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Presuppositions of classical logic : presuppositions of classical physics

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PRESUPPOSITIONS OF CLASSICAL LOGIC PRESUPPOSITIONS OF CLASSICAL PHYSICS

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1. INTRODUCTION

There are hidden assumptions in Classical Logic (*CL*) and there are hidden assumptions in Classical Physics (*CP*). The hidden assumptions of *CL* presuppose a type of arbitrariness concerning the parts (propositions, concepts, predicates) in valid logical principles of *CL*. This arbitrariness is uncovered if *CL* is applied outside logic and mathematics, i.e. to empirical sciences [Weingartner 2001][1]. The hidden assumptions of *CP* presuppose a type of arbitrariness concerning the parts (concepts, properties, physical systems) in fundamental laws of nature. This arbi-

[1] By *CL* we mean classical two valued propositioned calculus (*CPC*) and classical predicate calculus of first order with identity. The problems in this essay are mainly concentrated on the propositional part of *CL*. By *CP* we mean physics without the Theory of Relativity (special and general) and Quantum Theory. The problems of this essay are mainly concentrated on Classical Mechanics (*CM*) as a part of *CP*.

trariness has been uncovered by the development of modern physics, especially by Quantum Theory and by the Theory of Relativity. Both types of arbitrariness have several similarities.

The subsequent sections will be concerned with the following presuppositions of *CL* and *CP*: In section 2 we shall deal with the ontological aspect of the principle of excluded middle and with Kant's principle of complete determination. Section 3 will be concerned with the assumption of fusing arbitrary propositions and arbitrary predicates. Section 4 will deal with arbitrary combinations of predicates (properties) and consequently with the problem of commensurability. Section 5 will be concerned with the assumption of universal distributivity. Section 6 will be devoted to the hidden assumption of possible replacement of parts as propositions, predicates, coordinates, parameters, of a principle or of a law by other parts of the same category. In section 7 a many-valued logic will be proposed, which avoids the strong assumptions and does not lead to difficulties when it is applied to empirical sciences and to physics.

2. THE ONTOLOGICAL ASPECT OF THE PRINCIPLE OF EXCLUDED MIDDLE

2.1. THE PRINCIPLE OF EXCLUDED MIDDLE

The principle of excluded middle can be expressed in versions of different strength [Rescher 1969, 148 ff]. The most restricted version rules out any many-valued logic. It can be expressed by the following two conditions:

BV1 There are only two single (truth-)values *T* (true) and *F* (false)

BV2 Every proposition must be either true (*T*) or else false (*F*).

Both principles together make up the principle of *bivalence*. Observe that *BV2* alone does not exclude every type of many-valued logic. It excludes logical systems which have additional values besides *T* and *F* as *indeterminate* or *intermediate*. But it does not exclude many-valued systems which have more than on value for truth (T_1, T_2 ... T_n) and/or more than one value of false (F_1, F_2 ... F_n).[2]

A more tolerant version of the principle of excluded middle is this:

PEM A proposition cannot be true and its denial fail to be false and vice versa. Or: If a proposition is true its negation (contradictory or denial) is false and vice versa.

As one can see, whereas *BV2* excludes 3-valued systems as those of Lukasiewicz, Kleene and Bochvar[3], *PEM* permits (in its two formulations) them and others with more than one intermediate value and with more than one value for true and for false. In this sense the expression "excluded middle" is misleading.

An ontological version of the principle of *bivalence* (*BV1* + *BV2*) is this:

BV3 In our universe *U* and in every subsystem *S* of *U* it holds: Of the pair of propositions *p* and (its negation) $\neg p$ at least one is true. Or: Of the pair of states of affairs represented by *p* and by $\neg p$ at least one (state of affairs) obtains.

An analogue of *BV3* can also be formulated for properties:

BV4 For every object *x* and every property *P* in *U* or *S* it holds: *P* belongs to *x* or *P* does not belong to *x*.

If we define the opposite of a property *P* by its complement $\overline{P}$ of *P* (the complement certainly has to be relativised to some domain) then *BV4* may be formulated thus:

BV5 For every object *x* and every pair of properties *P* and $\overline{P}$ in *U* or *S* it holds: at least one of *P* or $\overline{P}$ must belong to *x*.

