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Hilbert and Bernays on definite descriptions

Studia Philosophiae Christianae 47/4, 19-29

2011

Studia Philosophiae Christianae
UKSW
47(2011)4

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HILBERT & BERNAYS ON DEFINITE DESCRIPTIONS

Key words: descriptions, D. Hilbert, P. Bernays

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1. INTRODUCTION

In their *Grundlagen der Mathematik* [GdM, vol. 1, §8, 1934] Hilbert and Bernays develop a theory of definite descriptions within a logical and mathematical framework.[1] The following quote of Hilbert and Bernays captures the focus of this paper (my translation; original in footnote[2]): "In order to set the rules for the use of the new 'ι-symbol'

[1] Since we shall for the most part only refer to the 1st volume of the 1st edition of *GdM*, we always refer to that particular volume as *GdM*.

[2] "Um die Verwendung dieses neuen '"-Symbols' in unserem Kalkül zu regeln, wollen wir uns möglichsteng an das tatsächlich im Sprachgebrauch und insbesondere auch in der Mathematik befolgte Verfahren anschließen, welches darin besteht, daß man einen Ausdruck wie 'dasjenige Ding, welches die Eigenschaft A hat', überhaupt nur dann verwendet, wenn bereits feststeht, daß es ein und nur ein Ding von dieser Eigenschaft gibt. Wir lassen demgemäß einen Ausdruck $\iota x A(x)$ erst dann als Term zu, wenn die zu $A(a)$ gehörigen Unitätsformeln abgeleitet sind. Außerdem müssen wir noch zum Ausdruck bringen, daß in dem genannten Fall der Term $\iota x A(x)$ eben ein solches Ding darstellt, auf welches $A(a)$ zutrifft. So kommen wir zur Aufstellung folgender Regel für den Gebrauch des ι-Symbols, die wir kurz als die 'ι-Regel' be-

in our calculus, we want to comply as closely as possible with the actual use of language and in particular with the common procedure in mathematics. In this procedure the use of the expression such as ‘the thing, that has the property A’ is only used when it is certain that there is exactly one thing with this property. Accordingly, an expression $\iota_x A(x)$ is an admissible term only if the uniqueness formulae for $A(a)$ are deducible. We also have to point out that in this case the term $\iota_x A(x)$ represents the thing to which $A(a)$ applies. Therefore, we now establish the following rule for the use of the ι-symbol, which we call the ‘ι-rule’ in short: if the uniqueness formulae for $A(a)$ are deducible, then $\iota_x A(x)$ (…) is a term from this point onwards, and the formula $A(\iota_x A(x))$ is a deducible formula according to the scheme:

$$\frac{\begin{array}{l}\forall x A(x)\\ \forall x \forall y (A(x) \wedge A(y) \rightarrow x=y)\end{array}}{A(\iota_x A(x))}$$

” (H.-B. *GdM*, p.384)

On H.-B.’s account, definite descriptions are seen as admissible terms. The admissibility of definite descriptions depends on whether or not the *uniqueness condition* (to which Hilbert and Bernays refer to in the quote as “the uniqueness formulae”) is provable in the formal system (**FS**) in question. In order to state the uniqueness condition in a proper manner some terminology is needed.

A definite description within a formal framework is depicted as: $\iota_x A(x)$, where ‘ι_x’ is called the *description operator* and ‘$A(x)$’ is called the *basis of the description*. ‘$\iota_x A(x)$’ is read as: the x such that x is A.

The uniqueness condition is split into two sub-conditions: the *existential* and the *definiteness* condition. Both can be stated formally as follows:

(UC1) $\mathbf{FS} \vdash \exists x A(x)$ (Existential Condition)

(UC2) $\mathbf{FS} \vdash \forall x \forall y (A(y) \wedge A(x) \rightarrow x=y)$ (Definiteness Condition)

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zeichnen wollen: Sind für die Formel A(a) die Unitätsformeln abgeleitet, so gilt von da an ιxA(x) (…) als Term, und die Formel A(ιxA(x)) gilt als abgeleitete Formel im Sinne des Schemas

$$\frac{\begin{array}{c}\exists x A(x)\\ \forall x \forall y (A(x) \wedge A(y) \rightarrow x=y)\end{array}}{A(\iota x A(x))}$$

.” *GdM*, p.384.

