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Formal assumptions and limitations of circular models in typologizing psychological and educational theories

Założenia formalne oraz ograniczenia modeli kołowych w typologizacji teorii psychologicznych i pedagogicznych

Abstract: The article examines the limitations of circular models used in psychology and pedagogy, focusing on their inherent constraints due to their two-dimensional geometric structure. It highlights that circular models face challenges in maintaining interpretability and accuracy with an increasing number of variables, prompting the exploration of alternatives. Proposed spherical models, by introducing a third dimension, address the issue of "dimension compression," enabling more precise representation of relationships between variables.
A practical application example is the use of Support Vector Machines (SVM) with a Radial Basis Function (RBF) kernel, which allows efficient data analysis in three-dimensional space. The article also discusses potential benefits and limitations of spherical models and outlines directions for future research to support the development of methodologies and analytical tools.

Keywords: spherical models, circular models, model verification, Support Vector Machine (SVM), Radial Basis Function (RBF) kernel.

Introduction

Circular models have been a popular tool in psychology and pedagogy for years. They are used to represent and classify types as well as to examine relationships between them within various theories, such as the theory of upbringing mistakes (Gurycka, 1990), value models (Schwartz, 1992), or the circular model of maternal behavior styles (Schaefer, 1959). In circular models, types are visualized based on variables that operationalize the traits or aspects of the analyzed phenomenon. Relationships between variables are geometrically represented as angles and distances ($r = \cos(\alpha)$), allowing for a precise depiction of interconnections between types. This process facilitates the organization of data and their clear presentation in a geometric space.

Their foundation lies in trigonometry, based on relationships between angles and correlations, which play a key role in describing connections between variables (Fabrigar et al., 1997). This article examines the methodological assumptions and limitations of circular models, analyzing their significance and potential drawbacks in the context of typologization in psychology and pedagogy.

The primary goal of circular models is the visual representation of theoretical types and their interconnections. This enables researchers to better understand complex relationships and draw meaningful conclusions from a geometric perspective (Lingoes, 1977). However, as the number of types increases, the angles between them decrease, making empirical verification of the models more challenging. This limitation becomes particularly evident when analyzing complex theories and multidimensional data.

In classical models representing the typology of theories, it is expected that they explain three main aspects: (a) relationships between types, (b) their affiliation to higher-order structures, and (c) relationships between higher-order structures (Heck & Thomas, 2009). The circle, as a geometric figure, is often used to visualize these aspects. However, applying this model may not be suitable for all theories, leading to limitations in the analysis of trait typologization.

This article analyzes the formal assumptions and limitations of circular models in the context of their application to the typologization of psychological and pedagogical theories. The term "formal assumptions" refers to the mathematical foundations of the models, such as trigonometric rules (e.g., correlations described by angles, $r = \cos(\alpha)$) and geometric principles for arranging dimensions in two-dimensional space.

The article also aims to justify the need to explore alternative solutions that can overcome the limitations of circular models. In this context, three-dimensional models are proposed, which, by adding a third dimension, offer the possibility of more precise representation of complex theoretical structures. This proposal aims not only to improve empirical verification but also to enable more advanced analyses of relationships between types within psychological and pedagogical data.

The spherical models proposed in this article represent an innovative solution, developed by the author as a response to the limitations of circular models.

Methodological and statistical assumptions of circular models used for typologizing theories

Circular models are used in psychology and pedagogy to describe various relationships and structures, often serving as graphical representations of typologies. They allow for a clear depiction of the connections between variables within a geometric space.

Relationships between types. The arrangement of types in circular models depends on the specific theory that the model represents. Typically, the types are organized in a manner that reflects their theoretical connections. In practice, the angle between adjacent types determines the expected correlation between them, and their placement on the circle enables researchers to quickly identify patterns.