Principle *BV5* comes close to the principle of complete determination (*Grundsatz* der durchgängigen Bestimmung) by Kant:

KT "Every thing, however, as to its possibility, further stands under the principle of thoroughgoing determination; according to which, among all possible predicates of things, insofar as they are compared with their opposites, one must apply to it" [Kant 1787, B600][4].

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[2] For such many-valued systems [Weingartner 1968] and [Weingartner 2009].

[3] For a lucid elaboration of these systems [Rescher 1969, 22 ff].

[4] The German text reads: "Ein jedes Ding aber, seiner Möglichkeit nach, steht noch unter dem Grundsatz der durchgängigen Bestimmung, nach welchem ihm von allen möglichen Prädikaten der Dinge, so fern sie mit ihren Gegenteilen verglichen werden,

2.2. CONSEQUENCES AND HIDDEN ASSUMPTIONS

Assume the property P in *BV5* is the property of a physical object or system x being in spatial position (with space coordinates x_1, x_2, x_3) where the space is finite. Then a consequence of *BV5* and of *KT* is that any such physical object or system always possesses a well-defined position in space. *BV5* and *KT* are formulated for every (possible) pair of properties. This consequence is an important hidden assumption of Classical Physics (*CP*) and especially of Classical Mechanics (*CM*). But it is no longer generally acceptable in Quantum Physics [Mittelstaedt, Weingartner 2005, 268].

We may also consider a kind of relativisation of the usual version of the principle of excluded middle in *CL*: $q \vee \neg q$.

If we relativise this to some assumption p then we shall get:

RBV $p \rightarrow ((p \wedge q) \vee (p \wedge \neg q))$

This principle is valid in *CL*. It is valid there even as a logical equivalence. We shall call it *relativised bivalence* (*RBV*). But when applied to physics, it leads to several difficulties. One of them is this: Let p be the statement that the property P belongs to a physical system x. Then the above principle claims: Any other arbitrary property Q is such that P conjoined with Q belong to x or P conjoined with non-Q belong to x.[5]

With respect to the double slit experiment this means: If the particle x hit the detector (photo-plate) behind the two slits – i.e. if p is the case – then, together with p, any arbitrary state of affairs q or non-q must be the case, regardless whether the obtaining or not-obtaining of such a state is observable (measurable) at all.

RBV leads to other difficulties concerning commensurability, since it claims universal commensurability on logical grounds (cf. section 4). In section 6 will be shown that it violates an important relevance criterion which protects from difficulties, when logic is applied to empirical

eines zukommen muß.“ Transl. by P. Guyer and A.W. Wood in Cambridge Edition of the Works of Kant.

[5] For a detailed discussion of *RBV* as *weak objectification* postulate see [Mittelstaedt 1989, 211 ff] and [Mittelstaedt 1998, 74 ff]. In the lattice L_q (p. 77) it holds only in the direction from right to left. This is the same in the restricted logic *RMQ* mentioned in ch. 6 below.

sciences. The problematic assumption of *RBV* and similarly of several other principles of *CL* is this: Under the assumption of p, p can be conjoined with any arbitrary proposition q or with its negation non-q. This is a consequence of a more general arbitrariness of *CL*, which allows replacement of parts in the consequent by arbitrary parts salva volidi-tate of several of its principles (cf. section 6).

We may weaken *BV5* in such a way that we obtain a consistency principle which is satisfied in both *CM* and *QP*:

BV5⁻ For every object x and every pair of properties P and $\overline{P}$ in U or S it holds: at most one of P or $\overline{P}$ can belong to x.

