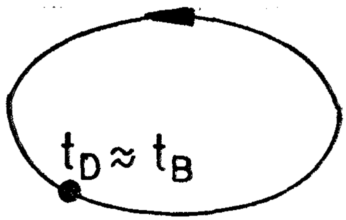

Fig. 2. Circular scale of time.

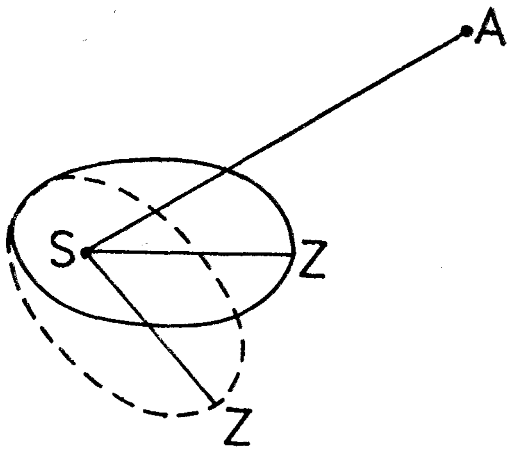

Fig. 3. The motion of Z along an ellipse about point S which is in one of the ellipse foci. The dashed orbit represents precession of the ellipse about the axis going across S and being perpendicular to the Figure plane.

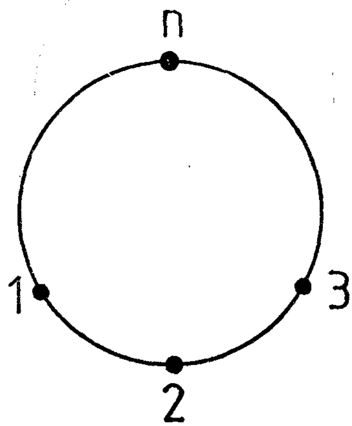

Fig. 4. An example of a cycle of collisions having three intermediate collisions with the perturbation potential V.

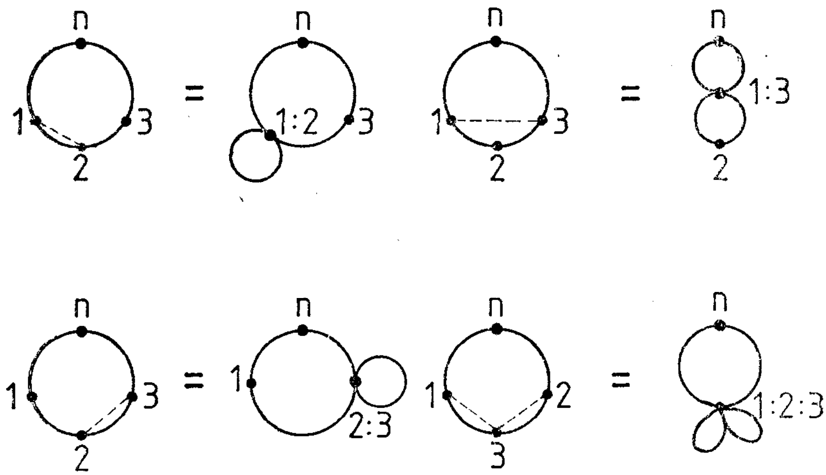

Fig. 5—5c. Cycles of three intermediate collisions with potential V in case when simultaneous collisions occur.

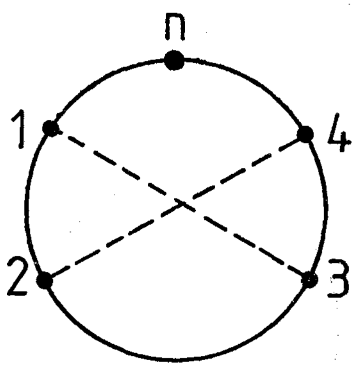

Fig. 6. An example of a forbidden set of simultaneous collisions for a cycle of four intermediate collisions.

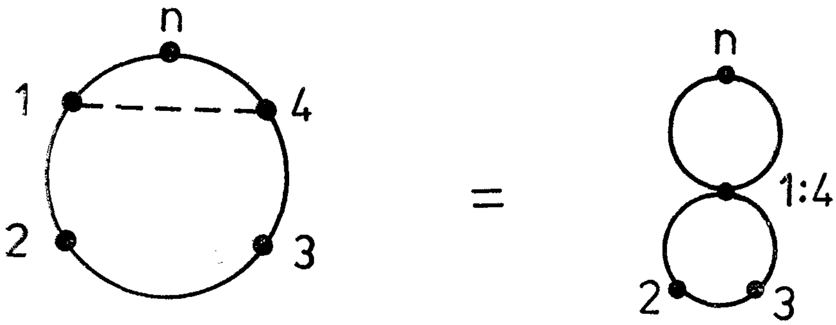

Fig. 7

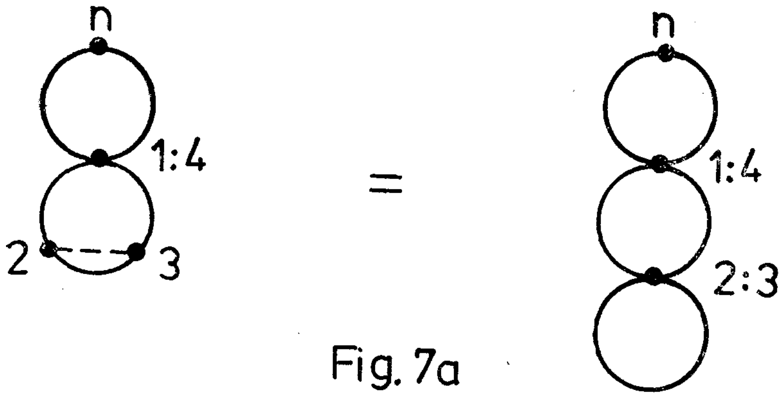

Fig. 7a

Fig. 7—7a. Examples of allowed sets of simultaneous collisions for a cycle of four intermediate collisions.