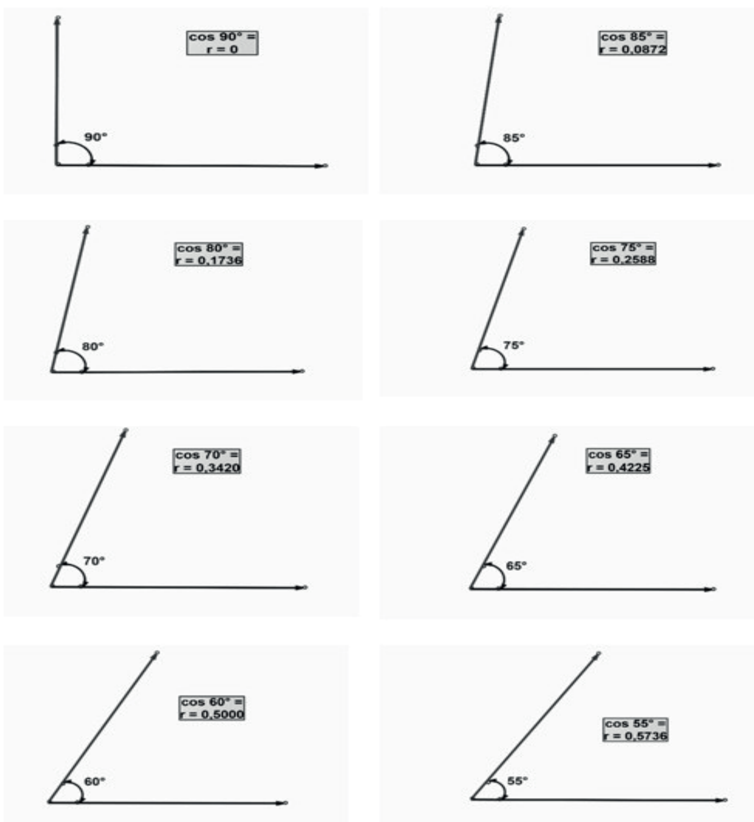
As Lingoes notes: "Since we are talking here about a factor model, it is worth remembering that the points in the factor space represent the ends of vectors originating from the coordinate system's origin, and the solution takes into account both the length of these vectors and the angles they form. In other words, $rij=hi\ hj\ cos\Theta ij$ (the reproduced correlation between variables i and j equals the product of the lengths of the respective vectors and the cosine of the angle between them). Similarly, $aik = hi\ hk\ cos\Theta jk$,, meaning the factor loading of k on variable i equals the product of the vector lengths (where $hk=1,\ k=1,2,\dots,m$ from the model's constraints) and the cosine of the angle between the test and the factor." (Lingoes, 1977, p. 104).

Trigonometric principles play a key role in describing the relationships between types. The angle between types indicates the expected correlation.

Regulating relationships using the cosine of the angle in circular models. The regulation of relationships using the cosine of an angle in circular models involves applying the cosine of angle α to determine the connections between variables representing types. Cosine values range from 1, indicating a strong positive correlation, to -1, signifying a strong negative correlation, with a value of 0 representing no correlation. This mathematical approach allows for the effective verification of model alignment with empirical data. It enables researchers to precisely define the relationships between types, ensuring that the circular model aligns with the empirical findings.

In theoretical analysis, the cosine of angle α is used to determine the strength of the relationship between variables in circular models. Variables that are more strongly related are positioned at smaller acute angles, while those with larger obtuse angles are negatively related. This structure facilitates the accurate representation of both positive and negative relationships between variables and the identification of independent dimensions.

Figure 1 illustrates how the cosine of angle α affects correlations between variables. A high cosine value at 0° indicates maximum positive correlation (r = 1). At 180°, where the cosine value reaches -1, it corresponds to maximum negative correlation. In summary, trigonometric assumptions in circular models form the foundation for their interpretation and analysis, enabling their application in multidimensional psychological and pedagogical research.