3. THE ASSUMPTION OF FUSING ARBITRARY PROPOSITIONS AND PREDICATES

CL makes the presupposition that two arbitrary propositions may be fused into a conjunction. According to Classical Logic the domain of meaningful propositions p, q, r, ... is truth functionally closed under the usual connectives[6] and thus also under conjunction $\wedge$. Thus if p and q describe physical states *CL* dictates that also $p \wedge q$ must describe a physical state. Or more specifically: If (under conditions r) proposition p describes the physical state P (that the position of a particle has a certain precise value) and (under conditions r) the proposition q describes the physical state Q (that the momentum of that particle has a certain precise value), then it is not the case that (under conditions r) the conjunction $p \wedge q$ describes a measurable magnitude at all. But according to classical logic it should, because the corresponding principle is a theorem in the underlying classical propositional logic:

$$[(r \rightarrow p) \wedge (r \rightarrow q)] \rightarrow [r \rightarrow (p \wedge q)] \qquad (1)$$

Observe however that this difficulty is not peculiar to physics. If p and q describe human actions, it does not follow that $p \wedge q$ describes a human action too. Similarly for states of animal behaviour:

Assume the proposition S represents (describes) the observable state of affairs that sexual excitement obtains, the proposition A represents (describes) the observable state of affairs that aggression obtains and

[6] The problem of universal logical closure is also stressed as an important problem by [Dalla Chiara, Giuntini 2001, 59]. Cf. also [Kreisel 1992] and [Weingartner 1996].

the proposition F represents (describes) the observable state of affairs that fear obtains. Then research about animal behaviour shows the following facts: $S \wedge F$ does not represent (describe) an observable state in male animals, but does represent (describe) an observable state in female animals. On the other hand: $S \wedge A$ does not represent (describe) an observable state in female animals, but does represent (describe) an observable state in male animals.

The general assumption for fusing arbitrary propositions in *CL* is the following one: From propositions p, q infer: $p \wedge q$. In fact, this kind of "principle" called *adjunction* is never formulated as an axiom when the classical two-valued propositional logic (*CPC*) is built up as an axiom system.[7] In the context of truth-values, adjunction is rather harmless. But as soon as the principle is applied to empirical situations – either in this simple form or relativised to a condition – the arbitrary fusing into a conjunction becomes a problem (see the examples above).

The problematic assumption of fusing presupposed by *CL* can be formulated as the principle *FC*. *FR* below restricts this too strong an assumption.

FC Any two propositions p, q can be fused into the conjunction $p \wedge q$ in the conclusion or consequent of an inference or implication iff

p, q appear somewhere as premises or as parts of the antecedent
(i) either unconditioned
(ii) or conditioned

According to *FC* one may infer $p \wedge q$ from two separate premises p, q (unconditioned) or conditioned in the form of the principle above.

FR Two propositions p, q can be fused into the conjunction $p \wedge q$ in the conclusion or consequent of an inference or implication only under special conditions which are specific w.r.t. a certain domain of application. For example *w.r.t.* the domain Quantum Physics (*QP*) commensurability of p with q is required; *w.r.t.* action theory compatibility or joint possibility of the respective actions is required.

[7] However it is used in some fragments of the classical two-valued propositional calculus (Schlechter 2005, 336); and systems of natural deduction.

4. THE ASSUMPTION CONCERNING COMMENSURABILITY

This assumption is closely connected with the assumption of fusing arbitrary propositions and predicates. But fusing need not be only fusing by conjunctions. Also *CL*'s permission of arbitrarily combining propositions and (with them) predicates, can lead to difficulties when *CL* is applied to empirical situations. This is evident from looking at *commensurability*. The propositional part of *CL*, classical two-valued propositional logic (*CPC*), presupposes that commensurability holds universally between propositions. Since propositions may also represent and describe measurement results. *CPC* makes the strong assumption that such states of affairs must always be commensurable. We may therefore say that *CPC* to a certain extent "dictates reality" or theoretically tries to do so. And thus an application of *CPC* to such real situations leads to conflict.

That *CPC* presupposes commensurability can be seen as follows. Commensurability between two propositions p and q (symbolized as $p \sim q$) is usually defined in one of the following ways (Mittelstaedt 1978, 30 ff):

$$p \sim q \leftrightarrow_{df} p \rightarrow [(p \wedge q) \vee (p \wedge \neg q)] \qquad (2)$$
$$p \sim q \leftrightarrow_{df} p \rightarrow (q \rightarrow p) \qquad (3)$$
$$p \sim q \leftrightarrow_{df} q \rightarrow (p \rightarrow q) \qquad (4)$$

It is easily seen that in *CPC* all the right parts, that is, the definientia of (2), (3) and (4) are logically true (are theorems). The definiens of (2) is the principle *RBV* discussed in ch.1 above. Therefore, according to *CPC*, commensurability is always satisfied on logical grounds. This is untenable in Quantum Logic, but also in any logic suitable for quantum physics (*QP*).