Now H.-B. proceed by claiming that if (UC1) and (UC2) are both provable in **FS** for a certain formula A, then $\iota_x A(x)$ can be introduced in the language.

Following their suggested procedure two questions arise:

(1) Is this procedure not potentially circular, since the set of provable formulas depends on the set of well-formed expressions and vice versa; and

(2) Given H.-B.'s procedure of introducing definite descriptions, does this procedure yield the decidability of the well-formed set of expressions of a given (formal) language?

Usually the (simultaneous) recursive definition of terms and formulas can be turned into an enumeration machine, such that the question of whether a given expression belongs to the language or not is decided within a finite number of purely mechanical steps. However, the answer within H.-B.'s context is not that straightforward. The decidability of the set of expressions depends on the mathematical context in which their procedure is embedded. If it is embedded, e.g., in *Presburger Arithmetic*, the set of terms and formulas of a given language is indeed decidable. Nevertheless, this does not hold if the mathematical context is richer, as in H.-B.'s system **Z** (H.-B. *GdM*, p. 371). But our investigation is set within **Z**'s frame.

The main focus of this paper is the question (1). Our approach to the alleged circularity is basically this: at the outset (Section 2) we shall state a simultaneous inductive definition for language **L*** of arithmetic with definite descriptions (i.e., ι-terms) and an inductive definition of a language $\mathbf{L_0}$ of arithmetic without ι-terms. **L*** will be enumerated by some fixed enumeration **E**, although this enumeration will not be stated explicitly. Next a formal system **HB** based on $\mathbf{L_0}$ (H.-B.'s system **Z**) will be presented. In order to extend $\mathbf{L_0}$ to $\mathbf{L_1}$ we make use of the enumeration of **L***. Let some formula A_i be the first formula of **L*** which is in $\mathbf{L_0}$ such that $\mathbf{HB_{L0}} \vdash \exists x A_i(x)$ and $\mathbf{HB_{L0}}$ **FS** $\vdash \forall x \forall y (A_i(y) \wedge A_i(x) \rightarrow x{=}y)$. Then $\iota_x A_i(x)$ is added to $\mathbf{L_1}$ and $\iota_x A_i(x)$ is a term of (extension-) level 1 and $A_i(\iota_x A_i(x))$ is a formula of $\mathbf{L_1}$. Then we define a provability relation for $\mathbf{HB_{L1}}$ and add a special instance of the H.-B.'s ι-rule.

Section 3 provides a semantical treatment of H.-B.'s account closely related to the methods of section 1. Section 3 states an elimination procedure (by proof-theoretic means). We would also like to mention that there are several accounts of H.-B.'s theory of definite descriptions in the literature.[3] All those approaches are "inspired" by H.-B.'s account but in our opinion they do not explicate some necessary details of the original theory.

2. SYNTAX

As mentioned above, we start with two languages, **L*** and $\mathbf{L_0}$. **L*** will contain ι- terms, and its enumeration **E** will serve as the basis for the extensions that will be carried out depending on the formal system **HB**, **E**, and the provability of the uniqueness condition of the basis of the ι-term.

2.1. LANGUAGE L*AND L_0

Alphabet

Individual constant: ○
a, *b*, *c*, … (with or without indices) are *free* individual variables.
x, *y*, *z*, … (with or without indices) are *bound* individual variables.
Logical signs: ¬, ∧, ∨, →, ↔, ∃, ∀, =, ι
Mathematical signs: ′, +, ×
Auxiliary signs: (,)

2.1.1. SIMULTANEOUS INDUCTIVE DEFINITION OF TERMS AND FORMULAS OF **L***

1) Every free variable and ○ is a term of **L***.
2) If s and t are terms of **L***, then s', $(s + t)$ and $(s \times t)$ are terms of **L***.
3) If s and t are terms of **L***, then $(s=t)$ is a formula of **L***.

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[3] [Lambert 1999; Lambert 2003], [Stenlund 1973], [Kleene 2000], [Kneebone 1965]. We note that our approach is similar to [Lambert 1999; Lambert 2003].