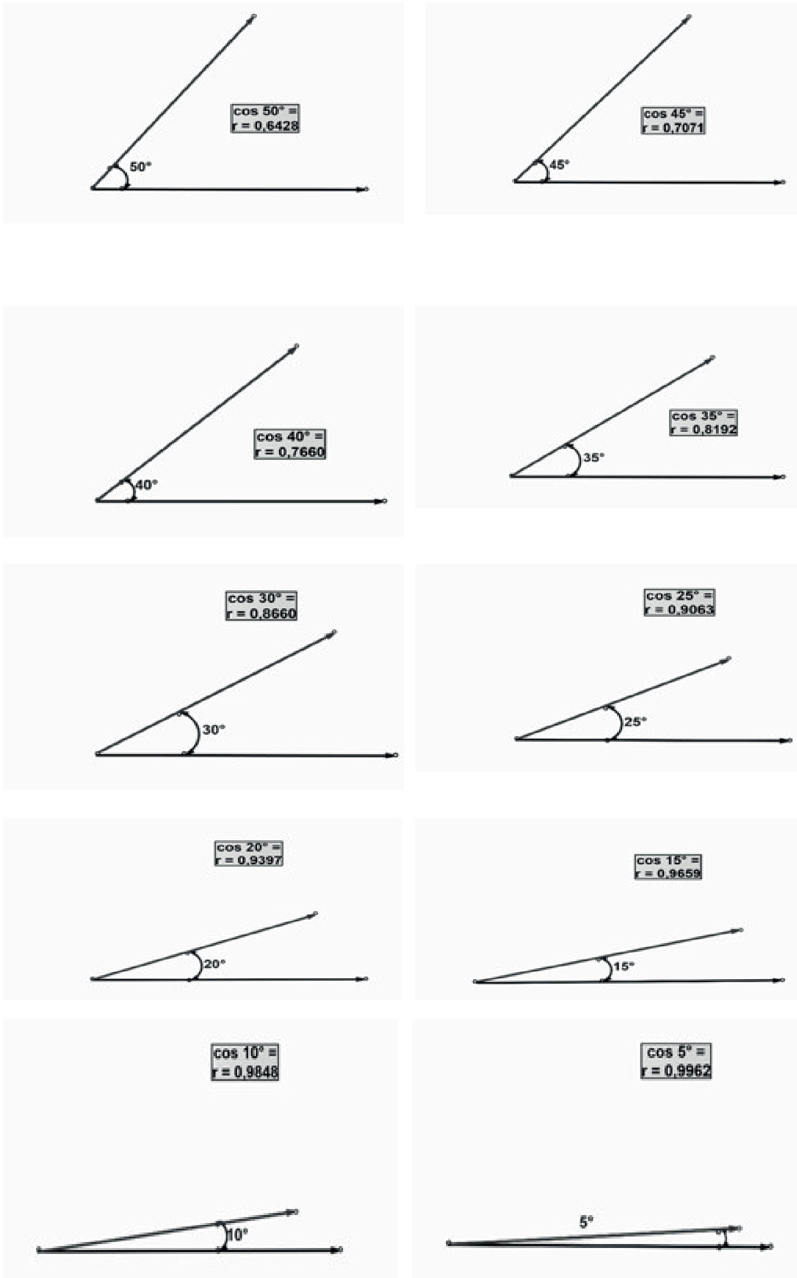


Figure 1. Cosine of angle alpha (α) and its corresponding correlation (r). The sharper the angles, the stronger the positive correlations. When angles are obtuse, the correlations are negative. The maximum positive correlation occurs at an angle of 0º (r = 1), while the maximum negative correlation occurs at an angle of 180º (r = -1). An angle of 90º indicates no correlation.

The belonging of basic variables to higher-order structures

In circular models, the belonging of basic variables to higher-order structures is described using axes that divide the circle into segments. This approach allows researchers to determine which variables belong to the same higher-order structures (Gurycka, 1990). Lower-order structures represent individual variables, such as specific parental mistakes (e.g., indulging, self-accentuation). In contrast, higher-order structures group these variables based on their shared characteristics, forming broader categories, such as warm mistakes and cold mistakes. This method enables researchers to trace how specific variables combine into more complex systems (higher-order structures), facilitating the process of analysis and interpretation.

For example, in the circular model of parental mistakes by Professor Antonina Gurycka, three axes divide the circle, allowing for the identification of parental mistakes and their belonging to different categories. The structure of this model is designed to illustrate how various parental mistakes are interconnected and to which categories they belong. Warm mistakes, as a higher-order structure, include lower-order variables such as indulging, self-accentuation, doing things for the child, and idealization. In contrast, cold mistakes encompass variables such as rigorism, aggression, constraining child's activity, and indifference.

Variables focused on the child and their tasks include doing things for the child, idealization, rigorism, and aggression, while mistakes centered on the parent and their tasks include constraining the child's activity, indifference, self-accentuation by the parent, and indulging.

According to Szymańska and Torebko (2015), this arrangement in the circular model allows for the identification of pairs of mistakes that can be considered twin mistakes, as they belong to the same areas in the geometric space defined by the axes of the circle. For example, rigorism and aggression fall within the space of cold mistakes as well as mistakes focused on the child. Examples of such twin pairs include: rigorism-aggression, constraining the child's activity-indifference, self-accentuation-indulging, and doing things for the child-idealization. These twin mistakes illustrate the relationships within lower-order structures as well as their belonging to higher-order structures, enabling a more comprehensive analysis of their interrelations (Szymańska and Torebko, 2015).

Circular models that use axes to divide the circle into segments provide researchers with a tool for a more detailed analysis of complex higher-order structures. This allows for a precise understanding of the connections and relationships between various variables representing types. Including these structures in the analysis not only facilitates the grouping of variables into categories (higher-order structures) but also enables the examination of the hierarchical dependencies among these categories and the detailed relationships between their components (lower-order structures).

Formal assumptions for verifying circular models

The verification of the accuracy of circular models is based on the assumption that theoretical relationships must be confirmed by empirical data (Wright et al., 2009). This means that the correlations predicted by the model should correspond to empirical reality, similar to methods used in structural equation modeling (Bartholomew et al., 2008; Hair et al., 2006; Heck and Thomas, 2009; Szymańska, 2016). Often, these models are verified using the method of multidimensional scaling (Biela, 1992).