4.1 PRESUPPOSED ONTOLOGY

The deeper reason for this difficulty in the application of *CL* lies in the fact that *CL* presupposes an ontology which is also assumed by *CP*. One important feature relevant here is this: Any physical object or system S_i possesses elementary properties $P_1 \ldots P_n$ which belong or do not belong to S_i, regardless of the possibility for observation or measurement; i.e. for all such properties it is objectively decidable whether they are there or not (see ch.1. above). This classical assumption has to be weakened in Quantum Physics: Quantum objects possess only a subset of all possible (and classically assumed as available) elementary properties $P_1 \ldots P_n$ either as P (positively obtaining) or as $\overline{P}$ (negatively obtaining). Those of them that can be possessed simultaneously by an object are called *mutually commensurable* [Mittelstaedt 2011, 7 and 51].

4.2. MONOTONICITY

Independently of these ontological presuppositions there is a further hidden assumption in the definentia of the definitions of commensurability (3) and (4): Assuming a proposition p we can get it from any premise q whatsoever; q in turn can be replaced by any other proposition (premise) salva validitate. This is expressed in a more straightforward way by

$(p \rightarrow q) \rightarrow (p \wedge r \rightarrow p)$ and by $(p \Rightarrow q) \rightarrow ((p \wedge r) \Rightarrow q)$

These are valid principles (theorems) of *CL* which represent *monotonicity*, an important feature of *CL*. *Monotonicity* is the property that if a conclusion *C* follows (by *CL*) from premises $P_1 \ldots P_n$ then it always follows from these premises independently of increasing knowledge by adding new premises. This amounts to a kind of *rigid* deduction. The deduction is *stable* or *rigid* in the sense that new premises or new information cannot change the validity of the deduction.

5. THE ASSUMPTION OF UNIVERSAL DISTRIBUTIVITY

The propositional part of *CL*, *CPC*, makes the assumption that for any three propositions p, q, r distributivity *w.r.t.* disjunction and conjunction holds universally in both directions. That means that the following equivalences are theorems of *CPC*:

$$[p \wedge (q \vee r)] \leftrightarrow [(p \wedge q) \vee (p \wedge r)] \qquad (5)$$
$$[(p \vee q) \wedge (p \vee r)] \leftrightarrow [p \vee (q \wedge r)] \qquad (6)$$

That distributivity cannot generally be satisfied was already clearly pointed out by Birkhoff and von Neumann [Birkhoff, v. Neuman 1936, 831; cf. Finkelstein 1979]. It can be shown as follows: Assume a particle in a box. We are concerned with the complementary pair of position and momentum of the particle. For a rough estimate let the position be replaced by the determination of whether the particle is in the left part of the box (L) or in its right part ($R = L'$) and the momentum be replaced by the determination of whether the particle is in an even (symmetric) state (E) or odd state (E'). This is determined by the parity of a certain quantum number n (even or odd) proportional to the momentum which means that the wave function is either invariant or is changed. Then the respective instance of the distributive law is this:

$$E \cap (L \cup R) = [(E \cap L) \cup (E \cap R)] \qquad (7)$$

Since $L \cup R$ means $L \cup L'$ and gets the value 1 in a Boolean algebra, the left side of the equation reduces to E. Translated into classical logic (*CPC*), $L \cup L'$ means that the particle is either on the left, or the particle is not on the left, which is a tautology. As to the right side of the equation, evenness (for momentum) and leftness or rightness (for position) are incommensurable properties such that the subspace of the wave functions of such a particle vanish, that is, they get the value 0. That is $E \cap L$ and also $E \cap R$ get the value 0 and so the whole right side gets the value 0. Translated into classical logic (*CPC*), this means that "a particle s has evenness and particle s has rightness" is contradictory. Thus the left side of the distributive law gets the value E (i.e., a certain positive value) and the right side the value 0, which shows the violation. The above instance of the set theoretical version of the law of distribution corresponds to the following instance in *CPC*:

$$p \wedge (q \vee \neg q) \leftrightarrow [(p \wedge q) \vee (p \wedge \neg q)] \qquad (8)$$

It is derived from the general form of law (5) above.