4) If A and B are formulas of **L***, then $\neg A$, $(A \wedge B)$, $(A \vee B)$, $(A \rightarrow B)$, $(A \leftrightarrow B)$ are formulas of **L***.

5) If $A(a)$ is a formula of **L*** such that the bound variable x does not occur in
$A(a)$, then $\forall x A(x)$ and $\exists x A(x)$ are formulas of **L***.

6) If $A(a)$ is a formula of **L*** such that the bound variable x does not occur in
$A(a)$, then $\iota_x A(x)$ is a term of **L***.

7) Nothing else is a formula or a term of **L***.

The notation $A(a)$ is explained in the following way: the free variable a marks the occurrences in A at several places (not necessarily all and maybe none). $A(x)$ is the formula which is obtainable from $A(a)$, if each free variable a is substituted by the bound variable x on the mentioned occurrences. For our reconstruction of H.-B.'s account of ι-terms, we think of some fixed enumeration of the expressions of **L***. The importance of this enumeration will be seen in section 2.4.

2.1.2. INDUCTIVE DEFINITION OF $\mathbf{L}_0$

Terms of $\mathbf{L}_0$

1) Every free variable and ○ are terms of level 0 and in $\mathbf{L}_0$.

2) If s and t are of level 0 and in $\mathbf{L}_0$, then s', $(s+t)$ and $(s \times t)$ are of level 0 and in $\mathbf{L}_0$.

3) All the terms of $\mathbf{L}_0$ are of level 0 (and no other terms are in $\mathbf{L}_0$).

Formulae of $\mathbf{L}_0$

1) If s and t are terms of $\mathbf{L}_0$, then $(s=t)$ is a formula of $\mathbf{L}_0$.

3) If A and B are formulas of $\mathbf{L}_0$, then $\neg$A, $(A \wedge B)$, $(A \vee B)$, $(A \rightarrow B)$, $(A \leftrightarrow B)$ are formulas of $\mathbf{L}_0$.

4) If $A(a)$ is a formula of $\mathbf{L}_0$ s.t. x does not occur in $A(a)$, then $\forall x A(x)$ and $\exists x A(x)$ are formulas of $\mathbf{L}_0$.

5) Nothing else is a formula of $\mathbf{L}_0$.

2.2. HB BASED ON L_0

The formal system called **HB** is similar to H.-B.'s system **Z** (*GdM*, p. 371).

Logical Axioms

(LAx1) Every Tautology.
(LAx2) $\forall x A(x) \rightarrow A(r)$
(LAx3) $A(r) \rightarrow \exists x A(x)$
(LAx4) $\forall x(x=x)$
(LAx5) $\forall x \forall y(x=y \rightarrow (A(x) \rightarrow A(y))$

Mathematical Axioms

(MAx1) $\neg\exists x(x' = \circ)$
(MAx2) $\forall x \forall y(x'=y' \rightarrow x=y)$
(MAx3) $\forall x(x+\circ=x)$
(MAx4) $\forall x \forall y(x+y')=(x+y)')$
(MAx5) $\forall x(x\times\circ=\circ)$
(MAx6) $\forall x \forall y(x\times y')=(x\times y)+x)$
(MAx7) $A(\circ) \wedge \forall x(A(x) \vee \rightarrow A(x')) \rightarrow \forall x A(x)$

Rules of Inference

(Det) $\dfrac{A,\ A \rightarrow B}{B}$ (α) $\dfrac{B \rightarrow A(a)}{B \rightarrow \forall x A(x)}$ (β) $\dfrac{A(a) \rightarrow B}{\exists x A(x) \rightarrow B}$

The free variable a must not occur under the inference line in (α) and (β).

2.3. PROVABILITY IN HB BASED ON L_0

A formula is an *immediate consequence* of one or two other formulas written above the line, if it has the form shown below the line of (Det), (α) or (β).

We shall write '$\mathbf{HB_{L0}}$' instead of the longer phrase: '**HB** based on $\mathbf{L_0}$'.

2.3.1. INDUCTIVE DEFINITION OF PROVABLE FORMULA IN $\mathbf{HB_{L0}}$

(P01) Every axiom is *provable* in $\mathbf{HB_{L0}}$.