In circular models, trigonometry serves as a key tool for determining relationships between types and their belonging to higher-order structures, using angles to visualize and analyze the level of correlation. A circular model assumes that correlations between types, represented by angles, should align with the cosine values of angle α. For instance, variables separated by a 45° angle should exhibit a positive correlation ($r \approx 0.707$), while variables located at opposite angles (180°) should exhibit a correlation close to -1. If empirical correlations deviate from these values—e.g., features at a 45° angle show a correlation of -0.3 or opposite features at a 180° angle show a correlation of -0.4—this may indicate errors in the model. Such discrepancies can result from improper operationalization of features, measurement errors, or the model's inadequacy in representing reality. For the model to be considered valid, it requires that correlations align with the predicted cosine values of angle α.

In summary, the verification of circular models requires a rigorous approach, including precise analysis of correlations and angles to compare empirical results with theoretical assumptions. Statistical tools, such as factor analysis, can support this evaluation process by comparing correlation matrices to ensure that the theoretical model accurately reflects observed phenomena. Only models that meet these criteria can be considered valid and reliable.

The issue of the number of variables (types) in circular models

Circular models, despite their utility in psychology and pedagogy, face limitations related to the number of variables that can be effectively represented within a circle. As the number of variables increases, the angles between them decrease, potentially causing distortions and challenges in empirical verification. These models, being two-dimensional geometric figures, have limited space for representing types. An excessive number of variables can lead to changes in correlations and hinder accurate analysis.

For instance, if a circular model includes eight variables, the nearest neighbors will be separated by an angle of 45°, corresponding to a correlation of $r = 0.707$. As more variables are added to the circular model, the angles between them become smaller, leading to higher correlations and potential difficulties in interpretation.

Theoretically, one could place as many as 360 variables within a circle, each separated by an angle of 1°, corresponding to a correlation of $r = 0.9994$. However, practical psychometric limitations suggest that at such high correlations, measurement accuracy becomes problematic.

Therefore, it is recommended to limit the number of variables in a circular model to approximately 10–12, which allows for maintaining appropriate geometric relationships and interpretability. Correlation values ranging from $\cos 36° = 0.7986$ to $\cos 30° = 0.8572$ seem to represent the upper boundary of contemporary psychometric capabilities, ensuring analytical coherence and minimizing the risk of interpretive errors.

Spherical models: a future alternative to circular models

In response to the significant limitations of circular models, alternative approaches such as three-dimensional spherical models should be considered. These models introduce an additional spatial dimension, allowing for a more precise representation of relationships between types and eliminating issues arising from the compression of dimensions in traditional two-dimensional models.

Spherical geometry differs from flat geometry in several key aspects. While circular models operate within a constrained two-dimensional plane, spherical models add an extra dimension, providing greater flexibility in the arrangement of variables. This allows data to be distributed in three-dimensional space, enabling a more accurate depiction of relationships among various variables.

The addition of a third dimension in spherical models makes it possible to include a larger number of types while maintaining appropriate distances between them. This approach eliminates the problem of "crowding" variables, a common issue in circular models that often results in a loss of interpretability.

Transitioning from circular to spherical models also opens up new possibilities for empirical verification. Circular models face constraints imposed by geometric relationships in two-dimensional space. As the number of variables increases, their proximity can lead to misinterpretations of correlations. In spherical models, this issue is mitigated, as the third dimension provides additional space for arranging types.

Spherical models may be better suited for representing complex theories, especially when the number of types exceeds the level that can be effectively verified in circular models. By adding a third dimension, researchers can more accurately represent relationships among structures (types) and achieve more consistent empirical results.

Adopting spherical models, however, introduces new challenges. Researchers must understand spherical geometry and how to arrange data within three-dimensional space. This requires modifications to existing analytical methods and the development of new tools tailored to three-dimensional geometry.

Despite these challenges, spherical models offer several new opportunities. The three-dimensional structure enables more faithful representation of relationships among variables, leading to more accurate and reliable interpretations of complex theories.

Researchers can leverage the additional dimension to create models with greater precision and higher reliability, which are difficult to achieve in traditional circular models.

Spherical models represent a forward-thinking alternative to circular models, particularly when the number of types exceeds the capacity of two-dimensional planes. While they demand a new approach to analysis and empirical verification, their application provides the potential for more comprehensive representation of complex theories. However, realizing the full potential |of this approach requires the development of new analytical and visualization tools to accommodate the unique demands of three-dimensional modeling.

Verification of spherical models using SVM and RBF kernel

The verification of three-dimensional spherical models using Support Vector Machines (SVM) with a Radial Basis Function (RBF) kernel enables the introduction of an additional dimension to the analysis, transitioning from circular models to three-dimensional spherical models in psychology and pedagogy. Such a spherical model is an advanced tool for representing and analyzing complex relationships among diverse variables, where features or variables are depicted in a three-dimensional space, forming the shape of a sphere. Their positions relative to one another reflect relationships such as similarity or opposition.

Spherical models are verified using SVM with an RBF kernel, which effectively handles nonlinear and three-dimensional data. The RBF kernel employs a function that maps data from a lower-dimensional space to a higher-dimensional space, facilitating the identification of linear separations. This approach is particularly crucial in cases where relationships among variables are complex (Prasad et al., 2010).