Now in *CPC*, the right part and the left part of this instance are logically equivalent to p. That is in *CPC* the following are theorems:

$$p \leftrightarrow p \wedge (q \vee \neg q); \quad p \leftrightarrow [(p \wedge q) \vee (p \wedge \neg q)] \qquad (9\text{ a,b})$$

Thus in *CPC* $[p \wedge (q \vee \neg q)] \leftrightarrow [(p \wedge q) \vee (p \wedge \neg q)]$ reduces to $p \leftrightarrow p$. The responsible theorems for this reduction are (9a,b) above. They have features which will be discussed below. It should be observed however that only one direction of the distribution laws (5) and (6) leads to difficulties in application. It is the direction from the left to the right whereas the other direction from the right to the left does not lead to difficulties and is acceptable in QL and in the application to Quantum Physics. That means that the following forms of the distribution laws (5) and (6) are acceptable:

$$[(p \wedge q) \vee (p \wedge r)] \rightarrow [p \wedge (q \vee r)]] \qquad (10)$$
$$[p \vee (q \wedge r)] \rightarrow [(p \vee q) \wedge (p \vee r)] \qquad (11)$$

It is easily seen that the right directions are those leading from disjuncts to conjuncts whereas the false directions are those leading from conjuncts to disjuncts.

The strong assumption of *CL* concerning both directions of the law of distributivity can be formulated by principle *DS*. *DR* restricts the equivalence to the correct implication.

DS All laws of distribution concerning $\wedge$ and $\vee$ are universally valid in both directions. According to *DS* the principles (5) and (6) above hold as logically valid equivalences in *CL* (*CPC*).

DR The laws of distribution concerning $\wedge$ and $\vee$ hold only in one direction. It is the direction which leads from disjuncts to conjuncts. According to *DR* only the principles (10) and (11) hold universally.

6. THE ASSUMPTION OF POSSIBLE REPLACEMENTS

Classical Logic (*CL*) permits special types of replacements salva validitate of its principles. Classical Physics (*CP*) permits special types of replacements salva validitate of its laws.

6.1. REPLACEMENT INVARIANCE IN CLASSICAL LOGIC

RI In *CL* some of its theorems which have the general form $A \rightarrow B$ are replacement invariant. $A \rightarrow B$ is *replacement invariant* iff some propositional variable (some predicate) in B is replaceable on some of its occurrences by any arbitrary propositional variable (any arbitrary predicate) salva validitate of $A \rightarrow B$.

RR $A \rightarrow B$ is *replacement restricted* iff it is not the case that a propositional variable (or predicate) is replaceable in B on some of its occurrences by any other propositional variable (or predicate) salva validitate of $A \rightarrow B$.[8]

Examples:

The traditional principles of *CL*: modus ponens, modus tollens, hypothetical syllogism (transitivity of implication), contraposition, disjunctive syllogism, double negation, simplification, commutation and association are all *replacement restricted.* They usually do not give rise to paradoxes and difficulties, when applied to empirical sciences.

On the other hand, principles which introduce an arbitrary variable as *ex falso quod libet* principles like $\neg p \rightarrow (p \rightarrow q)$ or the principles of addition and explosion $p \rightarrow (p \vee q)$ and $(p \wedge \neg p) \rightarrow q$ satisfy *RI* and are ruled out by *RR*. As it can be seen very easily the variable q in the above principles can be replaced by an arbitrary other variable salva validitate of the principle. The same holds for *RBV* (the relativized bivalence), further for the definentia of commensurability (2), (3) and (4) (ch. 3) and for the left right implications of the principles (9a,b) (ch. 4), which justify both directions of the instance of the distribution law.