(P02) If A is *provable* in $\mathbf{HB_{L0}}$, and B is an immediate consequence of A, then B is provable in $\mathbf{HB_{L0}}$.

(P03) If A and B are *provable* in $\mathbf{HB_{L0}}$, and C is an immediate consequence of A and B, then C is provable in $\mathbf{HB_{L0}}$.

(P04) A formula is provable in $\mathbf{HB_{L0}}$ only as required by (P01)–(P03).

If a formula A is provable in $\mathbf{HB_{L0}}$ we write: $\mathbf{HB_{L0}} \vdash A$.

2.4. EXTENSION FROM $L_{N\text{-}1}$ TO L_N

In the following the important clause is 3).

1) If t is a term of level $\boldsymbol{n}$-1 and t is in $\mathbf{L}_{n\text{-}1}$, then t is a term of level $\boldsymbol{n}$ and is in $\mathbf{L}_n$.

2) If A is a formula of $\mathbf{L}_{n\text{-}1}$, then A is a formula of $\mathbf{L}_n$.

3) Let A_i be the first formula in the enumeration of $\mathbf{L}^*$, which is in $\mathbf{L}_{n\text{-}1}$ such that

$\mathbf{HB_{L\mathit{n}\text{-}1}} \vdash \exists x A_i(x)$ and $\mathbf{HB_{Ln\text{-}1}} \vdash \forall x \forall y(A_i(x) \wedge A_i(y) \rightarrow x{=}y)$ then $\iota x A_i(x)$ is a term of level $\boldsymbol{n}$ and is in L_n.

4) If s and t are terms of level $\boldsymbol{n}$ and s and t are in $\mathbf{L}_n$, then s', $(s+t)$ and $(s\times t)$ are of level $\boldsymbol{n}$ and in $\mathbf{L}_n$.

5) If s and t are terms of $\mathbf{L}_n$, then $(s{=}t)$ is a formula of $\mathbf{L}_n$.

6) If A and B are formulas of $\mathbf{L}_n$, then $\neg A$, $(A \wedge B)$, $(A \vee B)$, $(A \rightarrow B)$, $(A \leftrightarrow B)$ are formulas of $\mathbf{L}_n$.

7) If $A(a)$ is a formula of $\mathbf{L}_n$ s.t. x does not occur in $A(a)$, then $\forall x A(x)$ and $\exists x A(x)$ are formulas of $\mathbf{L}_n$.

Remark

If there is a term like $\iota_x A(x) + \iota_x B(x)$ or $\iota_x A(x) \times \iota_x B(x)$ such that the level of one term is greater than the level of the other, then the level of the term $\iota_x A(x) + \iota_x B(x)$ or $\iota_x A(x) \times \iota_x B(x)$ is that of the greater term.

2.5 HB BASED ON L_N

First, we simply have to restate every axiom and inference rule of $\mathbf{HB_{L0}}$ for $\mathbf{HB_{L\mathit{n}}}$. Second, we add the ι-rule as a further rule of inference of $\mathbf{HB_{L\mathit{n}}}$, whereas A_i is a formula of $\mathbf{L}_n$:

(ι-rule) $$\frac{\exists x A(x) \quad \forall x \forall y(A_i(x) \wedge A_i(y) \rightarrow x{=}y)}{A_i(\iota_x A_i(x))}$$

3. SEMANTICS

In this section we shall give, in addition to the syntax, a semantical treatment (i.e., we provide the standard model for Peano Arithmetic) of our reconstruction of Hilbert and Bernays' procedure. This will be done closely along the lines of the syntax, thereby making use again of the enumeration **L***. It should be borne in mind that due to Gödel's theorems the semantics is by far stronger than **HB**'s proof theory.

3.1. BASIS: $\langle N, \Phi_0 \rangle$ FOR L0

We begin with the *structure* $\langle \mathbf{N}, \varphi_0 \rangle$ for **L0** as the basis for the semantical treatment of H.-B.'s view. Instead of writing $\circ'^{(n\text{-times})}$ we write simply $\underline{n}$; e.g. $\circ = \underline{0}$, $\circ' = \underline{1}$, $\circ'' = \underline{2}$, etc.