The RBF kernel in the SVM method introduces an additional computational dimension, meaning that for two-dimensional models (e.g., circular models), it can project data into three-dimensional space, and for three-dimensional models (e.g., spherical models), it enables analysis in four-dimensional or higher-dimensional spaces. In this way, the RBF kernel remains a versatile tool for various geometric structures, supporting the analysis of nonlinear relationships regardless of the initial dimensionality of the data. These capabilities open new perspectives for analyzing psychological and pedagogical data, paving the way for more complex and precise models.

When analyzing three-dimensional data, the SVM algorithm seeks to find a hyperplane that optimally separates data into different classes. This transformation into higher-dimensional space allows for the application of the so-called kernel trick, enabling computations regarding similarities in this high-dimensional space without directly modeling the higher dimension. As a result, computational complexity is significantly reduced, enhancing the efficiency of analyses.

The verification process of the model occurs in three stages:

1. **Data transformation**: Data representing various psychological types are transformed using the RBF kernel function, allowing them to be represented in higher dimensions, which facilitates the discovery of more complex relationships.

2. **Hyperplane optimization**: The SVM algorithm aims to identify a hyperplane that separates data classes with the maximum margin, minimizing classification errors.
3. **Classification and verification**: The classification system enables understanding and interpreting relationships between different psychological types in three-dimensional space, offering new perspectives on variable relationships that may have been challenging to detect previously.

The application of SVM with an RBF kernel in three-dimensional models in psychology and pedagogy opens new possibilities for researchers, enabling a deeper understanding of complex interpersonal and psychological relationships. Examples of applications for this model include personality data analysis, where traits such as extraversion, neuroticism, and openness to experience are examined in the context of their influence on interpersonal behaviors. Another example could be studying the relationships between various emotional states and their connection to decision-making abilities.

Through the presented tools and methodologies, spherical models supported by SVM techniques with RBF kernels create opportunities for advancing our understanding of complex psychological and pedagogical dependencies, contributing to a deeper comprehension of human psyche and behavior.

Based on Figure 2, the visualizations illustrating the application of the SVM method with an RBF kernel for data separation can be explained as follows:

1. **Figure 2a**: The first illustration depicts two data classes (green and pink spheres) in a two-dimensional space. The data points from both classes are shown to be very close to each other in some areas, with some elements even "mixing." This arrangement makes it challenging to clearly separate the classes using simple classification methods.
2. **Figure 2b**: The second illustration attempts to separate the two data classes (green and pink spheres) by shifting some of the spheres in the two-dimensional space and introducing a straight line as a boundary. This adjustment allows for better separation of the classes, yet it does not fully resolve the issue of complete separation, highlighting the need for introducing a third dimension to achieve more thorough data separation.

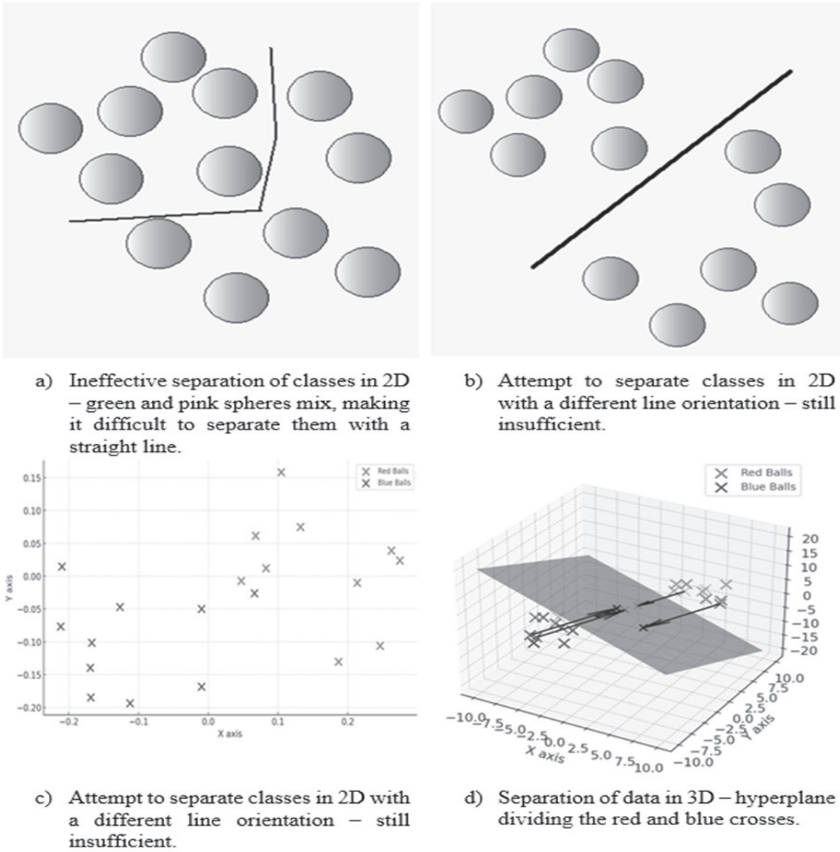


Figure 2. Illustration of the verification of spherical models using SVM with an RBF kernel, based on (Prasad et al., 2010).