A further example, which satisfies replacement invariance, are the different forms of Bell's inequalities. These are usually formulated in set-theoretical form. But it is easy to find out that variable C can be replaced on two of its occurrences salva validitate of the principle:

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[8] The *RR* criterion was originally proposed in [Schurz, Weingartner 1987]. There it was called *relevance criterion* (*RC*) later in [Weingartner 2009] *replacement criterion* (*RC*), *replacement restriction* in [Weingartner 2010].

$(A \cap B) \subseteq [(A \cap C) \cup (B \cap -C)]$ (12)

If we convert this into propositional logic, we have to replace $(A \cap B)$ by $x \in A \wedge x \in B$ and represent it by $p \wedge q$. Then we get the propositional analogue to this form of Bell's inequality as:

$p \wedge q \rightarrow [(p \wedge r) \vee (q \wedge \neg r)]$ (13)

Here the variable r can be replaced on two of its occurences.

Looking at the probabilistic forms of Bell's inequalities, shows that they are also *replacement invariant* (Weingartner 2010, 1584). As a final example we take the classical postulate of *weak objectification*, which is the relativized bivalence (*RBV*) in both directions. The problematic direction is that of *RBV*, the other is harmless:

$B \leftrightarrow (A \wedge B) \vee (\neg A \wedge B)$ (14)

If A and B state that the values a and b respectively belong to the system, then this classically valid equivalence holds. With the help of Kolmogorov's axioms one obtains from (14):

$p(C, B) = (C, A \wedge B) + p(C, \neg A \wedge B)$ (15)

where $p(C, A \wedge B)$ states the probability of obtaining both value a and value b (under condition C). Observe that in both principles (14) and (15) A can be replaced by any other variable on both occurrences salva validitate of (14) and (15).

It should be observed that in all cases of replacement invariance a special type *irrelevance* is involved. Since that part of the conclusion which can be replaced by any part (salva validitate of the inference) cannot be a relevant part of the inference.

Furthermore most principles satisfying *replacement invariance* lead to difficulties when applied to empirical domains.[9]

[9] Such domains are: Theory of confirmation, explanation, law statements, disposition predicates, epistemic logic, deontic logic, physics including quantum physics and quantum logic.

These difficulties seem to show that "nature" (i.e. the application to some empirical domain) does not permit such kinds of arbitrary replacements. This can also be seen from another perspective as the next section will show.

6.2. REPLACEMENT INVARIANCE IN CLASSICAL PHYSICS

In this section, it will be shown that there are several analogies between *replacement invariance* in *CL* and replacement invariance in *CP*. In analogy to definition *RI* (*replacement invariance* in *CL*) we define *replacement invariance* of physical systems (objects).

RIL A physical law is *replacement invariant* iff the values (pairs, triples ... of values) of some properties of the physical system for which the law holds, can be replaced by any other arbitrary value (pair, triple ... of value) of that property salva validitate of the law.

RIS A physical system S is *replacement invariant* iff the values (pairs, triples ... of values) of some properties of S can be replaced by any other arbitrary value (pair, triple ... of value) of that property.

Depending on what the physical system (physical object) is, we receive different types of invariances as instances of *RIL* and *RIS*:

(1) If the physical system S is an object of *CM* (Classical Mechanics), then one instance of *RIL* is *Galilei Invariance*; that is the position x_1 of S can be replaced by an arbitrary position x_2
its angle of orientation w_1 by any other w_2
its velocity v_1 by any arbitrary velocity v_2
the point in time t_1 (for any state of S) can be replaced by any other point in time.

(2) If the physical system S is identifiable by position and momentum, then an instance of *RIL* is *reidentifiable* in space and time. That is the triple $x_1\, p_1\, t_1$ (solely describing S) can be replaced by any other triple $x_2\, p_2\, t_2$ (solely describing S) salva validitate of the law which describes the trajectory.

(3) If the physical system S is a laboratory (an observer), then an instance of *RIL* is *observer invariance*; that is a laboratory (observer) can be replaced by any other laboratory (observer) moving or not moving, distant or not distant relative to the original one.