1) N is the set of natural numbers (including 0), i.e. $\{0, 1, 2, 3, \ldots\}$
2) φ_0 is an interpretation function such that the following conditions hold:

2.1) $\varphi_0(\circ) = 0$,
2.2) $\varphi_0(a) \in \mathbf{N}$, for each free individual variable a of $\mathbf{L_0}$.
2.3) $\varphi_0(')$ is the *successor function* from **N** to **N**, such that: $\varphi_0(t') = \varphi_0(t)+1$,
2.4) $\varphi_0(+)$ is the *sum function* from $\mathbf{N}^2$ to **N** such that $\varphi_0(s+t) = \varphi_0(s) + \varphi_0(t)$,
2.5) $\varphi_0(\times)$ is the *multiplication function* from $\mathbf{N}^2$ to **N** such that: $\varphi_0(s\times t) = \varphi_0(s) \times \varphi_0(t)$,
2.6) $\varphi_0(=)$ is the *identity relation* in $\mathbf{N}^2$ such that: $\varphi_0(s=t) = \mathbf{T}$ iff $\varphi_0(s) = \varphi_0(t)$,
2.7) $\varphi_0(\neg A) = \mathbf{T}$ iff $\varphi_0(A) = \mathbf{F}$,
2.8) $\varphi_0(A\rightarrow B) = \mathbf{T}$ iff $\varphi_0(A) = \mathbf{F}$ or $\varphi_0(B) = \mathbf{T}$,
2.9) $\varphi_0(A\wedge B) = \mathbf{T}$ iff $\varphi_0(A) = \mathbf{T}$ and $\varphi_0(B) = \mathbf{T}$,
2.10) $\varphi_0(A\vee B) = \mathbf{T}$ iff $\varphi_0(A) = \mathbf{T}$ or $\varphi_0(B) = \mathbf{T}$,
2.11) $\varphi_0(A\leftrightarrow B) = \mathbf{T}$ iff $\varphi_0(A) = \varphi_0(B)$,
2.12) $\varphi_0(\forall x A(x)) = \mathbf{T}$ iff $\varphi_0(A(\underline{m})) = \mathbf{T}$ for each natural numbert m,
2.13) $\varphi_0(\exists x A(x)) = \mathbf{T}$ iff $\varphi_0(A(\underline{m})) = \mathbf{T}$ for some natural numbert m.

It is easily seen that $\varphi_0(\underline{m})=m$ holds.

3.2. EXTENSION FROM L_{N-1} TO L_N

1) For every term t of level n-1 in $\mathbf{L}_{n-1}$: $\varphi_n(t) = \varphi_{n-1}(t)$.
2) For every formula A of $\mathbf{L}_{n-1}$: $\varphi_n(A) = \varphi_{n-1}(A)$.
3) Let A_i be the first formula in the enumeration of **L***, which is in $\mathbf{L}_{n-1}$, such that $\varphi_{n-1}(\exists x A_i(x)) = \varphi_{n-1}(\forall x \forall y(A_i(x) \wedge A_i(y) \rightarrow x=y)) =$ **T**, then $\mathbf{L}_{n-1}$ is extended to $\mathbf{L}_n$ and $\iota_x A_i(x)$ is of level n and in $\mathbf{L}_n$ and $A_i(\iota_x A_i(x))$ is a formula of $\mathbf{L}_n$.
4) If $\varphi_{n-1}(\exists x A_i(x)) = \varphi_{n-1}(\forall x \forall y(A_i(x) \wedge A_i(y) \rightarrow x=y)) =$ **T**, then there exists in the standard interpretation an $m \in \mathbf{N}$, such that $\varphi_{n-1}(\underline{m})=m$ and $\varphi_{n-1}(A(\underline{m}))=\mathbf{T}$. We take this m and define $\varphi_n(\iota_x A_i(x)) = m$.

Finally we (could) define a structure $\langle \mathbf{N}, \varphi_\omega \rangle$ based on all structures $\langle \mathbf{N}, \varphi_n \rangle$; a formula A is *valid in* $\langle \mathbf{N}, \varphi_\omega \rangle$ iff $\varphi_\omega(A)=\boldsymbol{T}$.