3. **Figure 2c**: The third illustration presents a graph with data points represented as crosses in a two-dimensional Cartesian space. Red and blue crosses represent two different data classes. In this scenario, a straight line might suffice for separation; however, the absence of a third dimension limits the potential for a more comprehensive separation.
4. **Figure 2d**: The fourth illustration displays a three-dimensional model where data are represented by crosses positioned in a three-dimensional space. A hyperplane (green) is employed to effectively separate the data into classes (red and blue crosses). The addition of a third dimension enables the creation of a clearer division, which is not achievable in two-dimensional space.

The use of Support Vector Machines (SVM) with an RBF kernel to introduce higher dimensions in data analysis enables more effective separation of data, which may be challenging to separate in lower-dimensional spaces due to geometric constraints. This highlights the importance of applying appropriate mathematical techniques to analyze complex relationships within the data.

Verification of circular models using SVM with RBF kernel: prediction of variables in Gurycka's parental mistakes model

To empirically verify the assumptions of Gurycka's circular model of parental mistakes, a Support Vector Machine (SVM) with a Radial Basis Function (RBF) kernel was applied. The analysis aimed to assess the extent to which variables located in the closest proximity within the circle allow for the prediction of the value of the dependent variable. It should be noted that the introduction of the third dimension through radial functions in the SVM method is computational in nature. The data in the circular model remain two-dimensional in a geometric sense, but the three-dimensional computational space enables a better representation of relationships between variables.

For each variable in the circular model, its results were predicted based on two neighboring variables within the circle's structure.

Table 1 presents the results of the SVM regression analysis with the RBF kernel for each dependent variable in Gurycka's circular model of parental mistakes. The correlation coefficient values between the results predicted by the model and the actual results ranged from r=0.234 to r=0.495, indicating moderate predictive accuracy.

Table 1. Results of the SVM Regression Analysis with the RBF Kernel

Dependent Variable	Predictors	Number of Support Vectors	Mean Squared Error (Test)	Std. Dev. Ratio (Test)	Correlation Coefficient (Test)
Rigorism	Aggression, Idealization	224	13.546	0.914	0.427
Aggression	Rigorism, Constraining Child's Activity	244	14.153	0.934	0.404
Constraining Child's Activity	Aggression, Indifference	212	9.649	0.94	0.371

Indifference	Constraining Child's Activity, Self-Accentuation	246	7.191	0.967	0.257
Self-Accentuation	Indifference, Indulging	239	17.996	0.927	0.379
Indulging	Self-Accentuation, Doing Things for the Child	252	21.601	0.926	0.418
Doing Things for the Child	Indulging, Idealization	206	18.122	0.905	0.486
Idealization	Doing Things for the Child, Rigorism	225	14.944	0.949	0.33

The analysis showed that the number of support vectors in predicting variables within Gurycka's model of parental mistakes was moderate, ranging from 206 to 252, indicating a relatively complex nature of the data. A moderate number of support vectors suggests that the algorithm had to account for a significant number of decision points to accurately separate classes and predict the dependent variable values. These results highlight that the relationships between variables in the model are nonlinear and therefore require more advanced modeling.

Interpretation of SVM model results with RBF kernel in light of circular model assumptions

The circular model assumes that the distance between variables in a geometric structure (expressed as an angle in the circle) corresponds to the strength of their mutual relationships. Theoretically, variables located in close proximity (e.g., at a 45° angle, where cos(45°) = 0.707) could potentially explain up to 50% of the variance in outcomes (based on the classical assumption that a correlation of $r = 0.7$ leads to $R^2 = 0.5$) (Kinnear & Gray, 2009).

In the SVM model, results are evaluated based on the correlation between the values predicted by the model and the actual values. This correlation serves as an indicator of the model's predictive accuracy. For the variables in the parental mistakes model, correlation coefficients ranged from $r = 0.257$ to $r = 0.486$, indicating moderate (and in some cases low) predictive ability of the model to predict variables based on their neighbors in the circular structure (see Table 1). A correlation of approximately $r \approx 0.4$ suggests that the model achieves moderate effectiveness, which, in the context of the circular space, can be interpreted as confirmation that the variables are arranged in a manner consistent with the circular model's assumptions.