(4) If the physical system *S* is a measurement rod, then an instance of *RIL* or *RIS* is the *rigid measuring rod* which is freely movable in space; that is the space time coordinates of the measuring rod can be replaced by any other space time coordinates. Or: position and velocity of the measurement rod can be replaced by any different one.

(5) If the physical system *S* is a clock, then an instance of *RIL* or *RIS* is *universal time measurement*; that is the space time coordinates of the clock can be replaced by any other space time coordinates. Or: position and velocity of the clock can be replaced by any different one.

(6) If the physical system *S* is a pair of synchronized clocks, then an instance of *RIL* or *RIS* is *universal simultaneity*; that is the distance and the velocity of each clock can be replaced by any other distance or velocity. Or: the space time coordinates of the clocks can be replaced by arbitrary different ones.

Neither of these types of replacement invariances are universally true; i.e. they do not hold in modern physics. That is they are violated by Quantum Physics or by the Theory of Relativity.

Thus in (1) the replacement by *arbitrary velocity* is too liberal, the respective classical assumption is too strong. The *reidentifiability invariance* of (2) is not generally satisfied in Quantum Physics. The *observer invariance* of (3) is not generally satisfied in the Theory of Special and General Relativity. In (4) "*freely movable*" is too free; measurement rods are physical bodies, which undergo length-contraction according to Special Relativity. The *universal time measurement* in (5) is a very strong assumption of Classical Physics with several classical consequences; it is untenable according to Special and General Relativity. *Universal simultaneity* (6) is a similarly strong assumption; it is not generally satisfied (not satisfied beyond *Einstein—synchronization*) according to Special and General Relativity.

The above remarks clearly show that *replacement invariance* of laws (*RIL*) and of physical systems (*RIS*) is by far too strong an assumption of Classical Physics. In other words: the replacement of values of certain properties of physical systems by arbitrary different values is much too liberal to be permitted by nature.

7. RESTRICTED LOGIC

Different proposals have been made in order to avoid too strong assumptions of *CL* in the first place and to avoid the difficulties coming up when *CL* is applied to empirical sciences and especially to physics in the second. Concerning the second case, further complications arise by the fact that additional principles, which do not belong to *CL*, are also influenced by the underlying logical system. This especially holds for the Theory of Probability, according to which different theorems depend on whether the axioms of Kolmogorov are conjoined with *CL* or with a more restricted logic.

One proposal for a restricted logic which avoids the strong assumptions of *CL* and the difficulties in the applications to modern physics is the Quantum Logic *Lq* by Peter Mittelstaedt, which he elaborated in many of his writings (Mittelstaedt 1978; Mittelstaedt 2004; Mittelstaedt 2011, 64 ff.; Mittelstaedt, Weingartner 2005, ch. 13). This Quantum Logic is based on a winning strategy semantics given by a dialogical logic. Most of the problematic principles of *CL* which make too strong assumptions are not provable in *Lq*; relativized bivalence, *KT*, *BV5*, all principles claiming commensurability on logical grounds, universal distributivity (distributivity is only satisfied if commensurability is), strong and weak objectification and others.

Another proposal is that of Dalla Chiara and Giuntini [Dalla Chiara, Giuntini 2001; Dalla Chiara, Giuntini, Greechie 2004]. It also avoids the strong assumptions of *CL* concerning commensurability and distributivity.

A different proposal is the many-valued system *RMQ* developed by the author (Weingartner 2009). It also avoids the too strong assumptions of *CL* and those which give rise to difficulties in the application to different domains (see 8) – 10) below). It has the following properties:

1) *RMQ* is a 6-valued matrix system (3 values for truth, 3 for falsity) and therefore it contains its own semantics. Every well-formed formula of *RMQ* is unambiguously determined by a particular matrix which contains 6n values for n (n = 1, 2 ...) different propositional variables.

2) *RMQ* is motivated by two criteria called replacement (*RC*) and reduction (*RD*) which avoid difficulties in the application of logic (see below: 8 – 10)

3) *RMQ* is consistent and decidable.

4) *RMQ* has the finite model property.