It is easily proved that every axiom and every rule of inference of $\mathbf{HB}_{\mathbf{L}_n}$ is valid in $\langle \mathbf{N}, \varphi_n \rangle$

4. CONCLUDING REMARKS

H.-B.'s main intention is that every singular term denotes. This is ensured within this circular-free reconstruction. We have presented H.-B.'s account in the context of a mathematical framework. Neither $\mathbf{HB}_{\mathbf{L}_n}$ nor the set of all well-formed expressions is decidable. For example Carnap [Carnap 1956][4] admits that H.-B.'s approach might be convenient for practical work with a logico-mathematical system, even though he concedes that following H.-B.'s theory leads to awkward conclusions. For instance the set of well-formed expressions depends on the contingency of the world, when the background theory is not a mathematical (as we did in this paper) but a physical theory. Scott [Scott 1990] challenges H.-B.'s view by stating that their

[4] [Carnap 1956, 34]. He continues: "For systems also containing factual sentences, the disadvantage would be still greater, because here the question of whether a given expression is a sentence or not would, in general, depend upon the contingency of facts."

approach does not really justice to the mathematical practice, since even mathematicians introduce ι-terms without proving the uniqueness condition first.

Nevertheless, we think that the approach presented here has its merits, such as the guarantee that an introduced ι-term refers. Furthermore, it might be interesting to investigate a proof for the existence of god (like Anselm's argument) in a way analogous to the one presented here.

Lambert [Lambert 1999; Lambert 2003] also provided reconstructions of H.-B.'s account. However, we hold the view that our reconstruction is advantageous in at least two respects: (1) As it has been constructed here, the theory is embedded in some other theoretical framework that exceeds pure logic, e.g. Peano Arithmetic. And this point is noteworthy since whether a definite description can be introduced in a theory depends the on the strength of its provability relation. (2) It is not very clear if e.g. Lambert (Lambert 1999, especially pp. 275f.] does really abandon the alleged circularity with which Hilbert and Bernays account is confronted with.

We want briefly turn to the relationship between ι-terms and ε-terms.[5] First we adopt the formation rules for $\mathbf{L}_0$ (turning it into a simultaneous recursive definition of terms and formulas): If $A(a)$ is a formula such that the bound variable x does not occur in it, then $\varepsilon_x A(x)$ is a singular term. Second, we interpret the ε-term (informally) in the following way (supposing tacitly that the domain is the set of natural numbers): if there is at least one natural number n such that $A(\underline{n})$ is true, then $\varepsilon_x A(x)$ refers to some natural number with property A. ε-terms may be thought of *indefinite* descriptions. H.-B. chose the

$$(\varepsilon\text{-formula}) \qquad A(t) \rightarrow A(\varepsilon_x A(x))$$

as the new axiom for ε-terms.

Now if $\mathbf{HB_{L}}_{k\text{-}1} \vdash \exists x A_i(x)$ and $\mathbf{HB_{L}}_{k\text{-}1}\ \mathbf{FS} \vdash \forall x \forall y (A_i(y) \wedge A_i(x) \rightarrow x=y)$, then by the (ι-rule): $\mathbf{HB_{L}}_k \vdash A_i(\iota x A_i(x))$.

But in the light of the (ε-formula): $\mathbf{HB_{L}}_k \vdash A_i(\iota_x A_i(x)) \rightarrow A_i(\varepsilon_x A_i(x))$ and hence $\mathbf{HB_{L}}_k \vdash A_i(\varepsilon_x A_i(x))$.

This research is supported by the Alexander von Humboldt Foundation.

[5] [Cf. Hilbert, Bernays 1939; Leisenring 1969].

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DESKRYPCJE

Streszczenie

Rozważania skupione są na pojęciach deskrypcji określonych i nieokreślonych. Prezentuje się znaną teorię deskrypcji określonych sformułowaną przez Bertranda Russella w „Principia Matematica". Rozważa się przy tym niektóre z problemów związanych z tym podejściem. Mniej znane są badania Russella dotyczące deskrypcji nieokreślonych. W tym przypadku także przedstawia się problemy związane z koncepcją Russella i parę propozycji ich rozwiązań.

Słowa kluczowe: deskrypcje, D. Hilbert, P. Bernays