Based on the data presented in the table, the predictive accuracy of the SVM model is stable for most variables. Correlation coefficients ranging from r = 0.33 to r = 0.486 indicate that the relationships among variables in the circular space are consistent, aligning with the theoretical distribution of variables in the model. An exception is the variable "Indifference," for which a lower correlation value (r = 0.257) was obtained. This discrepancy may suggest specific properties of this variable or its weaker alignment with the spatial relationships in the circular model.

Nevertheless, it is significant that for most variables, the model achieves similar correlation values at around r = 0.4. This indicates that the variables are distributed in the circular space in a manner close to uniform, supporting the theoretical assumptions of the model. The closer the variables are located in the model, the stronger their predictive relationships appear, as confirmed by the SVM analysis results. The fact that variables such as "Rigorism" (r = 0.427), "Indulging" (r = 0.418), and "Doing Things for the Child" (r = 0.486) achieve similar correlation values demonstrates the consistency of the model's structure with its theoretical circular assumptions.

The lower value for the variable "Indifference" may be interpreted as a local specificity that requires further investigation. However, it does not significantly affect the overall conclusion that the variables in the model are distributed in the circular space in a manner consistent with theoretical assumptions.

From circular to spherical models – interpretation of results and future research directions

The analysis conducted in this article using SVM with a Radial Basis Function (RBF) kernel in Antonina Gurycka's model of parental mistakes introduced an innovative approach to analyzing data from circular models. It is important to note that the application of the RBF kernel in the calculations presented here introduces an additional computational dimension but does not alter the theoretical structure of the circular model, which remains two-dimensional in both geometric and theoretical terms. The radial function enables the modeling of nonlinear relationships between variables in a higher-dimensional computational space, enhancing data modeling and improving predictive accuracy.

The introduction of a third dimension via radial functions allowed for overcoming the geometric limitations of the circular model, particularly issues related to "dimension compression" as the number of variables increases. However, it must be emphasized that the three-dimensional nature of the analysis introduced by RBF pertains only to the computational space and does not imply a fundamental change in the theoretical assumptions of the model. The geometric structure of the data in the circular model remains two-dimensional, consistent with the original theoretical assumptions. Nevertheless, three-dimensional analysis provides a more precise representation of relationships between variables.

Each variable in the analysis was described by coordinates (x, y, z) in a three-dimensional computational space, enabling a more accurate mapping of relationships among variables. The primary goal of this analysis was not to transform the circular model into a spherical model but to extend the predictive capabilities based on data derived from the circular model.

By applying SVM with an RBF kernel, it became possible to capture the mutual relationships among variables in the model more effectively. However, this does not imply that the circular model has been converted into a spherical model. The interpretation of data in the three-dimensional computational space serves merely as a tool for more efficient modeling of nonlinear relationships without altering the original geometric assumptions.

Radial functions (RBF) introduced a third dimension reflecting radial distances between points representing variables. In this approach, variables are no longer confined to a plane in a computational sense but extend from the center of a sphere toward its surface in three-dimensional space. Each dependent variable was predicted based on the two variables located closest to it within the circle's structure.

A distinctive feature of this approach is that the third dimension introduced by RBF does not necessitate changes to the theory or modifications of the data. The analysis in the three-dimensional computational space is an analytical tool that facilitates the representation of geometric relationships among variables while preserving the original assumptions of the circular model.

One variable stood out from this harmony. The variable "Indifference" achieved the lowest correlation value, $r = 0.257$. The low correlation value for "Indifference" may suggest that the relationships in this part of the model require further analysis to understand why neighboring variables contribute less to its prediction.

Earlier studies conducted on a different sample revealed an even more intriguing phenomenon: variables that theoretically should be positively correlated, such as “Indulging” and “Self-Accentuation,” begin to correlate negatively when incorporated into a shared higher-order structure with “Indifference” (Szymańska & Torebko, 2015). This may indicate that the presence of “Indifference” disrupts certain mechanisms related to parent-focused behaviors, leading to internal tension among these variables. Although this finding requires further research, it opens intriguing perspectives for a deeper understanding of the mutual relationships among parental mistakes.

Transforming the circular model into a spherical model: the significance of variable placement in radial space

In a spherical model, variables are arranged within a three-dimensional space, where each variable finds its position along an axis radiating from the center of the sphere toward its surface. A key assumption is that the position of variables in spherical space reflects both their intensity and significance in relation to other variables. It is assumed that variables closer to the sphere’s center represent more integrated and universal traits or attitudes, while variables on the sphere’s surface indicate more extreme, destructive, or specific characteristics.

In practice, such a structure allows for the representation of a continuum ranging from constructive to destructive traits or attitudes. For example, if the model pertains to parental mistakes, “directing the child’s activity” might be located closer to the center of the sphere, symbolizing its more adaptive and balanced nature. Conversely, “constraining the child’s activity” would be closer to the surface, indicating its potentially destructive effects on the child’s development. This arrangement preserves the continuity between positive and negative traits while incorporating their intensity and specificity into the analysis.