5) *RMQ* has two concepts of validity: a weaker one (classically valid which is identical with materially valid) and a stronger one (strictly valid). All theorems of two-valued Classical Logic (Classical Propositional Calculus *CPC*) are at least classically valid, that is materially valid, in *RMQ*. Only a restricted class of them is strictly valid in *RMQ*.

6) The validity of a proposition is decided by calculating the highest value (*cv*) in its matrix. If $cv = 3$ the proposition (formula) is classically valid, that is materially valid. If $cv = 2$ the proposition (formula) is strictly valid.

7) *RMQ* is closed under transitivity of implication, under modus ponens, and under equivalence substitution.

8) The strictly valid theorems of *RMQ* almost completely approximate *replacement restriction* (*RR*), *distribution restriction* (*DR*) and *fusion restriction* (*FR*).[10]

9) The strictly valid theorems of *RMQ* avoid a great number of well-known paradoxes in the domain of scientific explanation, law statements, disposition predicates, verisimilitude, theory of human actions, deontic logic … etc.

10) The strictly valid theorems of *RMQ* avoid the well-known difficulties which arise when logic is applied to physics: Like *Lq*, they avoid *relativised bivalence*, all principles claiming commensurability on logical grounds, universal distributivity, strong and weak objectification. While *Lq* and the system of Dalla Chiara-Giuntini do not rule out Bell's inequalities, *RMQ* rules them out (or avoids them) with the help of *replacement restriction*, as all forms of Bell's inequalities, including its probabilistic forms, are replacement invariant. This is a typical feature of *CL*, which is too liberal in the sense of permitting replacement by arbitrary parts in the consequence.[11]

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[10] See the theorems of *RMQ* which are only materially (classically) valid in section 4.2 of [Weingartner 2009] and those which are strictly valid and satisfy replacement restriction *RR*, *FR* and *DR* in section 4.4 of (Weingartner 2009].

[11] See section 4.3 of [Weingartner 2009] and sections 2.1 – 2.4 of [Weingartner 2010].

11) *RMQ* contains a modal system with 14 modalities, which is close to the modal system *T* (of Feys) concerning the theorems with one modal operator (no iteration) applied to well-formed formulas.

12) If the negation of *RMQ* is strengthened while leaving the matrices of all other connectives unchanged, the result is the intuitionistic system *RMQI*, which incorporates many features of intuitionistic logic and where the excluded middle (and bivalence) is invalid [cf. Weingartner 2000]. If the negation of *RMQ* is weakened while leaving the matrices of all other connectives unchanged, the result is the weak paraconsistent system *RMQP*, which satisfies Da Costa's desiderata *DC2* and *DC3* and where the principle of explosion is strictly invalid (cf. Weingartner 2011]. Both systems *RMQI* and *RMQP* avoid most of the difficulties which emerge when *CL* is applied to empirical sciences and especially to Quantum Physics.

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PRESUPOZYCJE KLASYCZNEJ LOGIKI. PRESUPOZYCJE KLASYCZNEJ FIZYKI

Streszczenie

Klasyczna logika toleruje własności relacji dedukcji i prawdziwość implikacji, które prowadzą do trudności w zastosowaniu do nauk empirycznych. Takimi klasycznymi presupozycjami są: rozstrzygnięcie, że wielu inferencjach części zbioru konsekwencji mogą być zastąpione przez umowne części przy salva validate inferencji; przyjęcie arbitralnych połączeń zdań; presupozycje o współmierności podstaw logicznych; presupozycje dotyczące dystrybutywności w obu kierunkach. Wymienione presupozycje są tolerowane także przez klasyczną fizykę. Ponadto fizyka klasyczna przyjmuje dalsze założenia o własnościach obiektów fizycznych czy fizycznych systemów. Jednym z nich jest kantowski warunek określoności wartości. Innym założeniem jest sztywność obiektów spełniających przekształcenia Galileusza. Kolejnymi presupozycjami są te, które dotyczą jedyności, identyfikowalności związanej z upływem czasu i niezależności od obserwatora. Jak pokazano w artykule, wymienione presupozycje powinny zostać osłabione by móc je stosować w naukach empirycznych i we współczesnej fizyce.

Słowa kluczowe: zastosowania logiki, presupozycje, fizyka klasyczna