The transition from a circular to a spherical model necessitates a reinterpretation of the relationships between variables. In a circular model, variables are arranged in a two-dimensional plane, where the angle between variables reflects the strength of their correlation. In a spherical model, the introduction of a third dimension allows for the additional consideration of variable intensity in a radial manner. In theory, this means that variables equidistant from the center in a circular model may occupy different depths within the sphere depending on their universality or extremity.

For instance, in a model based on parental mistakes, variables representing adaptive attitudes, such as “directing the child’s activity” or “supporting independence,” might be closer to the center of the sphere. Meanwhile, variables like “constraining the child’s activity” or “indifference toward the child" would be located in the outer layers of the sphere. This division accounts not only for the continuum of positive and negative traits but also for their intensity and influence on other variables.

To effectively implement a spherical model, appropriate scales must be developed to accommodate the three-dimensional nature of the space. These scales need to reflect both direction (e.g., positive-negative) and the depth of a variable in radial space. Dichotomous divisions, such as the continuum from constructive to destructive traits, can be helpful but are insufficient on their own. It is crucial to include gradations between variables and their mutual influence.

For example, in a spherical model, variables could be positioned such that both "constraining the child's activity" and "directing the child's activity" are represented at opposite poles of one axis, with their intensity expressed as the distance from the sphere's center. In this framework, more complex or specific variables could be situated between these poles, creating smooth transitions between various attitudes.

Additionally, radial functions can be utilized to calculate distances and relationships between variables in spherical space. This enables precise representation of interactions among variables, facilitating a more detailed analysis of complex relationships.

Example of a spherical model based on the parental mistakes model

In Gurycka's model, transforming the circular model into a spherical one would mean extending the interpretation of the continuum from positive attitudes to parental mistakes by adding a dimension of intensity and universality. In the spherical model, attitudes such as "constraining child's activity" could be placed on the surface of the sphere, while more adaptive attitudes, like "directing the child's activity," would be closer to the center. Meanwhile, intermediate attitudes, such as "excessive control," could be positioned in layers between the center and the surface.

This arrangement would allow for a better representation of not only the strength but also the nature of the relationships between variables. For example, variables like "doing things for the child" and "indulging" could be positioned closer to each other within the same layer of the sphere, reflecting their similarity in terms of their impact on child development, while their differences could be expressed by their distance from the center.

Summary and discussion: transforming circular models into spherical models and future research perspectives

Spherical models offer greater precision and flexibility in analyzing complex relationships between variables, eliminating the constraints of two-dimensional space. The addition of a third dimension allows for the resolution of issues related to the "compression" of variables that arise in circular models. In spherical models, variables are arranged radially, enabling the consideration of both the intensity of influence and the positive or negative relationships between them.

Despite their advantages, spherical models present certain challenges. Introducing a three-dimensional space requires adjustments to existing scales and analytical tools. New analytical tools must account for the complexity of three-dimensional space, which involves additional costs and time. While spherical models provide greater precision, they may be less intuitive for researchers accustomed to two-dimensional models. Developing intuitive visualization tools could facilitate the transition from circular to spherical models.

Although spherical models represent a significant advancement, researchers might also consider spherical surface models, which analyze data on the surface of a sphere. These models could be more suitable for analyzing cyclical or periodic phenomena, better reflecting their nature. The choice between a spherical model and a spherical surface model depends on the characteristics of the data and the research objectives.

Transforming circular models into spherical ones opens new avenues for research, such as developing scales that enable transitions between two-dimensional and three-dimensional analyses and creating intuitive visualization tools. Empirical validation of spherical models in various theoretical contexts, such as parent-child relationship analysis, could also be explored. In such contexts, spherical models might provide new insights that are difficult to capture with circular models.

Spherical models represent a significant step forward in analyzing relationships between variables, offering flexibility, precision, and the ability to account for data complexity. Their development could fundamentally alter researchers' approaches to analyzing complex theoretical structures.

In conclusion, the Radial Basis Function (RBF) kernel in the Support Vector Machine (SVM) method can be successfully used to validate existing circular models by introducing a third computational dimension. This tool allows researchers to more accurately model relationships between variables, overcoming the limitations of two-dimensional space. At the same time, it is worth encouraging the development of new spherical models, which not only better reflect complex theoretical structures but can also be validated using the RBF kernel, enabling analysis in higher-dimensional spaces, such as four or five dimensions.

In these cases, spherical models offer a new perspective, enabling more accurate and precise representation of relationships between variables. However, the construction of such models requires careful development of theoretical foundations, the logic of which has been outlined in this article. This points to a direction for further research and the development of analytical tools that could support researchers in creating and validating more advanced geometric models.

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